Title: Theorem of the heart for weight structures

Abstract:
A weight structure on a triangulated category is an object similar to a t-structure.
Its uses are also similar -- one wants to connect the properties of a category with a weight structure to the properties of the heart of the weight structure.

The category of perfect complexes over a connective E_{∞}-ring spectrum admits a weight structure whose heart is the minimal idempotent-complete and additive subcategory containing the free module of rank 1.
More generally, the category of finite cell modules over any additive ∞-category A admits a weight structure whose heart is equivalent to A up to the idempotent completion.

It turns out that any stable ∞-category C with a bounded weight structure w is of this form. We use it to construct maps $K_i(C) \to K_i(h(Hw))$ and prove that they are isomorphisms for $i\le 0$.

Another example of a weight structure is the Chow weight structure on the category of geometric (effective or not) motives over a field.
With this example in mind we use our result and a recent result of B. Antieau, D. Gepner, and J. Heller to show that non-vanishing negative K-groups of the additive category of Chow motives would yield an obstruction to the existence of the motivic t-structure.