A COMMON FRAMEWORK FOR COLLOQUIAL QUANTIFIERS AND PROBABILITY TERMS

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The importance of modal qualifiers for argumentative reasoning is investigated and it is shown that colloquial quantifiers and uncertainty expressions can be interpreted as fuzzy numbers in the interval $[0,1]$. Empirical procedures are suggested for the determination of these fuzzy numbers.

The empirical results reveal that for propositions on a defined level of abstraction colloquial quantifiers and probability terms can not only be expressed as fuzzy numbers but furthermore can be used in according to the rules for fuzzy combination numbers.

For specific areas of content well defined scope functions can be empirically determined. These influence the meaning of colloquial quantifiers systematically, that is, they catch the contextual meaning. A somewhat related effect is observed in probability terms for conditional propositions.

1. INTRODUCTION: Schemes for Reasoning and Argumentation

In the history of Western thought, starting with Aristotles Organon, mechanical procedures for reasoning have been devised serving as normative theories for human reasoning and at the same time as tools for the processing of evidence. The rise of psychological investigations of thought processes has debunked the notion of logic as an - albeit normative - theory of human reasoning. The question, however, if and how formal approaches of reasoning and human reasoning can be brought together, remains open. The approach proposed here is intended to close the gap somewhat by proposing what could be termed an approach to a formal theory of informal reasoning (Zimmer 1984a).

In order to make more specific what such an approach is intended to achieve, it seems appropriate to compare classical formal approaches, that is, predicate calculus and probability theory, with what is known about everyday reasoning. In Table 1 the positions of predicate calculus, probability theory, and everyday reasoning regarding central problems of reasoning are compared.

The inspection of Table 1 highlights the fact that, in general, predicate calculus and probability theory take very similar approaches towards problems and modes of reasoning despite their different structure. Exceptions, however, are the evaluation of partial or circumstantial evidence and in the weighing of evidence by probabilities or by divers colloquial qualifiers; here probability theory and everyday reasoning take similar positions. What distinguishes these points from the rest of Table 1? They apply to situations where only approximate solutions are possible, where the reasoning is invalid or does not lead either to the alternative "true/false", but where only degrees of plausibility or veridicality can be reached.

As Toulmin (1964) observes, standard logic has been developed with an eye on mathematics where such ambiguous situations are to be avoided at nearly any cost (but see, for instance, Kline, 1980). In order to liberate logic from this Procrustean bed, Toulmin suggests the reconstruction of logic

TABLE 1
Modes of reasoning

	Predicate calculus	Probability Theory	Everyday Reasoning
law of the excluded middle	valid without specification	valid in the definition of the event space	usually not valid exept for easily enumerable emsembles
Modes of reasoning: – deduction	valid without exception	valid without exception	valid but the result looses plausibility with the length of the deductive chain
– induction	not valid (the method of proof by complete induction is a deductive method)	not valid in classical approaches v. Mises, Popper) valid with restriction in non-standard approaches	valid if many convincing analogies can be brought forward
evaluation of partial or circumstantial evidence	not possible except for non-standard approaches (non-monotonic reasoning, default reasoning	possible by means of Bayesian or information-theoretic schemes	possible but biased because of heuristic and/or characteristic of memory (primacy and recency
use of qualifiers	only standard quantifier (all, some, not all, none)	not defined except for weighing by probability	applicable without restrictions

according to the model of legal argumentation. He suggests a scheme of syllogistic reasoning with the following components:

(i) *claims* propositions that are supposed to be true or to be at least plausible to a certain degree

(ii) *grounds* or reasons for believing the claims to be valid (the usual form is that of explicitly or implicitly quantified statements)

(iii) *warrants*, statements about the relations between grounds and claims (e.g. causality, necessity, sufficiency, contingency)

(iv) *backing* commonly shared knowledge (Smith, 1982) which provides the rules for a combination of grounds and warrants in order to justify the claims (e.g., rules of syllogistic reasoning or statistical inference)

(v) *modal qualifications* general quantifiers and uncertainty expressions, such as possibly, usually, necessarily. They apply to to the propositions and to the inferential process.

(vi) *rebuttals* alternative claims which can also be inferred from the grounds, warrants, and the backing because of the modal quantification of propositions and inferential rules. Rebuttals can be overcome by either showing that they imply a smaller set of consistent propositions than the claim or by comparing the overall modal qualification of the rebuttals with the evaluation of the claims.

These components are combined as shown in Figure 1 (Toulmin, 1964, p. 104; the figure has been slightly changed in order to avoid inconsistencies).
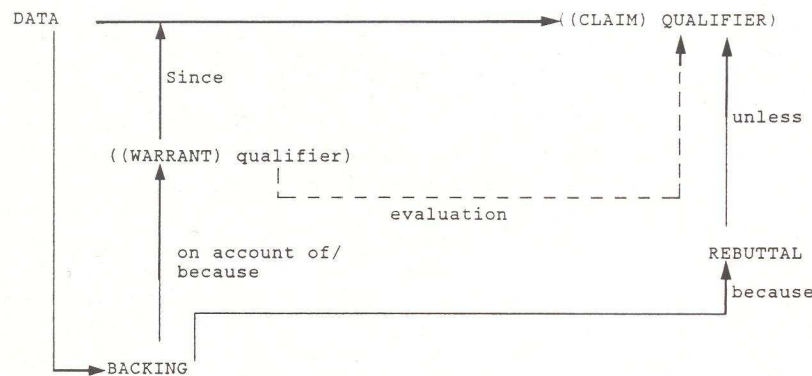

FIGURE 1

A modified version of Toulmin's (1964) model for syllogisms in argumentation. Toulmin's original model is indicated by upper-case letters and bold lines.

An example for this kind of syllogistic reasoning using an analysis of the chains of arguments in the determination of the probable price of a used book (see Figure 2).
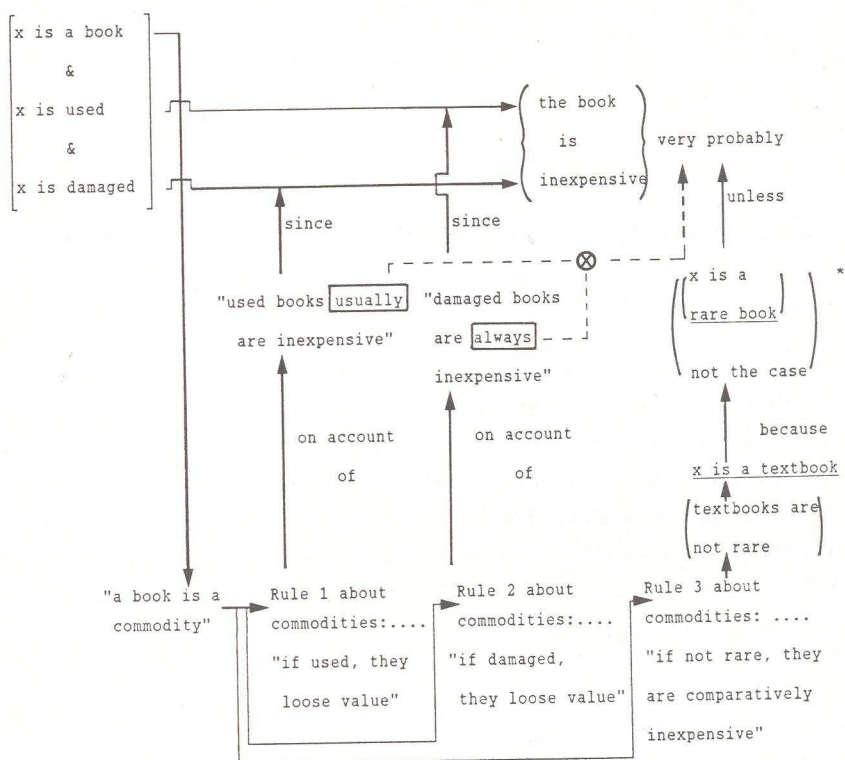

FIGURE 2

The application of the modified Toulmin model.

Figure 2 especially reveals the importance of implicit (...) or explicit (usually, always etc.) quantifiers, for the evaluation (qualification) of the claim. In order to develop a formalized version of Toulmins approach to plausible reasoning, it is necessary to develop a common framework for the interpretation of explicit and implicit quantifiers and furthermore an algorithm for their concatenation. Zadeh (1983) has suggested the interpretation of quantifiers as fuzzy numbers in the [0,1] interval and the use of the operations of fuzzy numbers (Dubois & Prade, 1980) as the algorithm for their concatenation.

2. FUZZY NUMBERS AND QUALIFIERS

The meaning of quantities like "about 50 %" or "slightly below 0.3" (Smithson, 1987) and adding "about 50 %" and "a bit more than 10 %" with the result "probably somewhat more than 60 %" seem to make sense immediately. However, it is not clear how this intuitive meaning is reflected in the formal definitions of fuzzy numbers and their rules of concatenation (Dubois & Prade, 1980). The formal definitions allow for the proving of abstract theorems in fuzzy number theory and for checks of consistency but these formal definitions do not provide any guidelines for the mapping of imprecise observable quantities into the different types of fuzzy numbers (Π, s, z, s/z, or z\s numbers) and for the setting of parameters. On the other hand, Smithsons (1987) and others purely empirical approach characterizing a fuzzy number by a listing of relative frequencies is not sufficient either, because he does not propose empirically testable rules for the concatenation of these numbers. Such rules however can be derived from results on approximate calculation in the areas of foreign exchange (Zimmer, 1984b) and of the stock market (Zimmer, in preparation). For merely illustrative purposes, let us start with fuzzy numbers of the form "standard number + qualification" (e.g. "approximately 0.7").

Figure 3(a and b) represents this fuzzy number where the core (0.7), and the fuzzy upper and lower boundaries, (the fuzziness due to the qualification) can be discriminated. The fuzzy number can now be represented by the following triple: lower boundary relative to the core, core, upper boundary relative to the core (0.1/0.7, 0.7, 0.15/0.7).
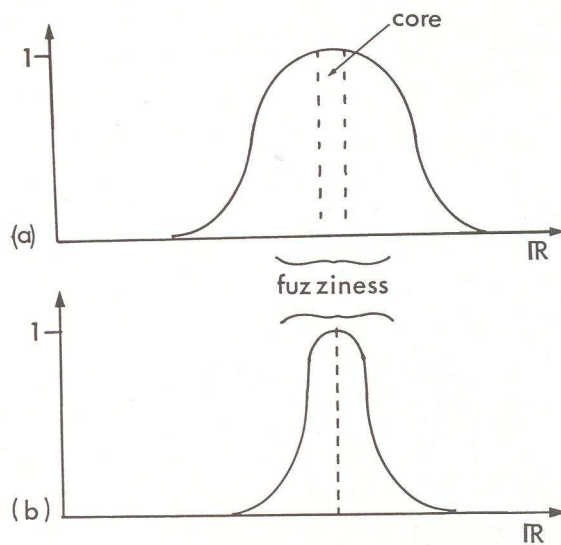

FIGURE 3

Fuzzy numbers consisting of a core (or prototypical) meaning of 0.7 and fuzzy upper and lower boundaries. (a) is a fuzzy number with core interval, (b) is a fuzzy number with a point-wise core.

Any two fuzzy numbers can be concatenated following these steps: (i) calculating the resulting core by means of standard arithmetics, by (ii) averaging the respective upper and lower boundaries, and by (iii) determining the resulting boundaries from the a veraged boundaries in relation to the resulting core. The operations with fuzzy numbers corresponding to the standard operations in arithmetics are illustrated in Figure 4(a-d).
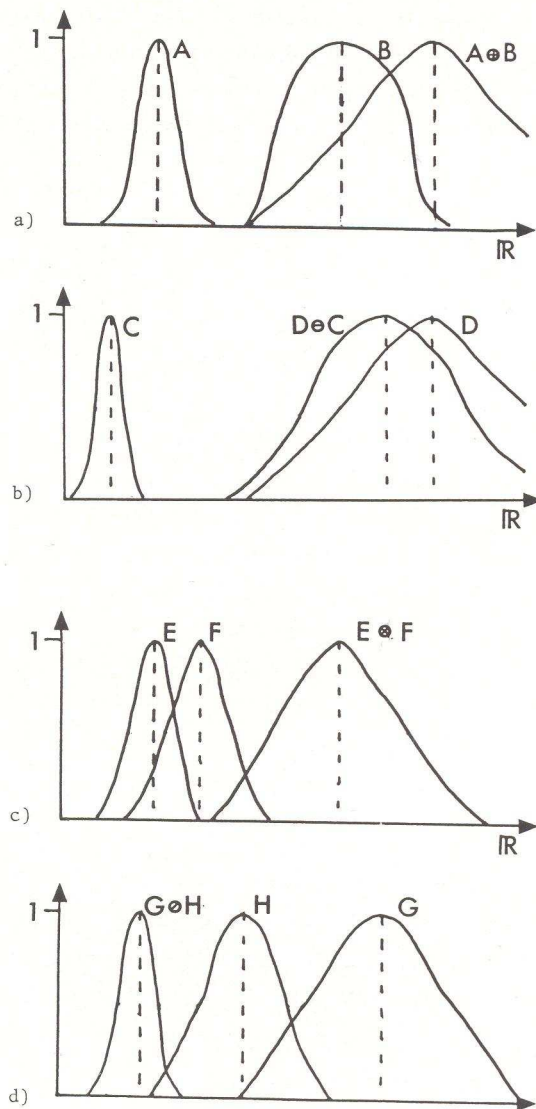
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FIGURE 4
Operations with fuzzy numbers
a/ fuzzy addition ⊕
b/ fuzzy multiplication ⊗
c/ fuzzy substraction ⊖
d/ fuzzy division ∅

The described approach of handling the core and the fuzziness seperately has been established empirically. Specifically, the think-aloud protocols of the subjects reflect this procedure. For fuzzy numbers like several , most or likely the same procedure can be applied provided the core has been determined empirically.

3. FUZZY ARITHMETIC AS A MODEL FOR REASONING

Starting with the experimental studies by Zimmer (1982, 1984a, 1984b) empirical evidence has been amassed for Yagers (1980) and Zadehs' (1983ab,(1984) claim that fuzzy numbers can be used for the representation of generalized quantifiers (Barwise & Cooper, 1981; Peterson, 1979) and furthermore that human reasoning with these quantifiers can be modelled according to the operations with fuzzy numbers. As noted above, further experimental studies have led to modifications in the definition of fuzzy num bers as well as of operations with them. These modifications, however, are not crucial for the general claim.

From a formal point of view, quantifiers expressed as fuzzy numbers in the interval [0,1] and uncertainty expressions represented as fuzzy probabilities are comperable. Furthermore, in chains of argumentation (see Figure 2) both kinds of qualification can be found and should therefore be represented in a common framework for the use in intelligent or expert systems (Zadeh, 1983c). Empirical analyses of uncertainty expressions (Zimmer, 1983, 1986a; Wallsten, Budescu, Rapoport, Zwick, & Forsyth 1986; Zwick & Wallsten, 1987) have consistently shown that verbal expressions like "probable", "likely", or "toss-up" can be expressed as fuzzy numbers. Different experimental techniques (e.g. pair comparison vs. staircase estimation), different forms of display s (e.g. circle segments vs. random dots), and different samples of uncertainty expressions (all expressions of a language community vs. only those expressions that a subject has in his/her personal active vocabulary) have led to seemingly conflicting results about the consistency of estimates

and therefore the applicability of verbal uncertainty expressions in decision support or expert systems. There are two solutions for this problem: One consists in the Wallsten et al., (1986) approach of determining the fuzzy numbers for a complete lexicon of uncertainty expressions. This leads to *averaged meanings* that can be assumed to be valid for an entire language community. By means of iterative methods (Zimmer, 1986b), ambiguous meanings (fuzzy numbers with more than one peak) can be resolved. The problem with this approach is that the individuals lexicon of uncertainty expressions might differ from that of the language community. The important advantage of this approach, however, is its generality. The other solution for the problem consists in concentrating on the *individual's* lexicon of uncertainty expressions.

Calibrating individual vocabularies of uncertainty expressions by means of staircase methods with random-dot displays Zimmer and Körndle (1987) has resulted in fuzzy numbers that can be represented by (i) single-peaked membership functions, (ii) of compar able shape and (iii) with the tendency towards a proportional relation between the value of the core and the fuzziness. To be more precise: in contrast to fuzzy numbers without interval bounds, the fuzziness in the closed interval [0,1] is relative to the smaller distance of each core from the upper or lower limit. These qualitative aspects of the fuzzy numbers are consistent with the model described in Part 2 above. It should be kept in mind that (iii) contradicts one of the theoretical assumptions of Zimmer (1982, 1983), namely the assumption of equal informativeness on the entire scale of judgment, and therefore equal fuzziness for all uncertainty expressions in an individual active vocabulary. However, the consequence of the unequal informativeness (low in the central part and high in the extremes) is in accordance to the results reported by Wallsten and his group (Wallsten et al., 1986).

The major disadvantage of this individualistic approach, specifically its lack of generalizability, can be overcome by the procedure described in Zimmer (1986b). Starting with the individuals expressions but then mapping them into the general lexicon. If a mapping does not result in a single-peaked fuzzy number or if the fuzziness is excessive, it is iteratively searched for the non-degenerate expression which captures best the initially intended meaning.
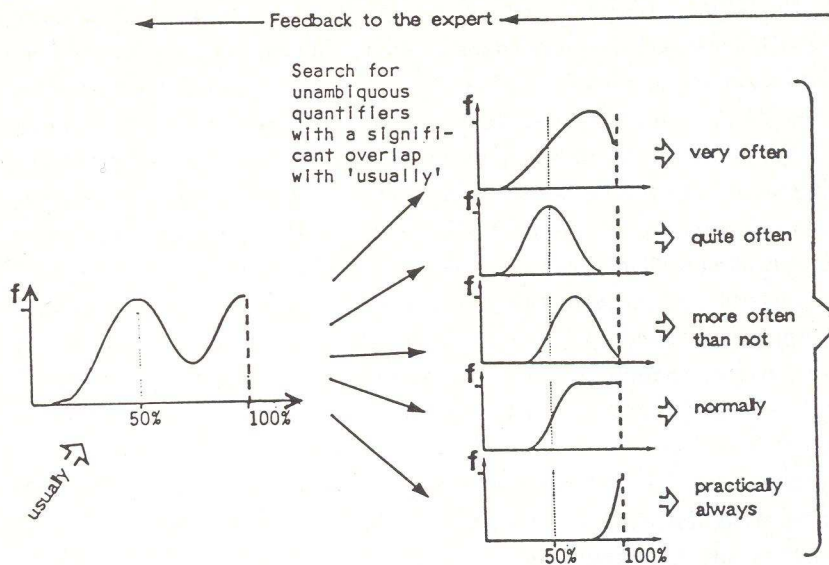
FIGURE 5

Interactive detemnination of unambiguous uncertainty expressions
(Zimmer 1986b).

During and by the interaction with this computer-controlled procedure, subjects learn to use only those expressions which have a meaning meaning in accordance with that of the language community. However, this method restricts the expressive power of the indivial lexicon.

The common framework for quantifiers and uncertainty expressions as established by the assignation of fuzzy numbers in [0,1] has to be complemented by a comparison of the algorithms of inference. The standard algorithms, that is, syllogistic resolution and Bayesian weighing of evidence are seemingly incomparable. However, since Zadeh (1983a) has shown that any form of syllogistic resolution can be modelled by fuzzy quantifiers and fuzzy operators (addition, multiplication, and conjunction), it is possible to use the same operators for Bayesian inference. There is only one additional

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operator necessary, division, needed for working with conditional probabilities. On the first glance this operator does not fit into the reasoning with quantifiers. However, as Hörmann (1983) and Zimmer (1986a) have shown, in the colloquial usage of quantifiers these are quite often conditioned on the background knowledge about the situation. For instance, the utterance "many of the convertibles" can only be properly modelled if the general meaning of many is taken into account as well as the fact that convertibles form a very small subset of all cars. The implicit reasoning runs as follows: if the cars in question are convertibles, then even a small proportion of all cars fulfills the condition of applying "many". This construction of a conditional quantifier is completely compatible with the notion of conditional probabilities. Using this result, it is now possible not only to assign fuzzy numbers to the qualifiers in Toulmin's model (Figure 1 and 2) but also to interp and their combination as fuzzy evaluations of operators (e.g. $\otimes$ means fuzzy multiplication). Furthermore, the relations between the backing and the warrants becomes straightforward fuzzy arithmetic.

4. CONTEXT SPECIFICITY OF QUALIFIERS

Yagers (1980) as well as Zadehs (1983a) models for the interpretation of quantifiers in natural language as fuzzy numbers assume implicitly that there is a one-to-one relation between quantifiers and fuzzy numbers (for redundant sets of quantifiers the relation might be many to one). One major experimental result of Zimmer (1982, 1984b) was that for sufficiently rich contexts (natural sciences vs. social sciences and everyday events) this simplifying assumption does not hold. In these contexts the standard meaning of quantifiers (see Figure 6) is modified by the subjects knowledge about the normal scope of discourse in these contexts, specifically, how often events are mentioned with an occurrence rate of x %. It has been shown that the scope functions for contexts can be determined independently from the quantifiers (Figure 7a) and that the context-denedant quantifiers (Figure 7b)

result from the fuzzy conjunction of the standard quantifiers and of the respective scope functions (the MIN-operator).
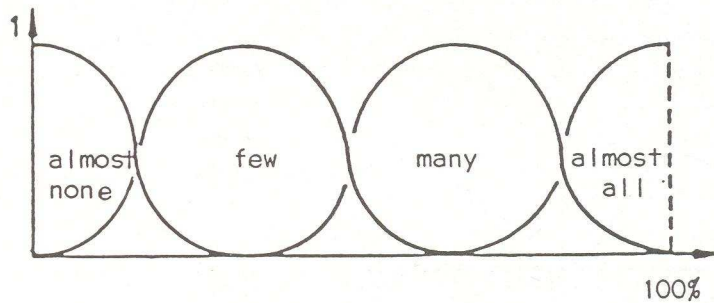

FIGURE 6
Standard meaning of colloquial quantifiers (Zimmer, 1984b).

Slightly different is the situation for conditional quantifiers (see above) where the background knowledge is taken care of by dividing the fuzzy number for the standard meaning of the quantifier question by the fuzzy munber for occurence rate for the eve nt in question. As mentioned above, the notion of conditional quantifiers bridges the gap between the apparently disjoint types of modal qualifiers, namely, quantifiers and uncertainty expressions or qualitative fuzzy probabilities. The context dependability of explicit or implicit conditioning. The procedures of Wallsten et al., (1986) as well as of Zimmer (1983) and Zimmer and Körndle (1987) assume implicitly that uncertainty and frequency expressions can be estimated independently from contextual inf luences. This is apparently true for impoed contexts like circle segments and random dots but remains - at least - questionable for more realistic situations.
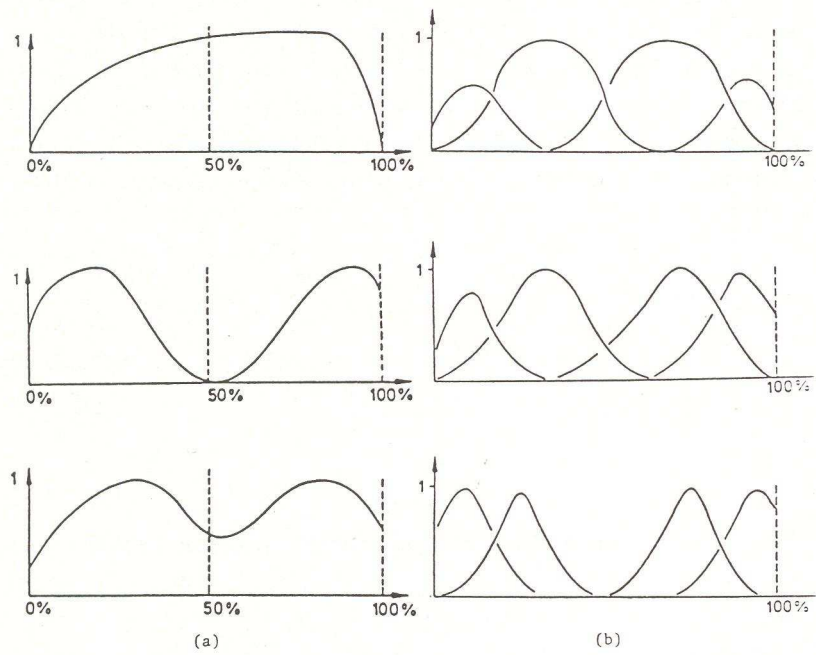
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FIGURE 7

Context dependability of quantifiers: (a) scope functions for contexts /from above: everyday events, natural sciences, social sciences/ (b) resulting context-dependent quantifiers /contexts as above/

From Hörmann s (1983) results one might conjecture that in situations where the individuals have background knowledge, the meaning of their observable lexicon is the result of conditioning the observable frequency on the possible frequency. Despite the fact that the operators are different (fuzzy conjunction vs. fuzzy division) the result for quantifiers and uncertainty expressions is comparable: The observable usage of modal qualifiers can be deduced from a fuzzy operation on the standard meaning and the occurrance rate of the background knowledge. In Figure 1 this context dependability is indicated by the arrow from DATA to BACKING which results in data-dependent constraints on the background knowledge (e.g. the rules for commodities).

5. CONCLUSIONS

The common framework for the two major kinds of modal qualification allows for a modelling of intricate nets of arguments (see Figure 2) by means of fuzzy arithmetics. Furthermore, the variability of meaning caused by individual differences and different context can be taken care of by the generalization of this framework. This consists in a decomposition of the observable meaning into the standard meaning and into the contextual or individual constraints.

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