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Pseudo-differential operators and applications

Exercises 8

1. *Examples of wave front sets*

   (a) Determine the wave front set of the delta distribution on $\mathbb{R}^n$.

   (b) Let V be a linear subspace of $\mathbb{R}^n$ with the induced volume element dv. Let δ_V be the distribution defined by $\langle \delta_V, f\rangle = \int_V f\, dv$. Determine the wave front set of δ_V.

   (c) As above, but let V be a submanifold of $\mathbb{R}^n$ or of a manifold with an arbitrary volume form.

2. Verify that the definition of a wave front set on a manifold does not depend on the choice of parametrization, see Corollary 4.15-1)-a).

3. Let $U_1 \subset \mathbb{R}^n$ and $U_2 \subset \mathbb{R}^m$ be open subsets. Show for $u_1 \in \mathcal{D}(U_1)$ and $u_2 \in \mathcal{D}(U_2)$ and some obvious rearrangment in the coordinates we have

$$WF(u_1 \otimes u_2) \subset (WF(u_1) \times WF(u_2) \;\cup\; ((\operatorname{supp} u_1 \times \{0\}) \times WF(u_2))$$
$$\cup\, (WF(u_1) \times (\operatorname{supp} u_2 \times \{0\}))$$

4. Show that $\delta(|x|^2 - 1)$ is well-defined as the pullback of the delta distribution under the map $\mathbb{R}^n \to \mathbb{R}$, $x \mapsto |x|^2 - 1$. Investigate whether this distribution is related to a distribution studied in Exercise 1.