Seminar
Approaches to quantization of space-time

Sommersemester 2012

Prof. Bernd Ammann, Prof. Felix Finster

Time and location

Thursday 10-12, M101

Dates

- Non-commutative geometry: 19.4., 26.4., 3.5., the talk on 10.5.is transfered to 15.5. 10-12 Sitzungszimmer,
- Quantization of fields: 24.5., 31.5., 14.6., 21.6.
- Fermionic projector: 5.7., 12.7., 17.7., 19.7.

1 Non-commutative geometry

The first 3 talks were condensed into two sessions 819.4. and 26.4.) in ordert ot have adequately much time for the longitudinal index theorem whose presentation is extended to two sessions.

Talk 1. *Non-commutative geometry à la Alain Connes.*
Overview. Metric aspects of non-commutative geometry, from riemannian spin manifolds to spectral triples and back, examples of non-commutative spaces (non-commutative tori), recent developments [2],[3],[4].

Talk 2. *The Einstein-Hilbert functional as a spectral action.*
Recall of heat kernel asymptotic for the Dirac operator, Weyl asymptotic of eigenvalues, Dixmier trace, Wodzicki residue as a non-commutative analogue of the Einstein-Hilbert functional [1], [3, 7.3-7.6].

Talk 3. *Examples of non-commutative spaces.*
Various examples, sketch why foliated spaces provide important spectral triples, The Dirac operator of Connes-Moscovici. Discussion of longitudinal, transversal and mixed signature operators.

Talk 4. *Foliated spaces I.*
Foliated spaces provide examples of non-commutative spaces. [2, I.5, and the needed preliminaries from I.4], [5].

Talk 5. *Foliated spaces II.*
Continuation of the previous talk.

References to this part

[1] B. Ammann and C. Bär, *The Einstein-Hilbert action as a spectral action*, Noncommutative geometry and the standard model of elementary particle physics (Hesselberg, 1999), Lecture Notes in Phys., vol. 596, Springer, Berlin, 2002, pp. 75–108.

[2] A. Connes, *Noncommutative geometry*, Academic Press Inc., San Diego, CA, 1994.

[3] J. M. Gracia-Bondía, J. C. Várilly, and Héctor Figueroa, *Elements of noncommutative geometry*, Birkhäuser Advanced Texts: Basler Lehrbücher. [Birkhäuser Advanced Texts: Basel Textbooks], Birkhäuser Boston Inc., Boston, MA, 2001.

[4] F. Lizzi, *Non-commutative geometry, review of recent developments for non specialists*, PDF-Slides from a talk in Vietry, 2009.

[5] C. C. Moore and C. Schochet, *Global analysis on foliated spaces*, Mathematical Sciences Research Institute Publications, vol. 9, Springer-Verlag, New York, 1988, With appendices by S. Hurder, Moore, Schochet and Robert J. Zimmer.

2 Quantization of fields

This part of the seminar studies the local (functorial) approach and the construction of the quantum field, both in the bosonic and fermionic settings.

Talk 6. *Analytical and geometric backgrounds [3, Sec. 4.1-4.2], [1], [2, Sec. 2-3.1].*

First give a very brief review of C*-algebras (see also [1, Sec. 1.1-1.3]), Weyl systems, CCR-representations and recall the existence and uniqueness of CCR-representations (see also [1, Sec. 1.6]). Then survey the geometric and analytical backgrounds [2, Sec. 2] (globally hyperbolic spacetimes, Green-hyperbolic operators).

Talk 7. *Bosonic quantization [2, Sec. 3.1], [1, Sec. 1.4], [2, 4.1-4.2].*

Focus first on the construction of algebras of local observables associated to Green-hyperbolic operators [2, Sec. 3.1]. Then introduce states, GNS-representation, before defining and discussing the bosonic quantum field associated to (strongly) regular states, the n-point functions. Show that the quantum field and the n-point functions satisfy the CCR-relations.

Talk 8. *Fermionic quantization [2, Sec. A.1, 3.2 & 4.3].*

Introduce briefly CAR-representations [2, Sec. A.1], discuss the local approach to quantization for first-order Green-hyperbolic operators of definite type [2, Sec. 3.2] (it is not really necessary to discuss its "real" version) and construct the fermionic quantum fields associated to any state [2, Sec. 4.3]. Emphasize the differences with the bosonic version!

References to this part

[1] C. Bär and C. Becker, *C^*-algebras*, Quantum field theory on curved spacetimes, Lecture Notes in Phys., vol. 786, Springer, Berlin, 2009, pp. 1–37.

[2] C. Bär and N. Ginoux, *Classical and quantum fields on lorentzian manifolds*, Global Differential Geometry (C. Bär et al., ed.), vol. 17, Springer Proceedings in Mathematics, no. 2, 2012, pp. 359–400.

[3] Christian Bär, Nicolas Ginoux, and Frank Pfäffle, *Wave equations on Lorentzian manifolds and quantization*, ESI Lectures in Mathematics and Physics, European Mathematical Society (EMS), Zürich, 2007.

3 The fermionic projector

Literature for the all of the following talks: [1], [2], [3].

Talk 9. *An overview of the fermionic projector approach.*

Talk 10. *Construction of the fermionic projector in a globally hyperbolic spacetime.*

Talk 11. *Causal fermion systems and causal variational principles.*

Talk 12. *A Lorentzian quantum geometry.*

Talk 13. *The continuum limit and renormalization.*

References to this part

[1] F. Finster, *The Principle of the Fermionic Projector*, hep-th/0001048, hep-th/0202059, hep-th/0210121, AMS/IP Studies in Advanced Mathematics, vol. 35, American Mathematical Society, Providence, RI, 2006, ArXiv: `http://arxiv.org/abs/hep-th/0001048`, `http://arxiv.org/abs/hep-th/0202059`, and `http://arxiv.org/abs/hep-th/0210121`,.

[2] ———, *Causal variational principles on measure spaces*, J. Reine Angew. Math. **646** (2010), 141–194, ArXiv: `http://arxiv.org/abs/0811.2666`.

[3] F. Finster, A. Grotz, and D. Schiefeneder, *Causal fermion systems: A quantum space-time emerging from an action principle*, Quantum Field Theory and Gravity (F. Finster, O. Müller, M. Nardmann, J. Tolksdorf, and E. Zeidler, eds.), Birkhäuser Verlag, Basel, to appear 2012, ArXiv: http://arxiv.org/abs/1102.2585.

Web page:

http://www.mathematik.uni-r.de/ammann/quant

More information on the web page of the seminar..