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SOME EXPERIMENTS CONCERNING THE FUZZY MEANING OF LOGICAL QUANTIFIERS

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ABSTRACT

Quantifiers are not only an indispensible element of logic, but they play an important role in natural language too. Starting from a comparison of quantifiers in logic and in natural discourse, empirical results led to the interpretation of natural-language quantifiers as values of a fuzzy linguistic variable 'percentage true', which is context dependent. Defining scope-diagrams for 3 contexts, it was possible to show that the membership functions for the context-dependent quantifiers are convolutions of the possibility functions for the context scopes and of a general fuzzy linguistic variable of relative truth.

The application of these results to inference schemata is demonstrated with two different models, and the empirical results support the proposed interpretation of natural language quantifiers.

Keywords: Natural-language quantifiers, fuzzy linguistic variable, context-dependent meaning, inference schemata.

INTRODUCTION

Usually in logically oriented studies of human reasoning two problems are mixed up

- what is the meaning of logical terms (connectives and quantifiers)?

- how does the human brain process inferences based upon propositions consisting of predicates, concepts, and logical terms?

The traditional approach to solve this dual problem by one decisive strike, is to assume that human reasoning is basicially like Aristotelian or propositional logic, only impaired by the vagueness of empirical predicates or concepts and the limitations of human information-processing capacity. From this point of view the meaning of logical terms is crisp and errors in processing are only due to random perturbations or misconceptions.

This traditional way to handle both problems simultaneously dates back to Aristotle and Plato and can still be found nowadays (e.g., in RUSSELL 1940). How influential this mood of argumentation is, can be seen in the works of psychologists like WERTHEIMER (1912, 1925) or HENLE (1962), who juxtaposed 'Natural' vs. 'Pure' logic but basically assumed that human reasoning is logic plus something else.

STRAWSON (1952) pointed out the difference in meaning of logical terms in logic and in ordinary language and GRICE (1967) investigated 'conversational implicatures' as opposed to logical inferences. Influenced by these developments and additionally by the works of PIAGET (1949) and GENTZEN (1935), JOHNSON-LAIRD (1969, 1975) and BRAINE (1978) defined reasoning as processing of knowledge by means of inferential schemata. In these theories too, vagueness is regarded as a source of error but not as a basic feature of human reasoning.

For a different school of thought the very idea of vagueness in natural language and natural reasoning is the central concept: ZADEH (1975) and GOGUEN (1969) developed a system of logic with fuzzy truth-values and compared it with human reasoning. Between these directions of research no fruitful interactions exist except for sometimes heated controversies (see JOHNSON-LAIRD, 1981; SMITH & OSHERSON, 1981), which is somewhat surprising, since both start from a functional analysis of human reasoning instead of the traditional normative approach.

The developments and experiments, which are reported in this paper, are intended to bridge this gap and to improve the communication. The starting point for these studies was the observation that not only in everyday language, but even in scientific communication very often quantifiers are used and apparently understood, when a logical analysis reveals that its usage is not appropriate. According to the conceptions above two different approaches can be followed to solve this problem:

#1 The meaning of logical quantifiers is basically the same in 'pure' logic and in human reasoning. Empirical constraints and random perturbations cause errors in their usage. Since people know about their limitations in processing and their error-proneness, they are able to communicate nevertheless by understanding them with a 'grain of salt' (formally: a crisp meaning mapped into a step function with a superposed error distribution).

#2 The meaning of quantifiers in discourse is determined by constraints in communication (what kind of meaning is optimal for sharing knowledge between people in a more-or-less defined contextual background?) and therefore is basically different from the meaning of quantifiers in logic, which is intended to function in formalized proof schemata.[1]

In the following paragraphs both approaches are explained in more detail and compared.
In order to compare different meanings of the same quantifier the concept of scope diagrams is introduced. A scope diagram is a function in the interval [0,100%] which indicates how many (in %) instances of a quantified proposition are necessarily true, possibly true of necessarily false (impossible).

Figures 1 and 2 depict the meanings of the universal affirmative ('all'), the particular affirmative ('some'), the particular negative ('not all'), and the universal negative ('none') in standard modal logic.

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insert figures 1 and 2 about here
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The quantifiers are characterized as step-functions in the interval [0,100%] with admissible values 'necessary', 'possible', and 'impossible'.[2] The interpretation of the meaning of quantifiers outside of the field of formal logic as loose generalizations of the meaning in logic (crips meaning + random pertubations) leads to scope diagrams like the fuzzy set diagram of 'some' in GOGUEN (1969, p. 370).

Similar results can be found in Adams (1974) for the quantifiers "almost all", where the colloquial quantifier is interpreted as a hedged logical quantifier, or for the fuzzy negation in Swedish and English(Tottie 1977). Figures 3 and 4 depict the characteristic functions for such 'loose' quantifiers for the case of correlated looseness/crispness and scope.

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The meaning of these quantifiers is equally precise, but their ranges of applicability differ markedly as revealed by the scope diagrams; whereas 'all' and 'none' apply only to a very restricted scope of propositions, 'some' and 'not all' apply nearly to the complete scope. From the point of view of pragmatic discourse this situation is clearly suboptimal, which is reflected in ANDERSON'S (1981, p. 318) results regarding the memory for propositions quantified by 'some'. Furthermore as STRAWSON (1952) and BRAINE (1978, p. 3) have pointed out, especially 'some' in colloquial English is ambiguous and its effective meaning tends to differ from its logical meaning.

For communicative purposes the situation as depicted in figures 1 and 2 or 3 and 4 is contrary to the rules of effective discourse as governed by the 'Principle of Cooperation' (GRICE, 1967); it especially violates the maxim of quantity: 'Make your contribution no more and no less informative than is required', since the quantifiers differ in the grade of transmitted information. They are either too precise (the crisp 'all' and 'none') with the consequence of non-applicability to empirical concepts or too wishy-washy as the loose 'some' and 'not all', which are furthermore indistinguishable for the major part of the scope.

Since ordinary discourse does not seem to be impeded too much by these problems, which originated from the formal analysis, it is suggested to start from the careful analysis of normal-language usage of quantified propositions. This analysis aims at the detection of regularities in the discourse with quantified sentences, thus allowing the decision, if there are meanings of quantifiers which obey the 'Principle of Cooperation' as well as the restrictions of human information-processing capacity.

The investigation of linguistic variables (ZADEH, 1975, LAKOFF, 1973; ZIMMER, 1980 a,b) can serve as a model for the treatment of quantifiers, which might be regarded as constituents of a linquistic variable for the possibility of quantified propositions given a certain scope.

The empirical investigation of quantitative judgments in normal language (ZIMMER (1980a) has revealed 3 points:

#1 subjects apply fuzzy linguistic variables to generate such judgments e.g., 'this article is boring', where 'boring' is a value of the fuzzy linguistic variable 'interestingness' in the context 'scientific communications'

#2 if the application of a linguistic variable in a given object context is highly learned, then the membership functions for different judgments are of the same shape and differ only in location

#3 the location of the fuzzy sets is such that it maximizes the transmitted information.

Point #3 needs some more elaboration: if for the given context of objects the possibility for the existence of an object is equal for all points, then the membership-functions are equally dispersed as in figure 5.

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if $R_1(x), R_2(x), \ldots, R_j(x), \ldots, R_n(x)$ represent n levels quantitative judgments about objects $x \varepsilon X$ and

$$\bigvee_{x,x'} \pi(x) = \pi(x')$$, that is, all possibility-functions are equal

then

$\delta(x_{max(j)}, x_{max(j+1)})$ is equal for all j where max(j) is the x with the highest membership-function on judgmental level j.

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[1] Hintikka (1977) has presented game-theoretic semantics of quantifiers in natural language. This approach, which is oriented at Montague grammars, is not presented here for lack of space. The reader is referred to Hintikka (1977).

[2] It should be noted that the particular affirmative has a surplus meaning which is not captured by this figure: it is necessarily true that at least one instance is true.

The assumption of a flat possibility function for the universe of discourse is granted only for special cases (ZIMMER 1980a); usually the possibility function is not flat due to different expectations (see FREKSA, 1981; YAGER, 1980). The suggestion of YAGER (1980) to combine the strength of the received signal R (x) with the possibility of x $\pi(x)$ by $\pi(R) = \sup_{x \in R} [R(x) \wedge \pi(x)]$ can easily be generalized to quantitative judgments. The fuzzy levels of quantitative judgments then become

$R_1(x) \wedge \pi(x), R_2(x) \wedge \pi(x), \ldots R_j(x) \wedge \pi(x), \ldots . R_n(x) \wedge \pi(x)$,

where the exact form of the operator $\wedge$ has to be determined empirically (YAGER, 1979; ZIMMERMAN, 1978; PRADE & DUBOIS, 1980).

If these results are to be applied to logical quantifiers like the ones discussed until now, it is necessary to take care for the ambiguity of 'some' and 'not all' due to the not only different but somehow diverging meanings in colloquial English and in logic. By introducing intermediate quantifiers instead of the existential quantifiers: 'few' for 'intermediately low' and 'many' for 'intermediately high', it is possible to disambiguate better than by applying 'some' and 'not all'. The results from fuzzy categorical judgments applied to the linguistic variable of logical quantification lead to regularly spaced fuzzy categories over the range of the scope.

It is not claimed that the actual form of the membership-functions is exactly like those in Figure 5; the important features of these interpretations of quantifiers are that

#1 the points of maximal membership are about equally spaced and that

#2 the slopes are approximately symmetric in respect to these points (see FREKSA, 1981 for a further discussion).

ZIMMER (1980a, p. 174) has shown that points #1 and #2 presuppose that the a-priori expectations for the occurrance of an instance are the same for every point of the range. That is, the expectation for the occurance of a proposition valid in 20% of the cases is the same as for a proposition valid in 95% of the cases. It appears to be reasonable to doubt the generality of this assumption, e.g., in statistics most of the applied approximations are either valid in nearly all cases or invalid in nearly all cases, and therefore the expectations differ over the scope.

If one regards the expectations for the occurrance as possibilities (ZADEH, 1978), one can combine these context-specific possibility functions with the general fuzzy meanings of quantifiers (see figure 5) in order to arrive at context-specific meanings of quantifiers. In a series of experiments such possibility functions have been determined for instances of ordinary life, for instances from the social sciences, and for instances of the natural sciences. Despite the apparent difficulty of this task the possibility-functions of different subjects were highly similar and revealed that the subjects shared the notion about which instances are possible in which context. The results are shown in figures 6 to 8.

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These possibility functions convoluted with the theoretical membership functions of figure 5 give the context dependent meanings of quantifiers (figures 9-11).

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These results have been taken to predict the behavior of subjects when they were asked to evaluate the meaning of quantified propositions from these contexts.

RESULTS OF EXPERIMENT 1

The sample of quantified statements used in this experiment consisted of sentences from Science, The new Scientist, Psychology Today, and Daedalus for the social-science and natural-science context, whereas the quantified statements about everyday life were drawn from the Stanford Daily, The Chronicle (S. F.), and The Examiner (S. F.).

Subjects either rated the percentage of instances directly or draw characteristic functions in the interval [1, 100]. Both techniques rendered about the same results (correlation of peaks $\tau = .95$, correlation of dispersions $\tau = .86$). These pooled results for the 3 different domains are shown in figures 12 - 14.

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The comparison of the context-dependent interpretations of quantified propositions in figures 12 - 14 with the predictions in figures 9 - 11 supports the theoretical assumptions about the generation of such quantified statements. The main difference between the predicted and observed curves consists in the smoother form of the observed results. This difference might be due to the choice of the applied connective for the predictions or to the averaging of data from different subjects and about different instances.

DISCUSSION

When interpreting these results and when comparing them to the predicted context-specific meanings two points have to be kept in mind:

#1 the precise form of the curves is not determined beyond the location of peaks, the width of the dispersions, and the support and

#2 the selection of instances is somewhat arbitrary despite the attempt to keep them comparable in syntactical constructions and vocabulary.

The introduction of the intermediate quantifiers 'few' and 'many' instead of 'some' and 'not all' was motivated by the (above mentioned) inherent ambiguity of the existential quantifiers, but many inferential schemata in logic as well as in ordinary language make use of them and therefore existentially quantified propositions from the contexts of physical science, social science and ordinary have been investigated in a second study.

The puzzling result was--contrary to GOGUEN's (1969) conjecture about 'some'--that the existential quantifiers are indistinguishable from the intermediate ones: 'some' is used in the same fashion as 'few' and 'not all' can be substituted for 'many'. From a psychological point of view this result indicates that the constraints on communication in order to maximize the amount of t ransmitted information overrule the logical meaning of the existential quantifiers. If one follows RUSSEL's (1940, p. 10) definition of 'proposition' as 'all sentences which have the same meaning as some given sentence', the gained information about fuzzy quantifiers should suffice to lay the foundations for a fuzzy propositional calculus, but stylistic peculiarities of normal language give rise to the conjecture that further modifications might be necessary.

Whereas linguistic hedges have been investigated in the framework of fuzzy set theory by different authors (e.g., ZADEH 1972, LAKOFF 1973), whose results can be applied to quantifiers too (e.g., 'almost all', 'virtually none'), the implicit form of the general assertive quantification (e.g., 'Germans are obedient' instead of 'all Germans are obedient') is a unique stylistic feature of quantified statements which therefore has not been investigated in this framework until now. Whereas in logic implicit and explicit quantification bear the same meaning, the question is, if this is the case in colloquial discourse too. From the point of parsimony in communication it seems plausible to assume that the development of this stylistic technique allows for a distinctive modification in meaning.

For the empirical investigation of implicit quantifiers the same sentences have been applied as for the explicit quantification. Pointwise as well as complete comparison reveal that implicit quantification can be characterized by an increase of dispersion and support, in short by a further fuzzification of the meaning of assertive quantifiers.

RESULTS OF EXPERIMENT 2

The goal of the reported studies was not only to clarify the meaning of quantifiers in colloquial usage but to push forward the formalization of inferential schemata which underlie human reasoning and argumentation. Such a formalization would allow to integrate evaluations by different experts without forcing them to state subjective probabilities or to give numerical confidence ratings.

ZIMMER (1980) proposed the following interpretation of reasoning with fuzzy quantifiers.

If $x \in \tilde{X}$ and $y \in \tilde{Y}$ then quantification with a quantifer $\tilde{Q}$ is a fuzzy mapping $\tilde{X} \underset{\tilde{Q}}{\rightarrow} \tilde{Y}$.

The elements of the mapping matrix are the randomly assigned values on the abscissa for the quantifiers in question, weighted with the respective possibilities. The application of the Max-Min-algorithm then allows the computation of the possibility for the conclusion.

The apparent weakness of this approach is the random assignment of elements in the mapping matrix. Nevertheless the empirical fit of this model was at least as high as for other models of human syllogistic reasoning.

The concept of typicality as developed by ROSCH and co-workers (for an overview see MERVIS & ROSCH, 1981) influenced a different approach to the problem of integrating fuzzy information in an inferential schema. The scope diagrams for context dependent quantifiers can be regarded as double-possibility plots, which can be coupled with the possibility of an instance to be regarded as a typical member of the category in question, e.g., if $x \in \tilde{X}$ and $\mu_{\tilde{X}}(x)$ 0.95 this can be interpreted as x having a high possibility (.95) to be regarded as a typical example for $\tilde{X}$.

Let U be a set, $\tilde{X}$ and $\tilde{Y}$ fuzzy subsets of U, and $\tilde{Q}$ a general fuzzy quantifier, which determines the possibility $\pi(x \in \tilde{Y})$: $\pi(\tilde{Q}\ x : x \in \tilde{Y} = \mu_{\tilde{X}}(x) * \pi(\tilde{Q})$

A suitable family of operatores for * seem to be the following

$$s\left(\mu_X(x),\ \pi(Q)\right) = \frac{\mathrm{MIN}\left(\mu_X(x);\ \pi(Q)\right)}{1 + p\left|\mu_{\tilde{X}}(x) - \pi(Q)\right|}$$

where $-1 \leq p \leq +1$.

P-values of -1, $^1/\mathrm{MAX}\ \ \mu_{\tilde{X}}(x);\ \pi(Q)$, and +1 have been investigated. The best fit was found for p = -1, which is the symmetric sum.

DISCUSSION

Loosely speaking, the restrictions posed upon $\tilde{X}$ by a quantified proposition are obeyed in dependence from the typicality of the members of $\tilde{X}$ in question: the more typical an x is for the category $\tilde{X}$ the more information about the membership of x in $\tilde{Y}$ is transmitted by the quantified proposition. Insofar the traditional syllogistic schemata remain valid for extremely typical members of categories.

CONCLUSION

Summing up the different results in this paper one is tempted to say that the findings tell nothing more than how prejudices about different domains of knowledge influence people's judgments and inferences. In a way that is exactly what motivated this approach: to find out for a restricted area of discourse which rules govern inferential judgments in ordinary language. The knowledge about these rules will allow to evaluate non-numerical expert opinions. Furthermore it allows a formal characterization of frames and/or judgmental schemata by the introduction of possibility functions over the scope of validity in different domains of knowledge.

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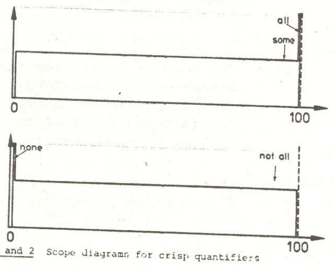
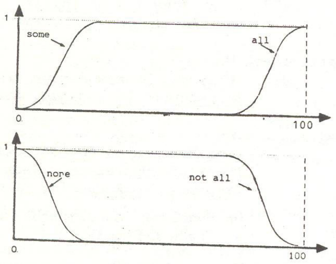


Figures 1 and 2 Scope diagrams for crisp quantifiers



Figures 3 and 4 Scope-diagrams for the fuzzified quantifiers of figures 1 and 2

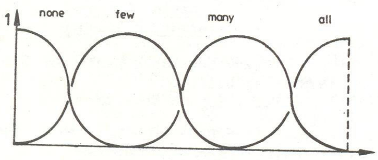


Figure 5 context-free fuzzy meanings of logical quantifiers

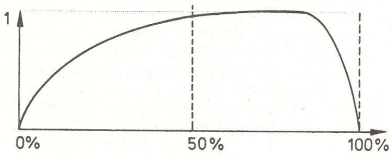


Figure 6 possibility-function for statements about everyday events

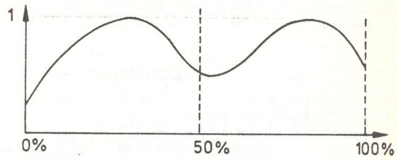


Figure 7 possibility-function for statements from the social sciences

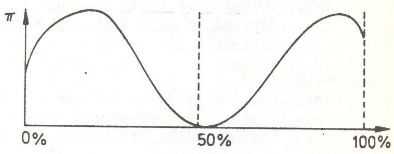
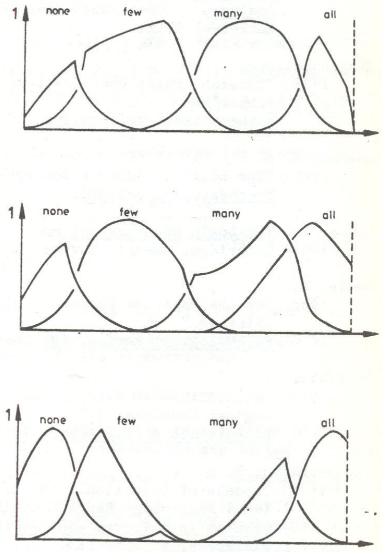
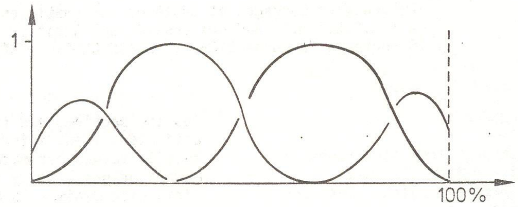
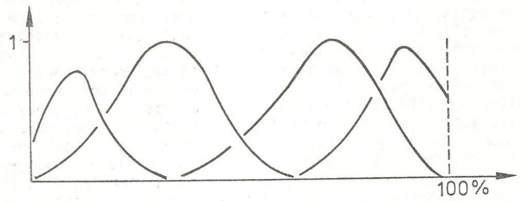


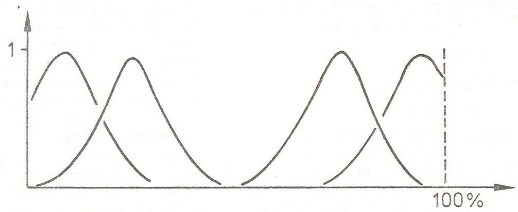
Figure 8 possibility-function for statements from the natural sciences



Figures 9 - 11 context-dependent meanings of quantifiers (everyday events, social sciences, natural sciences)







Figures 12 - 14 empirically determined fuzzy meanings of quantifiers for statements about everyday events, for social-science statements, and for natural-science statements.