

The rho-invariant and topological surgery

This is joint work with D. Crowley.

It is the goal of surgery theory to calculate the so-called structure set $S(X)$ of a given manifold X of dimension $n \geq 5$. This is a pointed set that organizes all n -manifolds homotopy equivalent to X up to h -cobordism. For example for $X = S^n$ the set $S^{DIF F}(S^n)$ is the group of homotopy spheres of dimension n , the superscript $DIF F$ indicates that here we are interested in smooth manifolds.

In the talk we will be interested in $S^{TOP}(X)$ for X an odd-dimensional manifold, the superscript TOP indicates that we will work with the topological manifolds. It is important to have calculable invariants of the structure sets, that means maps from $S(X)$ to somewhere. The rho-invariant produces one such map in the odd-dimensional case. It is a feature of the topological category that a-priori just a pointed set $S^{TOP}(X)$ carries the structure of an abelian group which is natural in some sense. The theorem I want to report on says that the rho-invariant map is a homomorphism of abelian groups with respect to this structure.

In the talk I will define the above concepts and I hope to devote some time to discussing the abelian group structure on $S(X)$ which is still mysterious in some sense.