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Are some Triangles Heavier than Others? Experimental Support for a Gravitational Model of Form Perception[1]

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Summary, Zusammenfassung, Résumé

The analyses of confusion matrices originating from experiments on the identification of forms (ellipses, triangles, unit circles in different r-metrics, and egg-shaped forms) resulted in asymmetric distance matrices even after the removal of the bias vector, which differentiates between prototypical and derived forms. The observed asymmetries fit well into a gravitational interpretation of the effects of "singularity" (Praegnanz), but contadict the predictions derived from the feature-model of similarity.

Sind einige Dreiecke schwerer als andere? Eine experimentelle Bestaetigung für ein Schwerkraftmodell der Formwahrnehmung

Die Analysen von Konfusionsmatrizen aus Formidentifikationsexperimenten (Ellipsen, Dreiecke, Einheitskreise in verschiedenen Minkowski-Metriken und eiförmige Figuren) resultieren nach der Extraktion von Bias-Vektoren in asymmetrischen Distanzmatrizen. Die Interpretation der Asymmetrien in den Distanzen stimmen mit einer gravitationsanalogen Interpretation der Praegnanzwirkungen der prototypischen Formen überein, widersprechen aber dem „feature"-Modell für asymmetrische Ähnlichkeiten.

Est-ce quelques triangles sont plus lourdes que d'autres? Evidence empirique d'une modèle de gravitation pour la perception des formes

Les analyses des matrices de confusion obtenues par des expériances sur l'identification des formes géométriques (des ellipses, des triangles, des cercles normales à r-métriques variées, et des contours oviformes) résultaient en matrices des distances

1 The problem of asymmetric similarity underlying this study was brought to my attention in W. Witte's research seminar in 1968. The then seemingly contrary claims in the assumptions of multidimensional psychophysics and of Gestalt psychology led to the first of the reported experiments. More experiments followed, while I was at the universities of Regensburg and Oldenburg. Without the stimulating experience of spending my sabbatical at the Department of Psychology/Stanford University 1980 the integrating conceptualization would never have been attained. I want to single out Roger Shepard and Jennifer Freyd to express my gratitude for many helpful discussions.

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asymmétriques, même quand les vecteurs des «bias» avaient été écartés. Les asymmétries obtenues correspondent à une modèle des gravitation pour l'interpretation des éffets de singularité (Praegnanz) mais contradisent le modèle presenté par Tversky.

On first glance the proposition "The distance from A to B equals the distance from B to A" seems to be selfevident, and therefore it is not surprising that in traditional geometry it was taken as granted. FRECHET for example took the relation of symmetry between two points for the second axiom in his theory of metric spaces. Everydays experience seems to corrobate this view too: road-distance tables or the fares for airline connections exhibit usually this characteristic of symmetry, but a closer look at many phenomena in our environment shows that violations of symmetry can easily be found too: e. g. migration patterns in the U. S., or the subjective impression of a walk downhill as compared to the climb upwards.

In multidimensional psychophysics of form perception initially the assumption of symmetry of distances was taken for granted (see e. g. ATTNEAVE 1950, TORGERSON, 1965). Especially the development of multidimensional scaling theoretically (SHEPARD 1962; BEALS, KRANTZ, and TVERSKY 1968) as well as computationally (KRUSKAL 1964) seemed to make this assumption necessary for the investigation of complex stimuli, because the symmetry of distances is a necessary condition for dimensional decomposability. Furthermore it was assumed that perceived similarities are monotonically decreasing functions of the distances in the cognitive space and that they are therefore symmetric too.

In Gestalt psychology a radically different position regarding the similarity of forms is held; for instance, METZGER (1968) argues that 'the derived figure is similar to the figure with a salient form (prägnante Form) but not vice versa' (p. translation by the author). This leads to various experimentally testable conditions which all follow KÖHLER's (1920) description: 'Under various circumstances which are all characterized by a certain instability in regard to the constituting factors of stimulation, phenomenal spatial forms tend to change into especially simple and salient ('prägnante') structures and thereby differentiating themselves from irregular forms' (1920, p. 259 translation by the author). GOLDMEIER (1937, p. 8) was the first to propose the metaphor of an electrical or gravitational field for the similarity space in which salient forms lead to asymmetric similarity relations. WERTHEIMER (1923) and RAUSCH (1966) developed the concept of a scale of saliency (Prägnanz) whch mirrors the forces of the salient forms on the adjacent, not quite salient forms; figure 1 (RAUSCH 1966, p. 910) gives saliency as a function of angular disparity.

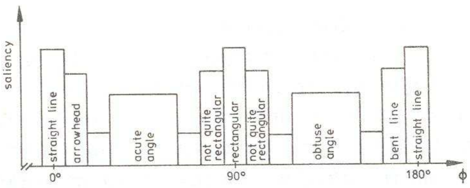

Fig. 1

Similar functions have been suggested for closed forms; for instance for elliptiod forms ($x^2/a^2 + y^2/\beta^2 = r^2$) with the most salient form when α equals β (the circle) and two lesser maxima for the golden selection proportion between axes $\beta : \alpha$ & $\alpha : \beta \approx 1 : 0{,}618\ldots$ (the same ellipse oriented vertically or horizontally[1].

The conditions for this saliency[2] effect have been investigated by various authors e.g., HOCHBERG and McALLISTER (1953), ATTNEAVE (1955), GARNER (1974), and PALMER (1980). By these analyses the vagueness of the original qualitative features of 'Prägnanz' (saliency): regularity, independence, integrity, simplicity, well-structuredness, meaningfulness (RAUSCH 1966; METZGER 1954) has been somewhat reduced (for a critical overview see PALMER 1980).

Gestalt psychologists assumed that these effects were purely perceptual and innate (see KOFFKA: '. . . greater survival value of better organization . . .', 1935, p. 507) and it seems that this assumption is held too – at least implicitly – by ATTNEAVE (1950), GARNER (1974), and PALMER (1980). An alternative hypothesis more in line with PYLYSHYN's (1980) notion of computational perception is that these effects are due to response factors. The assumption of an innate principle of parsimony (POPPER 1934), which governs the processing of perceived information, would allow predictions similar to those of Gestalt psychology: e. g., the minimization of the number of parameters necessary for an unambiguous identification of forms, or the preference for 'natural orientations' ('up and down', 'right and left' in a Cartesian framwork). If the latter hypothesis proves to be right, there is a simple way to reconciliate the two different conceptions of similarity held in multidimensional psychophysics and in Gestalt psychology: in form perception the similarity relation is

3 This proportionality of the 'golden selection' is assumed to render forms harmonic and pleasing since the days of classical Greek art.

4 GOLDMEIER (in press) uses the term singularity as translation for the German term 'Prägnanz'.

symmetric and thereby allows the construction of a dimensional perceptual space, whereas the higher order processing of forms is influenced by computational and decision processes for the effective selection of responses leading to asymmetric similarity relations on the response level.

KÖHLER's (1920) above mentioned assumption about the relation between saliency and the direction of change under instability of perception suggests identification experiments in order to investigate the question whether asymmetries in identification can be traced back to response factors. It is possible to develop various mathematical models of response behavior in such experiments, which allow to compute parameters of similarity and parameters of response bias statistically independent from each other (LUCE 1959; TACK 1963; TOWNSEND 1971). The general hypothesis for these experiments can be stated as follows: If the observed asymmetries in form identification can be reduced to mutually independent bias parameters and symmetric similarity parameters, then the posulated Gestalt effects on similarity are merely due to response factors. In that case the perceptual similarities could be assumed to be symmetric and they could be transformed into distances in a perceptual space. The application of the tests proposed in BEALS, KRANTZ, TVERSKY (1968) would then allow to decide, if this space is dimensionally decomposable.

In the first experiment the stimuli consisted of 19 ellipses of constant area but different proportions of axex, (α; β) ranging from 1 : 10 (stimulus #1) to 10 : 1 (stimulus #ı0) including the circle ($\alpha = \beta$, stimulus #10). All adjacent stimuli were about 2 jnd's apart. In each out of five presentations a series of 9 ellipses was shown for 15 msec. The sqeuence of stimuli was randomized under the constraint that each stimulus recurred five times. The series consisted of stimuli 6 – 14, 3 – 11, 9 – 17, 1 – 9, and 11 – 19. Series #1 included the maximally salient form (circle) plus the two minor salient forms (goldensection proportionality), series #2 and #3 included the circle and on eminor salient form, whereas #4 and #5 included only the minor salient forms. 10 undergraduate students of psychology at the University of Oldenburg/Fed. Rep. of Germany participated in ths experiment; all subjects had normal visual acuity. One main result of this experiment was that only in series #1 to #3 there was found a systematic (and statistically significant) asymmetry in the data, indicating that only the circle exerted an influence of saliency.

The separation of bias and similarity parameters was done by three different models: NAKATANI's (1972) confusion choice model, SHEPARD's (1978) decay model of confusability and a strength model of confusion. The main difference between these models is how the parameters are concatenated: In NAKATANI's model it is multiplicative, in SHEPARD's model it is exponential and in the strength model it is double exponential.

In all the models constraints on the data are made explicit, which allow an a-priori test of the applicability. The results of these tests were equally negative for the applied models. Since all plausible concatenations (see LUCE 1979, for a discussion of different forms of concatenation) of bias and similarity parameters were represented in the models, these results strongly favor the view that the observed asymmetry in identification is not merely due to response factors but also to perceptual regularities as claimed by Gestalt psychologists.

In order to confirm these results and to get a more detailed view of the relation between similarity and saliency a second experiment was run with 19 triangles of equal area and different angles (40° to 140°)[3]. The series were structurally like the ones of the first experiment but a sixth series was added, consisting of each third stimulus plus two more extreme ones with 30° and 150°[4]. In this experiment all confusion matrices were significantly asymmetric, the strongest asymmetry was found in series #6, followed by series #1, #2, and #3; the asymmetry in series #4 and #5 was definitely weaker but sill significant.

The six data matrices were analyzed by the confusion choice model, the decay model, and the strength model under two conditions: In condition A the variability of the bias parameters was not restricted, whereas under condition B all bias parameters were set equal. For the data matrices 1, 2, 3, 6 and for all models the fit between the observed and the computed confusion frequencies was better for condition A than for condition B taking into account the different degrees of freedom (table 1).

Table 1

Goodness-of-fit for the different response models under conditions #1 and #2

	models																	
	confusion choice						exponential decay						strength (double exponential)					
sets:	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
A	90	88	85	81	79	91	89	85	82	78	75	91	91	82	81	70	71	90
B	68	72	71	78	80	65	70	73	70	75	77	65	65	70	72	71	69	63

3 In 1975 the author had done an exploratory study on the identification of triangles with Antje Borchers at the University of Regensburg.

4 25 ss; undergraduates in psychology at the University of Oldenburg.

This table shows that the more asymmetric matrices (#1 and #6 gained most from the introduction of bias parameters, whereas for matrices #4 and #5 this influence seems to be negligable. Between the different models of response behavior there are no drastic differences, which is not very surprising if one looks at the underlying theoretical distributions (BAIRD & NOMA 1978, p. 167).

The magnitude of the bias parameters correlates highly with the saliency-scale predicted from the hypotheses of Gestalt psychology: The most saleitn form (the equilateral triangle, stimulus #10) has in those matrices the highes bias parameters, where it is member of the stimulus set; the second highest bias parameters were found for the rectangular triangles (stimuli #7 and #13). Contrary to the predictions from the saliency scale (RAUSCH 1966, see above) for the 'nearly salient' triangles (#8 and #10) the bias parameters were the smallest (table 2).

Table 2

bias parameters[1] for the stimuli in experiment #2 (triangles)

	models																	
	confusion-choice						exponential decay						strength (double exponential)					
sets:	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
A						09						01						12
1				18		12				06		07				10		15
2				20						09						09		
3		12		14				02		03				05		06		
4		10		19		15		07		07		09		09		09		17
5		15		15				06		06				10		07		
6	18	17		18			10	12		05			13	11		06		
7	20	25		18		25	25	27		04		33	21	25		07		30
8	16	18		18			09	17		07			16	15		09		
9	09	07	09	17			05	06	07	05			09	11	10	10		
10	38	35	36			45	42	38	37			52	39	37	35			48
11	07	10	05		15		06	05	03		05		09	11	09		07	
12	19		21		16		12		12		07		14		15		09	
13	28		29		12	27	29		31		03	35	27		29		06	29
14	20		21		19		15		20		10		19		18		13	
15			17		15				09		07				17		09	
16			15		20	19			08		12	07			13		17	11
17			11		18				03		10				10		15	
18					19						09						13	
19					20	05					13	08					17	12
B						09						02						09

1 all entries have to be divided by 100

The rectangular triangles (stimuli #7 and #13) have increased bias parameters only in those confusion matrices where the equilateral triangle is present; this result indicates that they are salient forms only in contrast to the equilateral triangel; FREYD (1980) coined for similar effects the bidirectionality hypothesis. That is the saliency-effect of the rectangular triangles is due to secondary, context-dependent factors which enhance the prototypicality of the equilateral triangle by giving them the status of typical counterexamples.

The comparison of the different models show that all of them are compatible; the small, but consistent differences between them reveal that SHEPARD's (1958), 1978) exponential decay model is least affected by the noise in the data and reveals the underlying structure most distinctly. Figure 2 shows the interpolated SHEPARD-bias function for stimulus sets #1, #2, and #3.

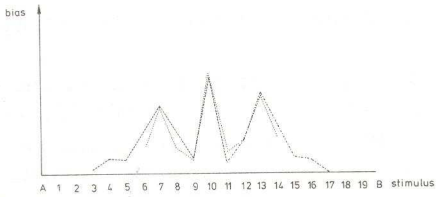

Fig. 2 Empirical bias function for the stimuli of sets #1 (*dotted*), #2 (*dashed*), and #3 (*dotted & dashed*)

The connecting lines in the graph above are somewhat misleading since they give rise to the impression of a gradual drop in saliency between the equilateral triangle and the adjacent stimuli. Until now there are no data, which would allow the computation of biasparameters for triangles wich are barely discriminable from the equilateral one. Under the assumptions of the validity

a) of the exponential decay model
b) of overall symmetry and
c) of an asymptotically unlimited extremum

at the differential limen for the equilateral triangle, the following hypothetical response bias function can be constructed (figure 3).

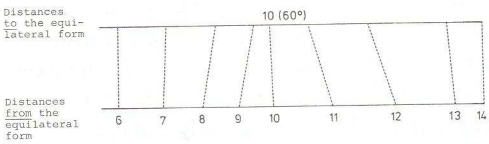

Fig. 3 Theoretical bias function of angular discrepancies from 60°

Even after removing the response bias effect, the matrices for the stimulus sets #1, #2, #3, and #6 remained asymmetric for the rows and columns containing the distances from and to the equilateral triangle. For the adjacent stimuli the distances *from* the quilateral triangle were greater than the distances *to* the equilateral triangle. In figure 4 there are plottet these distances against each other. The asymmetry decreases with the increase of angular difference to the quilateral (60°) triangle and vanishes for triangles with 90° and more.

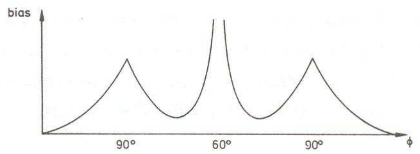

Fig. 4 The comparison of the distances *to* and *from* the most salient form (series #1)

The same result can be observed in series #2 and #3. These results provide some experimental support for GOLDMEIER's (1937) conjecture that salient forms distort the perceived relations among those and other similar forms as if they exert gravitational or electromagnetic forces. The analysis of distances between the non-salient objects compared to their distances to the equilateral triangle reveals another distortion which is in line with the gravitational metaphor (see figure 5).

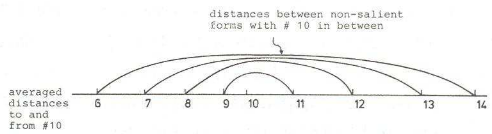

Fig. 5 The interpont distances of non salient forms localized on opposite sides of the equilateral triangle compared to the average distances from and to the equilateral triangle

Only a part of the pairwise distances are shown, but the overall pattern of the other distances is the same. Apparently there is a systematic violation of the triangle inequality. This violation too is in agreement with the gravitational metaphor, in which the shortest distance is not a straight line, but the geodesic following the path where the exerted gravitational force is cancelled out by relative speed. A two-dimensional manifold where these relations hold in a negatively curved surface embedded in a three-dimensional Euclidean space is shown in figure 6.

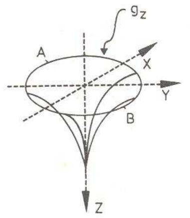

Fig. 6 A two dimensional manifold embedded in a 3-dimensional space, where z is the gravitational force, x and y are the Cartesian coordinates of the plane and g_z is a curve, where a constant speed cancels out z.

In this graph the planar coordinate axes of a two-dimensional Euclidean space are denoted by x and y, the third axis (z) depicts the gravitational force. Only movements on the negatively curved surface (the two-dimensional manifold) are possible, e. g., the shortest pathway from A to B is along the circle g_z[5]. Now it is possible to fit such a negatively curved surface to the obtained distances between triangles; the position of the triangles is on a linear crossection of the manifold, and the distances between them are geodesics on the manifold.

Figure 7 depicts these relations.

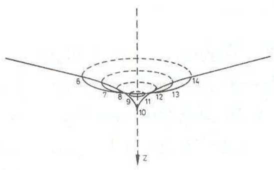

Fig. 7 The position of tht stimuli (set #1) on a negatively curved surface. The z-axis is stretched by a factor of 5.0

What happens to orthogonal coordinates on this surface is depicted in figure 8.

5 For non-technical overviews for the topic of curved spaces see PENROSE (1980) or RUCKER (1977).

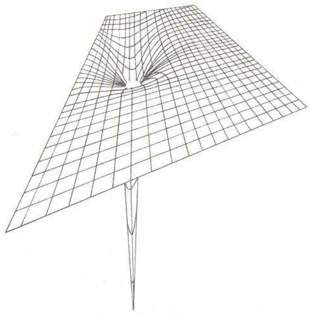

Fig. 8 Orthogonal coordinates projected on a negatively curved surface

One interesting feature of this surface is that, if turned upside-down, it represents exactly the two dimensional function of exponential decay postulated by SHEPARD (1958).

Similar effects like the ones described above have been found in another experiment, where the underlying one-dimensional variation was less obvious. The set of stimuli consisted of unit circles for different MINKOWSKI-metrics, which are defined by the following formula:

$$1 = [|x|^r + |y|^r]^{1/r}$$

9 different r's define the forms of the stimuli. They differed from 1 (a diamond-shaped form) to ∞ (a square) with 2 (a circle) lying perceptually in between. This series was augmented by concave variations of the diamond and of the square. The main difference to the triangle experiments was, that for this series there consist 3 stable salient forms (diamond, circle, square) resulting as well in high bias values as in asymmetric distances, and two context-dependent salient forms with increased bias parameters in between ($r \approx 1.35$ and $r \approx 3.2$).[6] This result makes it plausible that categorical perception is not only influenced by the prototype(s) but also by typical counterexamples. In contrast to the salient forms or prototypes the choice of typical counterexamples seems to be task dependent and therefore influences only the response behavior. A tentative interpretation of this cognitive process could be that it serves to diambiguate communication about forms.

The stimuli of the experiments reported until now were all variations of one parameter, and therefore they all could be represented by a linear

6 The subjects were the same as in the experiments with triangular forms.

cross-section of a negatively curved gravitational surface.[7] In order to check the validity of the curved-surface model a set of egg-shaped stimuli was constructed which vary in two parameters (elongation and direction), the most salient form being the circle. The most extreme of the 17 used stimuli are depicted in figure 9.[8]

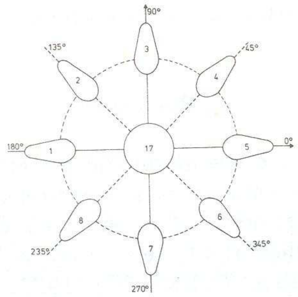

Fig. 9 Stimuli for the experiment of gravitational distortions in a two dimensional space

If for this set of stimuli the most salient form (the circle) exerts the gravitational force, which underlies the coordinate system of figure 8, then it is possible to predict certain distortions of distances between stimuli which do not lie opposite to each other. This feature differentiates this experiment from the former ones, where only distortions for opposite objects could be predicted. The curvilinear coordinate system of figure 10 shows the expected distortions graphically.

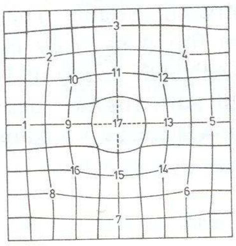

Fig. 10 Curvilinear coordinate system for the prediction of distance distortions around the most salient form

7 The formal structure of the stimulus generation procedures followed algebraic group structures. Only segments of the collections of possible objects were used in the experiments. It is planned to investigate the observed effects more thoroughly in different segments of the cyclical group of triangles. This incompleteness of the experimental design was pointed out by Jennifer FREYD/STANFORD.

8 This experiment was done with 20 students participating in a lab-course in experimental psychology at the University of Oldenburg.

The predicted distortions were only found in the distances between stimuli #9 – #16, where adjacent triples of stimuli formed linear arrays: e. g.:

$$d(11, 13) \approx d(11, 13) + d(12, 13)$$

and in the distances on paths crossing over point #17.

Discussion

In the reported experiments on the perception of geometric forms varying in one or two parameters asymmetric dissimilarities have been found even after the removal of response biases. The application of a gravitational model to these results seems to fit well. The alternative model for asymmetric similarities, TVERSKY's (1977) feature model does not hold for these stimuli, since the number of features remains constant for all stimuli.

The reported results give rise to the conjecture that in the perception of form two mechanisms enhance the prototypicality of regular forms. One mechanism can be identified as a response bias, which might be culturally determined. For instance in the Western culture Western people are used to perspectively distorted forms and are able to identify them as the same, since BRUNELLESCI's system of perspective (late 14th century), and especially since the invention of photography.

The second mechanism seems to be perceptually determined and enables the human perceiver via 'the suvival value of better organization' (KOFFKA 1937) to construct invariants. The gravitational interpretation of this mechanism can be found in many metaphorical expressions (as defined in LAKOFF & JOHNSON 1980) of English or German, e. g., 'it attracts attention', 'es fällt auf'. Insofar these experimental results only clarify common human knowledge, as it is found in everyday's language.

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