# Übungen zur Indextheorie

Universität Regensburg, Wintersemester 2016/17
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**Exercise Sheet 10, due to 13.1.2016**

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### Exercise 1

From the lecture we know that the Lie algebras $\mathrm{spin}(n)$ and $\mathrm{so}(n)$ are isomorphic. Describe explicitly the isomorphism between $\mathrm{spin}(n)$ as a linear subspace of $\mathrm{Cl}_n^0$ and $\mathrm{so}(n)$ as the subspace of antisymmetric matrices.

### Exercise 2

In order to provide the beginning of the inductive proof of Theorem 14.13, show that there is a $\mathbb{Z}_2$-equivariant diffeomorphism $D : \mathrm{Spin}(3) \to \mathbb{S}^3$.

### Exercise3

Conclude from the local formula (for $\psi = [\epsilon, \phi]$)

$$\nabla_X \psi = \left[\tilde{\epsilon}, \partial_X \phi + \frac{1}{4} \sum_{j,k=1}^{n} \Gamma_{Xj}^{k} E_j^b \cdot E_k^b \cdot \phi\right]$$

the local formula

$$R(X,Y)X\psi = \left[\tilde{\epsilon}, \frac{1}{4} \sum_{j,k=1}^{n} g(R(X,Y)e_j, e_k) E_j^b \cdot E_k^b \cdot \phi\right].$$

### Exercise 4

Conclude from the Exercise Sheet No 9, Exercises 2 and 3, that the spectrum of the Dirac operator is reflection-symmetric around 0 in dimension 1 mod 4.
**Hint:** Construct a complex-antilinear map commuting with $D$.