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Algebraic Geometry I

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Exercise sheet 5

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In all the following let k be an algebraically closed field

Exercise 1. Let $f, g \in k[X,Y]$ have greatest common divisor 1 (in the unique factorization domain $k[X,Y]$). Show:

a) There exists a $d \in k[X]$, $d \neq 0$, so that $d \in < f, g >$.
Hint: Write $k[X,Y]$ as $k[X][Y]$ and use the theorems of Gauß from algebra.

b) $Z(f,g)$ is finite.

(4 points)

Exercise 2. Let Z be an affine variety, whose underlying topological space is finite and irreducible. Show that Z contains only one point.

(4 points)

Exercise 3. Show: The prime ideals $\mathfrak{p}$ in the polynomial ring $k[X,Y]$ are the ideals of one of the following forms:

a) the zero-ideal (0),

b) a principle ideal (f) with an irreducible polynomial $f \in k[X,Y]$,

c) a maximal ideal $(X-a, Y-b)$ with $a, b \in k$.

Hint: The ideal $\mathfrak{p}$ contains an prime element, i.e. an irreducible polynomial. *(4 points)*

Exercise 4. Let X be a topological space. Show the equivalence of the following statements:

a) X is irreducible.

b) Every open non-empty subset $U \subseteq X$ is dense in X.

c) Every open non-empty subset $U \subseteq X$ is irreducible.

d) Every open non-empty subset $U \subseteq X$ is connected.

(4 points)