Evaluating meromorphic functions at poles: an ubiquitous problem in mathematics and physics

Both in mathematics and physics, one comes across the problem of evaluating divergent sums and integrals. Divergent sums may arise from "counting" discrete points on a cone, a problem relevant in toric geometry, while divergent integrals can arise from Feynman amplitudes which are building blocks in perturbative quantum field theory. To make sense of such divergent quantities, one can use what physicists call a "minimal subtraction scheme", which consists in "extracting divergences" (minimally) while retaining a (maximal) "finite part". This can be made precise using Laurent expansions, from which we extract the polar part while retaining the holomorphic part, which we then evaluate at the desired point.

I shall explain how by means of a multiparameter regularisation, one can implement such a minimal subtraction procedure for meromomorphic germs in several variables. I shall then present our main result which is the classification of the resulting evaluators thanks to what we call a Galois group.

This is joint work with Li Guo and Bin Zhang.