ALF ZIMMER

WHAT IS FORM?

The contributions of psychology to an old epistemological problem

Pertinax

Let chaos storm!
Let cloud shapes swarm!
I wait for form.

~~Alexander Pope~~
Robert Frost (1936)

1. INTRODUCTION: THE INTRICACIES OF THE FORM VERSUS MATTER DEBATE

One of the central topics of discussion in Western philosophy has been the precedence of form over matter, or vice versa. For this reason the question 'What is form?' has a strong traditional flavour and I have to admit that also my tentative answer from the vantage point of psychology, or to be more precise: from that of perceptual processes, is deeply rooted in philosophy. Democritus (fragment 13) distinguishes between genuine qualities in nature, like weight and size, and those which only emerge if nature is perceived, like colour and taste. In modern parlance,[1] this is the distinction between objects as they *are* and objects as they are *seen* from a specific point of view. In my opinion, the discussion about the primacy of either form or matter, especially in the Aristotelian tradition of thought, misses one important point: in perception neither objects are seen as isolated in space nor as spatial arrangements or geometrical projections unrelated to the projected objects. What is perceived are objects in situations or events which, except for very specific situations, are phenomenologically unique. Two examples can serve to illustrate this point. In Figure 1 an erratic line (A) is shown in two different contexts: while in (B) the oscillations are accidental and what is seen is a free-hand drawing of a brick, in (C) these same oscillations determine the meaning, namely, an undulating plane with one vanishing point – in a way, what is accidental in (B) is substantial in (C), but in both cases the percept is unique.

The degree to which a frame of reference determines the perception of an object is shown in Figure 2: the 'true' motion of the two points is given by two perpendicular vectors meeting in one point; however, what is perceived are two points colliding head-on while the scene is shifted orthogonally to the collision course. Actually, from the viewpoint of vector algebra (and from that of classical mechanics) both descriptions are equivalent but the perceived event corresponds only to the second – and that is phenomenologically unique.

L. Albertazzi (ed.), Shapes of Forms, 51–88.

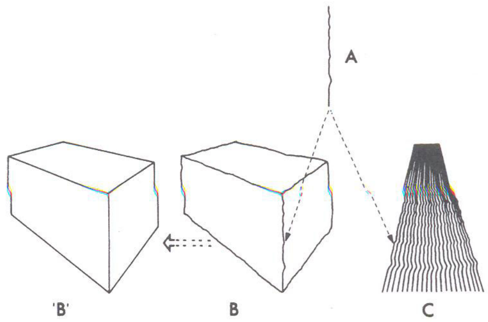


Figure 1. Context dependency of what serves as information and what as noise

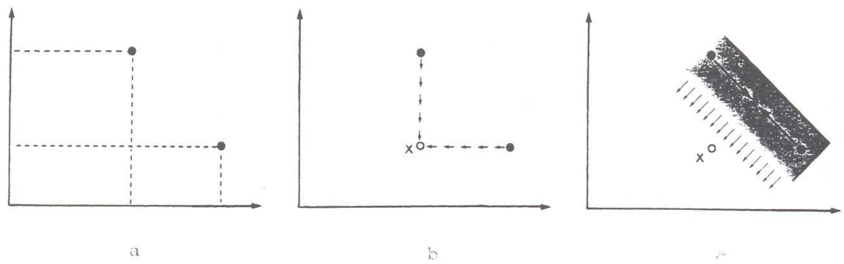


Figure 2. The emergence of a frame of reference for moving points

The question 'What is form?' and the concomitant matter-versus-form debate can also be regarded as a classic case of preventing insight by dissolving a complementarity into two mutually exclusive alternatives. In what follows, perceptual processes will be presented which show the intricate interaction and interdependence of objects 'as they are', corresponding to the Aristotelian 'substance', and 'as they are seen', that is, 'accidental' in his terminology. One could argue against equating these concepts, saying that it is misleading because everything that is perceived is only represented in what Democritus termed 'matter of opinion' as opposed to the 'matter of objects'. However, this would imply a Radical Constructivist point of view[2] which disregards the fact that perceptual processes have evolved subject to the constraints of the 'matter of objects'. Without going into too many details of this evolutionary process[3], I want to mention two examples for the naturalness of perceptual processes. One concerns sensory processes while the other concerns 'higher' perceptual processes. The sensitivity of the eyes of nearly all animals corresponds to the energetic maximum of electromagnetic waves on the surface of the earth;[4] if the sensitivity were different, higher levels of energy would be needed for a comparable discrimination. Moreover, the spectral sensitivity curves of the eyes in different species closely mirror the distributions of reflectances in the respective niches.[5] However, basic sensory processes are not the only ones that depend directly on what is materially given. Starting with the analyses of D'Arcy Thompson,[6] it has been demonstrated[7] that even beauty is not primarily 'in the eye of the beholder', as Hume assumed, but well founded upon material constraints. This becomes evident if one analyses the growth of plants, where a maximum of stability must be obtained with a minimal expenditure of mass. What one observes in branching sequences follows very closely the Fibonacci series (see Figure 3): 1,2,3,5,8,13,21 with the underlying construction rule $x_{n-2} + x_{n-1} = x_n$

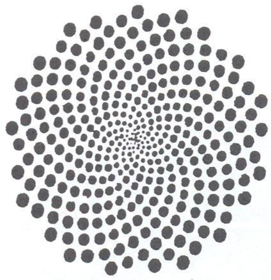


Figure 3. A sunflower

where the limit for $n \to \infty$ of the proportion $x_{n-1} : x_{n-2}$ is the Golden Section (approximately 1.618; see Figure 4);

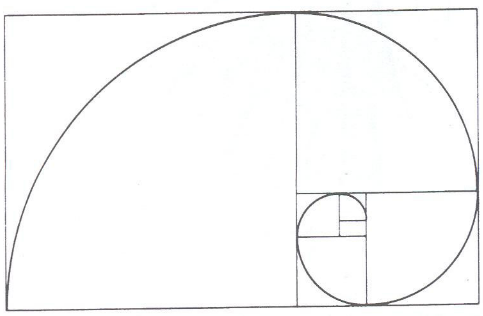
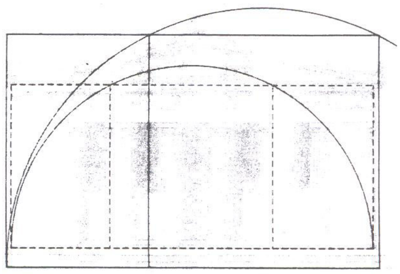


Figure 4. The generation of a sequence of Golden Sections

The Golden Section of this evolutionary process has in turn been the dominant generative rule in architecture and generally in visual art since the classical Egyptian age. For the ancient Greeks, the golden section together with the circle and the square constituted what was termed 'sacred geometry' (see Figure 5a and 5b).



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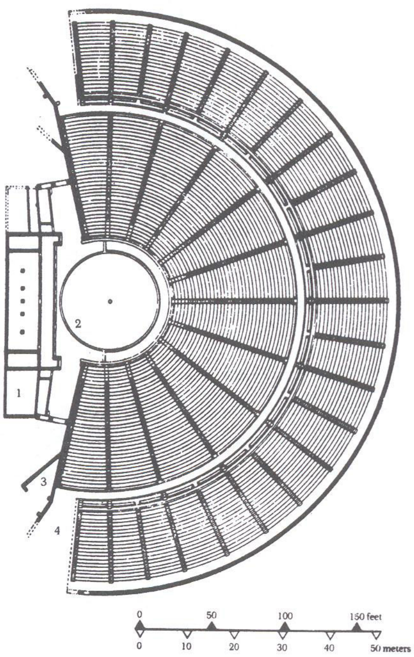

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Figure 5. (a) Front of the Parthenon with Golden Proportions. (b) The theatre of Epidauros (1. skene, 2. orchestra, 3. diazoma, 4. upper cavea); the numerical relations between the part approximate the Golden Section

The use of the Golden Section is documented for most cultures and styles. Even today it is pervasive in decorative art, in fashion and in the design of everyday objects. It is especially dominant in styles which refer explicitly to the organic, for instance Art Deco. The common denominator of all these variations of the Golden Section is the production of an impression of equilibrium in complexity. To this extent, beauty – at least partially – is an indicator of stability and therefore of evolutionary optimality.

D'Arcy Thompson made the point[8] that the interplay of forces is mirrored in form:

> The form, then, of any portion of matter, whether it be living or dead, and the changes of form which are apparent in its movements and its growth, may in all cases alike be described as due to the action of force. In short, the form of an object is a 'diagram of forces', in this sense, at least, that from it we can judge of or deduce the forces that are acting or have been acted upon it: in this strict and particular sense, it is a diagram – in the case of a solid, of the forces which have been impressed upon it when its conformation was produced, together with those which enable it to retain its conformation; in the case of a liquid (or of a gas), of the forces which are for the moment acting on it to restrain or balance its own inherent mobility. In an organizm, great or small, it is not rarely the nature of the motions of the living substance which we must interpret in terms of force (according to kinetics), but also the conformation of the organizm itself whose permanence or equilibrium is explained by the interaction or balance of forces, as described in statics.

This position was taken up by Köhler, who proposed[9] a variant of Gestalt theory founded on the conception of a generalized field theory. His treatise on forces in equilibrium expands on D'Arcy Thompson's approach to the mathematics of biological forms by describing the interaction of forces in a field, which gives rise to spontaneously self-organizing processes. Koffka, finally, has argued that due to this dependence of the perceptual processes on evolutionary constraints, perceived order and form are 'real' entities, not mentalistic Figments:

> [. . .] we see that without our principles of organization the objects could not be objects, and that therefore the phenomenal changes produced by these changes of stimulation would be as disorderly as the changes of stimulation themselves. Thus we accept order as a real characteristic, but we need no special agent to produce it, since order is a consequence of organization, and organization the result of natural forces.[10]

Thus the perception of order is not an interpretative act, but rather mirrors what is given in the world in reference to the perceiver. This is what Gibson describes as 'affordances'.[11]

That these perceptual processes really are phylogenetial in nature and do not result from experience becomes apparent in experiments with neonates[12] which reveal that objects and not features are the building blocks of neonate perception. Object and size constancy, together with the classification of objects according to their common fate, characterize infants' reactions towards objects prior to perceptual learning. In other words, sensitivity to invariants seems to precede sensitivity to isolated features; infant perception is the perception of forms (Gestalten). It should be noted, however, that not all Gestalt principles of perceptual organization seem to be independent of the influences of perceptual learning; for instance, the factors of symmetry and similarity – central to the concept of invariance in physics – appear relatively late in perceptual development. This is plausible because at least the perception of rotational and glide symmetry is strongly influenced by cognitive processes, as has been shown experimentally.[13] Further, what is perceived as similar depends not only on frames of reference but also on what might be called the perceptual attitude – that is, whether it is the global structure that is primarily taken into account or whether details are attended to with scrutiny.[14]

2. PERCEPTION: TOP-DOWN, BOTTOM-UP, OR BOTH AT THE SAME TIME?

Many theories of perception – for instance, all those originating from Helmholtz's theoretical point of view – postulate that the proper object of perception is constructed by starting with the raw material of sensory perceptions and going through a sequence of unconscious and later conscious cognitive operations. David Marr[15] postulates a sequence of filters and constructive processes which leads from sensory data to perceived objects. Gestalt theory, understood as a top-down theory, is often contrasted with these bottom-up theories; in such a top-down theory the form, termed 'Gestalt', determines the meaning of every detail, or a frame of reference makes a complex scene meaningful. If understood thus, a hierarchy of operations in perception can be found in Gestalt theory, too. If, however, Wolfgang Köhler's notion of perceptual fields[16] is taken to be the essence of the Gestalt theory of perception, the notion of a hierarchical organization of perception becomes questionable. I shall therefore posit the following hypothesis: perception consists of many processes of differing complexity. However, these are not hierarchically ordered but interact like forces in a field, and this interaction can be cooperative as well as complementary or competitive.[17]

In this respect, I agree. A network model of the multistable Necker cube can serve as an example for an interaction of processes in perception. Feldman has proposed such a model[18] which was subsequently refined by Zimmer.[19] The model starts from the notion that a Necker cube can be economically represented as an ordered set of 'forks' and 'arrows' (as favoured in 'Computer Vision'[20]) which induce a spatial impression if the following conditions are given[21]:

(i) A fork juncture is perceived as the vertex of a cube if and only if the measure of each of the three angles is equal to or greater than 90°.

(ii) An arrow juncture is perceived as the vertex of a cube if and only if the measure of each of the two angles is less than 90° and the sum of their measures is equal to or greater than 90°.

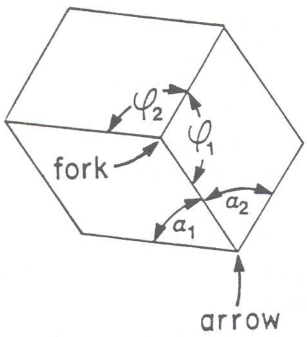

Figure 6. A cube with arrows and forks (Perkins 1973)

The constellations of arrow- or fork-like vertices of the Necker cube may have two different orientations, namely convex or concave. If one takes these vertices as the constituents of the Necker cube and connects them with excitatory or inhibitory arcs, as well as with autoinhibition, in order to model the saturation effect, the result is the following net (see Figure 7).

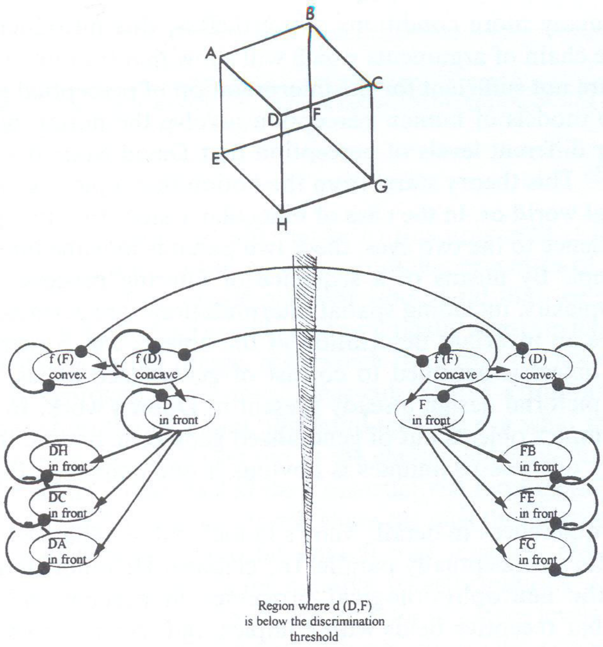

Figure 7. A network model of the Necker cube (Zimmer 1989). f(x) means fork at vertex X

The vertices with positive excitation are perceived as convex, and those with negative excitation as concave. Together with autoinhibition this results in a switching between two different percepts and hence in the bi-stability of the Necker cube. The three processes of excitation, inhibition and autoinhibition are competitive; since they are not hierarchically ordered, the phenomenon of spatial structuring, as well as of sudden re-structuring, is generated spontaneously and continually.

It should be noted that the network approach just demonstrated is not the only non-hierarchical interpretation for the effect of multistability. For instance, Ditzinger and Haken have suggested[22] a synergetic model for ambiguous or reversible Figures. In their model, multistability develops if there are two or more order parameters of about the same strength, in which case the enslaving of all but one order parameter is not stable because the perturbations due to attentional shifts are of the same order as the difference in the strength of the order parameters. Both models result in a behaviour very similar to that noted in human observers. However, it is difficult to decide which of these non-hierarchical theoretical accounts is preferable in regard to parsimony, because it is impossible to determine the relative complexities of these accounts.

Phenomena in real life are, without doubt, significantly more complex, and the perception of space is not only determined by rules like Perkin's laws but depends on many more conditions. Nevertheless, this introductory example illustrates the chain of arguments which will show that traditional hierarchical approaches are not sufficient for the interpretation of perceptual phenomenon.

Bottom-up models of human perception involve the notion of hierarchical processing or different levels of perception that David Marr developed in his seminal text.[23] This theory starts from the notion that a picture exists 'outside' in the physical world or, in the case of binocular vision, that two pictures exist in correspondence to the two eyes; these two pictures must be integrated into a unitary percept. By means of a sequence of filtering processes, features of different complexity, including spatial interpretations, are extracted until there finally evolves an invariant description of the objects which carries meaning. Usually, the object is assumed to consist of generalized cones,[24] which is a technique of pictorial design already present in Dürer's work. In 1500, Dürer composed complex objects out of generalized geometric bodies, usually cubes. The similarity of these techniques is obvious if one compares Figure 8a with Figure 8b.

In spite of differences in detail, Marr's model and its reliance on Hubel and Wiesel's results[25] conceptually parallel the classical Helmholtzian assumption concerning the neurophysiological processes in perception: namely, the assumption that receptive fields with complex and hypercomplex cells, up to gnostic cells,[26] act as analysers at different levels up to the cells which represent 'meaning': For example, according to Gross monkeys have a cell which fires only if there is a monkey's claw.[27]

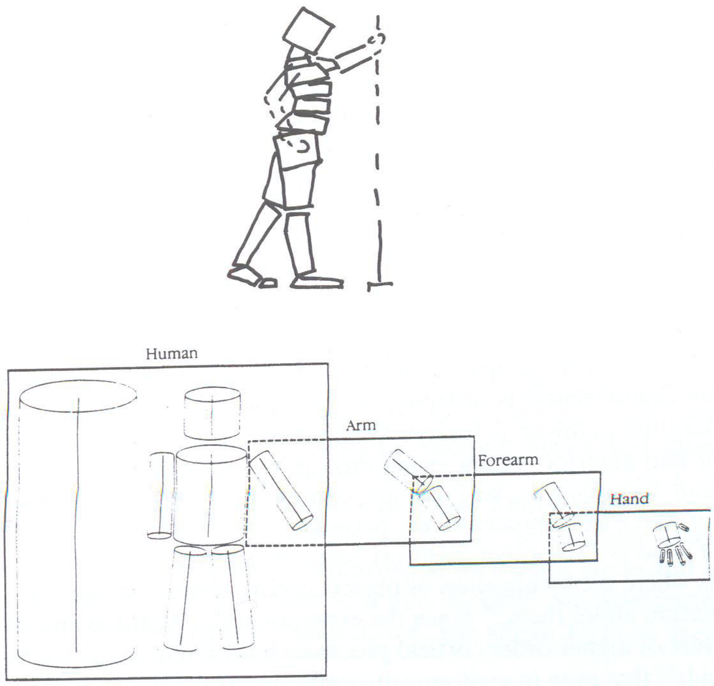

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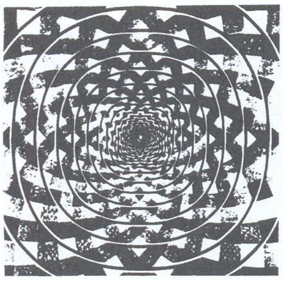
Figure 8. (a) Dürer's drawing of a body consisting of cubes. (b) the corresponding construction of a body out of generalized cones (Marr 1982)

The most radical theoretical alternative to this approach is 'Radical Constructivism').[28] According to this position, complex processes of construction, in particular self-reference, create *actuality* (the world that we can *act* upon) and which must be both theoretically and practically separate from *reality* (the world of transphenomenal things: *res)*, which is beyond the mind's grasp. The Constructivist approach is the most clear-cut case of a top-down theory of perception based on the idea that 'the mind tells the eye what to see'; an idea which derives from the results set out in the classic paper 'What the frog's eye tells the frog's brain'[29] that started the Constructivist movement.

The bottom-up and top-down models share the idea that perception is a kind of one-way road between pictorial stimuli and meaning. Moreover, both models assume that on each level massive transformations of the existing information take place in a sequence so that later or higher levels of perception never influence the activities of earlier or lower ones. This assumption has the important practical consequence that such structures are decomposable.[30] Without any doubt this is a highly desirable feature for theory building because it allows one to analyse and to model local processes without concern about the global structure in which they are embedded. However, investigations by Trevarthen showed[31] that even in his simplistic model the one-way characteristic of cortical processing is only true for one of the ascending pathways of visual information. That is, the pathway which leads from the retina through the *corpus geniculatum laterale* to the visual cortex (area 17) is linear, but for the other ascending pathway to the tectum the one-way characteristic no longer applies already in the *colliculus superior*. Here, eye movements are influenced by incoming information, so that the information that is 'picked up' now determines what will be 'picked up' as information next. Still, if one concentrates only on the former – that is, the purely ascending visual pathway in Trevarthen's model – a further and theoretically more serious deviation of the one-way characteristic is observed in the information processing behind area 17, where the pathway is forked. One pathway leads to the *cortex temporalis* inferior and anterior, where visual discrimination takes place (in the inferior part) and features (i.e., size, colour, texture, and form) are extracted (in the anterior part); these processes are necessary for the identification and recognition of objects. The other pathway leads in parallel to the anterior parietal regions where the localization of objects occurs, including auditory and tactile information about them.[32] Since the experiments by Mishkin and Ungerleider, the nature of higher-order cortical processes has been investigated further, with the result[33] that even in anatomically well-separated regions multiple functions can be found. In these seperated regions, the analyses of spatial relations, of object identification, and of feature discrimination, happen in parallel: "the inferior parietal lobule, in close conjunction with areas in the premotor and prefrontal cortex, provides a specialized set of semi-independent modules for the on-line visual control of action in primates".

The examples, demonstrations, and analyses discussed so far yield a picture which seems perturbing or contradictory only as long as one clings to the notion that the mind has to *translate* the sensory information into the mind's *language*. This implies, that '*Erkenntnis*' (attribution of meaning) does not refer to objects as they are but entirely to mental representations, as suggested by Fodor and by Pylyshyn.[34] If this position is correct, then these 'contradictions' of the visual perception should give rise to effects similar to those of meaningless sentences or paradoxes in language; that is, to a state in which understanding is no longer possible. However, in perception this is patently not the case: 'contradictions' abound, especially in the perception of space from two-dimensional displays. But instead of leaving the perceiving mind perplexed, they induce a strong spatial percept where only scrutiny[35] reveals that it relies on contradictory information. For examples, in the tower of S. Sidone in Turin, the architecture plays with constancy effects (of form as well as of size) in order to enhance the impression of height. Similar effects can be seen in S. Ignazio or the Palazzo Spada, both in Rome. The artful combination of local 'contradictions' results in an seemingly well-ordered perceptual world consisting of invariant objects and unique spatial relations. Projective geometry shows that both of these features cannot be realized at the same time; that is, an invariant object cannot be uniquely localizable and vice versa. However, if in perception both features are present at the same time, an interesting consequence for art results: the picture best representing what is perceived must be an impossible picture from the point of view of perspective geometry.[36]

Even if one regards the constraints of projective geometry on spatial perception as culturally imposed[37] or if one assumes that spatial knowledge is an amalgamation of tactual as well as visual information,[38] the intriguing question remains how local and global processes interact in space perception. Bottom-up theories require that the information obtained by early local processing must influence the final meaning of the perceived stimuli; by contrast, top-down theories assume that the constructive processes of the human mind start from something like Platonic ideals and impose their constraints on local analyses, thereby overriding contradictions in details, according to Goethe's dictum that "one only sees what one knows". Stimuli of the kind constructed by Frazer in 1908 can be used to test these contradicting assumptions. Figure 9a shows a compromise between a circle and a square; however, the details in Figure 9b reveal that in reality we have perfect cycles consisting of low level details which distort the global form of a circle to a square.



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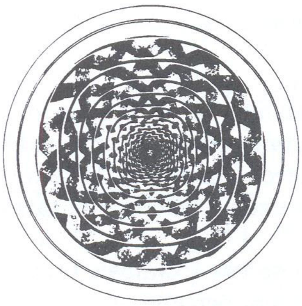

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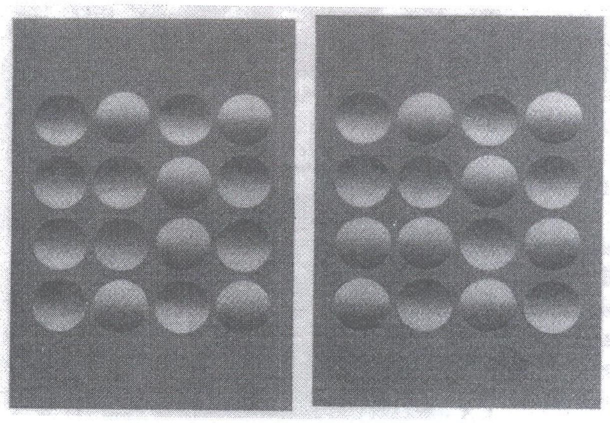
Figure 9. (a) Frazer's illusion. (b) Demonstration that the underlying form in (a) really is a circle

From the point of view of Gestalt theory, it is especially puzzling that the resulting perception contradicts the tendency towards good form because the circle is without any doubt a singular (*prägnant*) form,[39] and the form of a circle is present in the stimulus, whereas the alternative singular form of a square is not. However, what is perceived is none of these singular forms. Instead, a derived form prevails, a compromise between them. In such a case of two equally strong attractors, Gestalt theory would predict an alternation between these two forms. The top-down position predicts that the higher-order influences either unequivocally determine the percept of the circle or give rise to a perceptual paradox, namely, a bistability between two forms (cycle and square). As devastating as this result is for pure top-down models may be, the result of a visual demonstration by Ramachandran[40] is equally damaging for bottom-up models. If a subject is asked to decide in which of the two stimuli (Figure 10a or Figure 10b) a symmetrical form is imbedded, a) is always chosen despite the fact that here only a 'perceived object symmetry' is given, whereas in a pixel-wise fashion stimulus b) is horizontally as well as vertically symmetric but is not perceived as such.



a **b**

Figure 10. Ramachandran's stimuli for the perception of symmetry

The reason for this puzzling result seems to be that in this case the identification of form, apparently three-dimensional half-spheres, prevails over the directly detectable low-level symmetries. Here we have a case where what is seen is only what has been known before, and this is a clearcut case of top-down perception.

Joint consideration of these two examples reveals that not only has there been a contest between different perceptual mechanisms[41] in the phylogenetic process of evolution but also that every actual situation of perception involves some sort of contest between local and global analyses. In these examples the contest is competitive but it can also be complementary or cooperative.

In what follows, I shall demonstrate in three different areas of perception the thesis that it is not a hierarchical processing (top-down or bottom-up) which results in a world perceived as stable but a permanent interaction of processes or mechanisms on different levels of complexity. The examples are:

(i) The *phenomenon of spatial multistability* reveals that the strongest spatial effects do not occur if the parameters of basal features like symmetry, complexity or closure are simultaneously maximized or minimized, but only if the parameters are set at an intermediate level where no single feature is able to dominate the others.

(ii) The *perception of space from two-dimensional displays* which induce the strongest three-dimensional effect only if the invariant characteristics of perceived objects and the perspective distortions are equally strongly supported by the two-dimensional picture, resulting in a convincing spatial impression. This impression uniquely defines the position of the observer in relation to the depicted objects.

(iii) *The role of symmetry in the history of science* demonstrates how problematic the factor of 'symmetry' is for the adequate modelling of physical phenomena, at least as long as symmetry is understood as based upon perception and not as an abstract concept.[42]

3. MULTISTABILITY AS AN INDICATOR OF THE AUTOMATIC NATURE OF 3-D PERCEPTION

The Case of the Necker Cube

In Figure 11 six skeleton cubes are presented which differ to the extent that they induce 3-dimensionality (apparent depth) and in their degree of multistability. In both features, 11b induces the strongest effect because it has two equally strong semi-stable views which correspond to alternative spatial organizations.

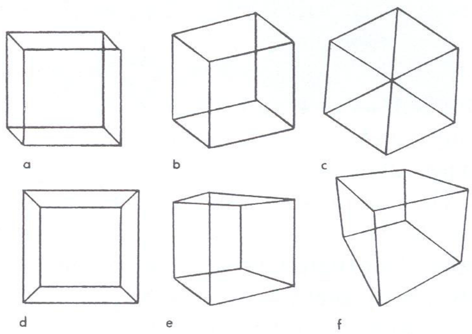


Figure 11. Six views of a skeleton or wire cube

Only slighly weaker is the effect in 11a, weaker because here a third transient state can be observed in which a 2-D pattern with a 45° axis of symmetry appears. This effect can be enhanced by rotating the axis of symmetry into a vertical position).[43] From an empirical point of view, only Figures 11d through f should give the impression of depth because only they correspond to real perspective projections with one, two or three vanishing points. By contrast, 11a is a perspectively impossible view of a wire cube, while 11b and 11c are parallel projections either without a vanishing point or with one that is infinitely far off; they are therefore not possible in a finite environment. On the other hand, because of their unique perspective specifications, 11d, e and f should induce a strong spatial effect but no multistability. As it turns out, they induce a weaker effect regarding apparent depth than do 11a and b, which is apparent from the fact that minor perturbations, namely removal of the vertices, destroy the 3D effect but nevertheless give rise to bistability, albeit with a bias towards the cube.

These results seem to defy the minimum principle of Gestalt theory[44] according to which deformations of forms exhibiting 'Prägnanz' (singularity) are avoided by perceptual mechanisms.[45] However, practically none of the closed geometrical forms in 11e or f displays the high degree of symmetry typical of stable 2-D forms.[46] The cube is a three-dimensional form with multiple axes of symmetry, and is therefore highly stable, but none of these characteristic features is preserved in the perspective drawings. The lines are not parallel, nor are they of equal length, and the angles are not orthogonal. These features are at least partially preserved in 11a, b, c and d, where this preservation plus the breaking of symmetry induces the strong three-dimensional effect, especially in 11a and b. If, by contrast, a two-dimensional projection of a cube exhibits maximal symmetry (6 axes of symmetry as in 11c and 8 in 11d), only a weak and transient depth effect, if any, is induced.

There still remains one puzzling example among the wire cubes of Figure 11, namely 11d. This preserves many features of the cube and it is a possible projection; nevertheless it appears to be flat, like a picture frame, not a cube. Perkins has used this instance[47] to derive his above-mentioned laws of the induction of 3-dimensionality in pictures.

These laws correctly predict that Figure 11d will be seen as flat. However, if the symmetries are broken, as in Figure 12, forms are generated which defy Perkins' laws, especially a and c.

Not only do the wire cubes in Figure 12 induce a depth effect, they also exhibit multistability. They can be seen as a cube from the inside or as a cut-off pyramid from the outside. There is no clear-cut preference for either of these perspective orientations. In experiments, the initial frequency of judgments made by subjects are about evenly split between a cube and a pyramid; afterwards subjects tend to stick to their initial percept. This is probably due to the fact that biases for the convex form (pyramid) and for the most regular form (cube) are about equally as strong, and a concept preserving transforma-

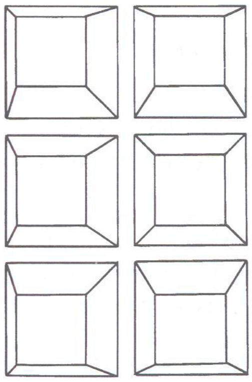


Figure 12. Breaking the symmetries in 11d

tion between these percepts is not possible. A similar 'real-world' example is provided by the view of the cupola of S. Giovanni degli Eremiti in Palermo, where the cupola is usually seen as concave but the pendentives tend to induce a convex orientation, despite our knowledge about the constructive rules for building cupolas over square groundplans (see Figure 13).

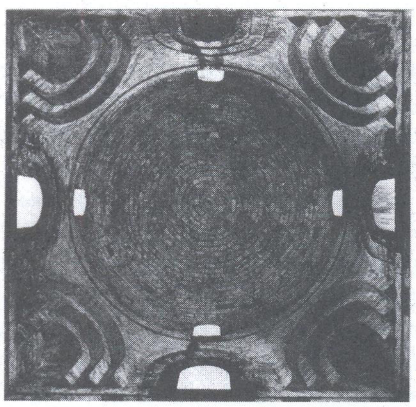


Figure 13. View of the cupola of S. Giovanni degli Eremiti in Palermo

There are several lessons about multistability and its role in depth perception to be learnt from this analysis of different projections of wire cubes:

(i) If the necessary conditions for perceiving a wire cube are given, namely 8 vertices with 3 connections each, plus convexity, apparent depth is induced if, and only if, the symmetry of the display is broken; an orientation of 30° relative to the fronto-parallel plane seems to be optimal.

(ii) The depth effect, as well as the frequency of perceptual switches, are strongest if as many features of the cube as possible are preserved in its projection.

(iii) Prior experience with such stimuli seems to play a negligible role, either in the sense that the subjects might have seen such a projection or in the sense that the rules of projective geometry are known and applied.

These results only partially agree with the results of a factor analytic study of "reversible-perspective drawings of spatial objects" carried out by Hochberg and Brooks. According to that study,[48] the apparent depth of a drawing depends on three factors: (i) simplicity versus complexity (measured by the number of angles); (ii) good continuation versus segmentation (measured by the number of line segments); and (iii) symmetry versus asymmetry (measured by the relative number of different angles). What this factor-analytic approach implies, however, is the additivity of these components: "the greater the complexity, the asymmetry, and the discontinuity of the projection of a given tridimensional object in two-dimensions, the more three-dimensional it will appear. We may, in reality, be dealing with only one dimension – 'figural goodness'".[49] This runs counter to the comparison of the induced depth effects in the cube drawings in Figure 11. The views of the cube in a and b have a much stronger effect than those in e and f, which implies that the interaction between the components identified by Hochberg and Brooks is not additive but that they must be modelled as competing processes producing the maximum joint effect if they all are of comparable magnitude. That this interpretation is not restricted to Figures 11c and b can be shown by analysis of generalizations of wire-cube drawings. For example, (i) repetitions and glide symmetries can be used to produce tilings with the Necker cube as constituting elements (Figure 14); (ii) the dimensionality of the generating spatial object can be increased from 3 to 4; a 4-dimensional hyper-cube is defined as consisting of 32 edges and 16 fourfold vertices (see Figures 15 and16); or (iii) the cube as one exemplar of the Platonic bodies can be exchanged for a more complex one, the dodecahedron (Figure 17).

In all cases, the factor-analytic criteria plus the convexity criterion of a generalized soap bubble[50] would predict that the spatial effect is even stronger than in Figure 11b. Inspection shows that this is not the case, and experimental results concerning the depth effect after an occlusion of the vertices support this finding.

In regard to the tilings in Figure 14, one may argue that, due to multistability, different perspective orientations cancel the overall depth effect but these different perspective orientations do not occur simultaneously. If one

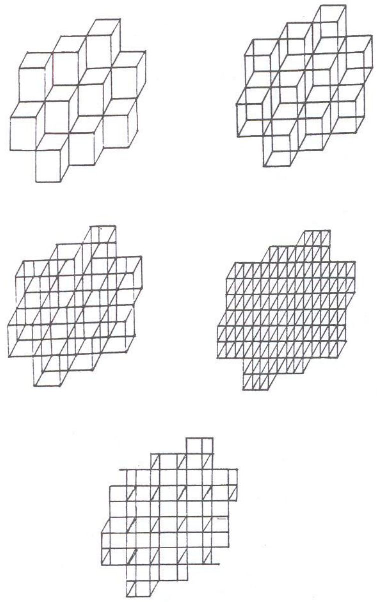


Figure 14. Tilings consisting of partial or complete Necker cubes

a

b

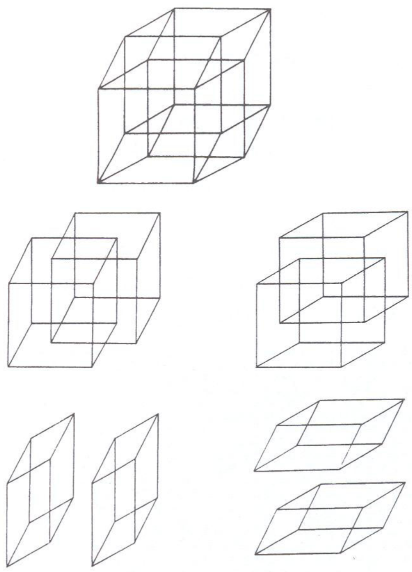


Figure 15. (a) A 2-D projection of a four-dimensional Necker cube. (b) 2-D projections of three-dimensional partitions of Fig. 15a

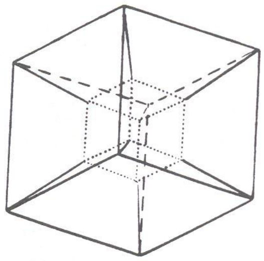


Figure 16. An alternative for Figure 15

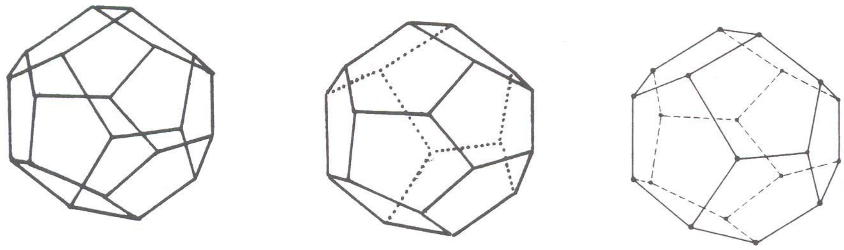


Figure 17. A skeleton dodecahedron

segment switches, all other segments become enslaved at such a speed that self-observation cannot determine the exact and detailed time course of the process or the subprocesses of enslavement. However, the theory of tilings offers one further alternative interpretation, which is that the patterns in Figure 14 are all periodic. In other words, there exists a subpattern which is repeated over and over again, resulting in glide symmetries; if the rhythm is broken, as in the non-periodic tiling of Figure 18, multistability is produced despite the complexity in the pattern. My conjecture is that non-periodicity, convexity, and symmetry breaking[51] are the decisive factors for the depth effect in drawings.

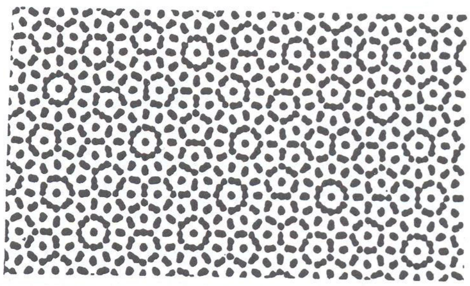


Figure 18. A non-periodic tiling (Stadler, Seeger, Raeithel, 1977)

In the following section, I shall show how artists have made use of these factors to produce strikingly realistic pictures which on closer inspection deviate from the directly projected transformation implied by the devices for perspective drawing invented by Alberti and Dürer. These deviations accentuate the factors of symmetry breaking and convexity, whereas non-periodicity is usually already given by the choice of the subject.

4. HOW TO INDUCE A COMPELLING 3D-EFFECT IN PICTURES

The bifurcation between the object as it is and the object as it is seen from a specific vantage point in a defined situation (or as Gibson put it,[52] the bifurcation between "no change" and "change") uniquely specifies the position of the observer in relation to the perceived object. However, this is only the case if the object and its environment comply with the experiences of the observer and are not at odds with the fundamental and time-independent features of objects. These features are three-dimensionality, partial convexity, rigidity, regularity (that is, showing a tendency towards good form), and closure of the surface. If all these features are present, a spatial orientation is possible, even with unknown objects. These benefits must be set against the implicit costs of these perceptual "hypotheses"[53] costs which consist in the tendency to succumb to spatial illusions, especially if a distorted environment induces a re-scaling of objects and thereby renders the entire scene a perceptual paradox, as in the case of the Ames room.

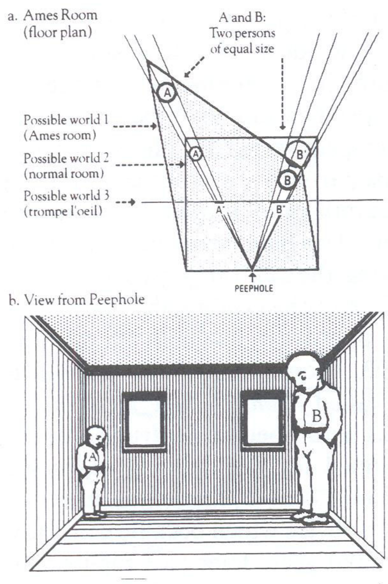


Figure 19. The Ames room

Panofsky claims that only the social and philosophical re-orientations caused by the emancipation of Italian cities allowed the emergence of a new iconographic schema in which objects were no longer depicted as they were 'known', or – in scholastic terms – according to their 'form', but as they were 'seen by a specific observer from his or her vantage point: the 'intuitus'.[54] In a later analysis,[55] Panofsky points out that there is an irreducible contradiction between the geometric construction and the subjective percept. In what follows, it will become apparent that this contradiction is in essence due to the multiple parallel processing of visual information, and that artists skilfully distort perspective to capture exactly these processes of perception and thereby make their spatial effects most compelling.

In his classic text, *De pictura* of 1435, Alberti explains the importance of his approach to perspective as follows:

> The instructions are such that anyone will grasp their utility, if they understand them and the [underlying] doctrine of pictorial expression. One should never assume that anyone can be a good artist who does not understand clearly what he is going to do (my translation).

It is apparent from this quotation that Alberti sets a standard for the evaluation of visual art, and specifically that expertise in perspective drawing is the necessary precondition for the production of genuine works of art. Alberti, however, does not confine himself to the theoretical aspects of perspective geometry, for he also gives practical advice on how to produce a drawing which is perspectively correct: "On the surface on which I intend to paint, I draw a rectangle of the size I want, this is regarded as an open window through which the object of my painting is viewed" (my translation). This is the definition of the 'Alberti window', which is sometimes called the 'da Vinci window' in the literature. In his treatise on architecture, Antonio di Piero Averlino gives closer specification to Alberti's technical instructions and develops the central perspective as the one which fits the human eye. His success explains – at least partially – why the central perspective played such an important role in the paintings of Raphael and other Renaissance painters. Leonardo da Vinci followed up these ideas and speculated about the physiology of the human eye, making the central perspective the only one suited to the human eye: " [. . .] in this way objects directly opposed to the eye impinge more strongly on the senses if they are in line with the respective nerves". Moreover, in manuscript A of 1492, Leonardo sets out rules on how to apply Alberti's rules correctly, the so-called '*Construzione legittima*'. In parallel with these theoretical elaborations and specific instructions, a number of technical instruments were developed for the drawing of perspective. The earliest of these was Brunelleschi's (1377–1446) apparatus, which consisted of a system of mirrors and a peephole; a device which became generally known as the 'camera obscura' about one hundred years later (see Figure 20). Finally, Dürer in his *Underweysung der Messung* (first published in 1524, but the third edition of 1538 is of special importance for the development of perspective) summed up what was then known about perspective, theoretically as well as practically. He also proposed the 'Dürer window', which became the standard instrument for perspective drawing until the advent of modern art. As regards the importance of form, it is thus interesting to note that perspective geometry originated in the intent of artists to protray objects as they are seen from a specific point of view. The implicit physical presuppositions, as well as the implications for mathematics, were analysed much later. For instance, in 1619 Schreiner showed experimentally that rays of light propagate linearly and that they cross in the camera obscura (see Figure 21).

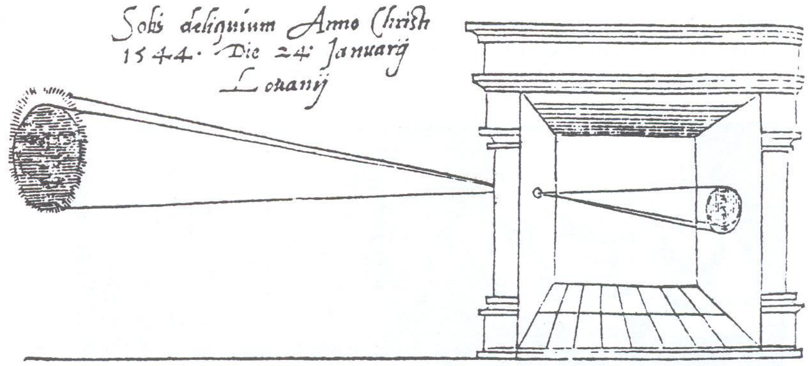


Figure 20. The camera obscura

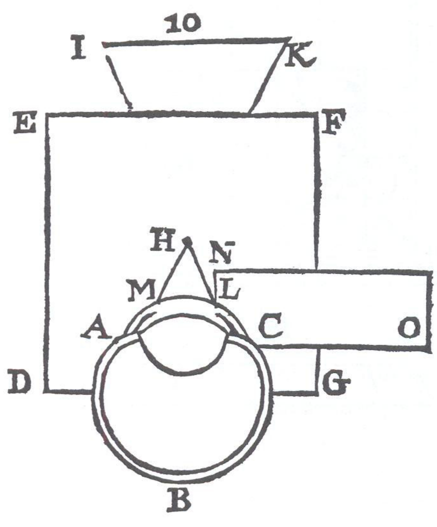
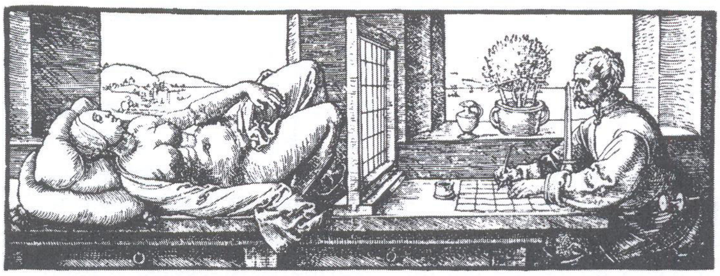


Figure 21. Schreiner's apparatus for the analysis of how the light travels in the camera obscura

It took even longer for mathematics to realize the implications of perspective drawing for geometry. As late as the second half of the eighteenth century, J. H. Lambert in his *Theorie der Parallellinien* (written in 1766 and published in 1786) finally concluded that any hypotheses on spatial relations which do not lead to contradictions offer a possible geometry; insofar as Renaissance perspective constituted one possible geometry.

One reason for this delay in basic research into the propagation of rays and of linear perspective may be that artists themselves systematically deviated from their own precepts. The third edition of Dürer's *Underweysung* (1538) contains a picture of an artist using the 'Dürer window' to draw a reclining women (see Figure 22a). However, if one uses this drawing to reconstruct how the artist really saw the reclining nude from his point of view, what results is shown in Figure 22b.

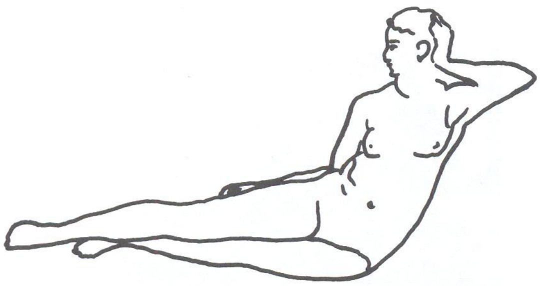


a

b

Figure 22. (a) Illustration of the Dürer window from the 'Underweysung' of 1538. (b) Approximation of the projection on the screen of the Dürer window

Nowhere in Dürer's *oeuvre* can one find such a view of a reclining woman. His drawings that most closely resemble the view of the artist in Figure 22 are the 'Reclining Nude' of 1509 (see Figure 22c) and the woman in the 'Family of the Satyr' of 1505 (see Figure 22d).



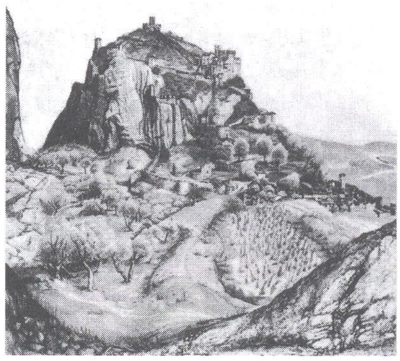
c

d

Figure 22. (c) 'Reclining Nude' by A. Dürer (1509). (d) Woman from the 'Family of Satyrs' by A. Dürer (1505) redrawn for reasons of comparability

When these Figures are compared, it becomes apparent that Dürer solves the problem of showing the objects as they are, and as they are seen from the artist's point of view, by rotating the reclining women of Figure 22c into a canonical position; from this perspective the distortions are minimal. In Figure 22d he uses an alternative approach which stretches or shortens the limbs towards their natural proportions: that is, he does not produce a real projective view but joins different views together in such a way that a global impression is formed. This impression takes account of the invariant proportions of the human body but at the same time gives the impression of a specific point of view. Dürer thus integrates what Penrose called the two opposite views of art:[56] showing what one sees, or showing what one knows.

Although the integration of these two opposing views can be regarded as convincing, in art criticism[57] these deviations from the geometrically correct projections are sometimes regarded as 'primitive'. Stadler makes this point when analysing Dürer's watercolour of the 'Castle of Arco' (see Figure 23a).



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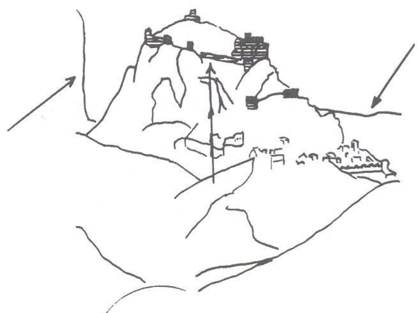


c

Figure 23. (a) The 'Castle of Arco' by A. Dürer (watercolour 1494). (b) A photograph of the castle of Arco (Leber, 1988). (c) The different vantage points of Figure 23a

The watercolour in Figure 23a shows marked differences with respect to the photograph in Figure 23b. Figure 23c shows from which points of view the different segments of the watercolour have been taken and how they have been integrated. Only the buildings on the top of the mountain (hedged) correspond exactly to the view in the photograph; a view which is nevertheless implied in the watercolour. This is in spite of the fact that most parts of the *aquarelle* are not seen from this point of view. If one compares the details and the proportions of the photograph with those of the watercolour, systematic differences appear in addition to the integration of different perspectives:

(i) the size of those objects serving as points of orientation for the observer have been enhanced;

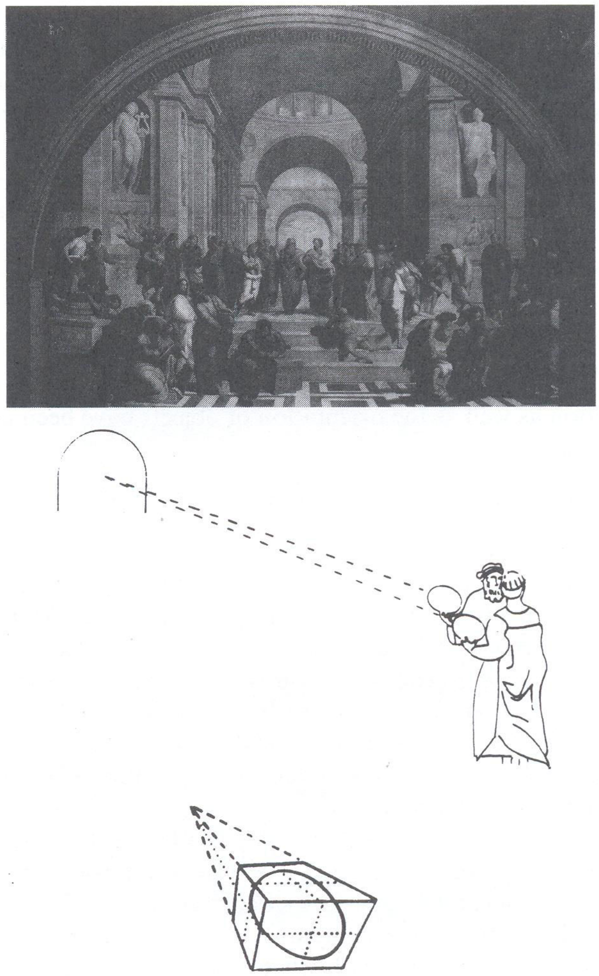
(ii) the position as well as the orientation of objects have been adjusted for a maximal spatial effect, and

(iii) the relation of height to width has been massively distorted.

More detailed analysis is conducted by Leber, who also provides further illustrations and examples.[58] In the eighteenth century, Piranesi used these systematic distortions in his *Vedute di Roma* to produce the spatial effect that corresponds most closely to the subjective view.[59] A series of experiments[60] have shown that the proportions in 'distorted pictures' are regarded as more realistic than those that correspond to 'perspectively correct' photographs. Therefore, contrary to what Stadler claims,[61] these are the results not of a 'primitive level of construction' but of an optimal synthesis of the 'no change' and 'change' in Gibson's terminology.

One further example of this synthesis is provided by Raphael's 'School of Athens' (see Figure 24a). The assembly of Greek philosophers in a central perspective view immediately produces the impression of a natural spatial constellation. However, if one compares the rendering of the people and geometric bodies in the picture with what a camera obscura would produce, systematic distortions become apparent. The most striking instance is the perfectly circular spheres held by Euclid and an accompanying person in the right part of picture. LaGournerie was the first to point out[62] that the spheres should appear as ellipses (see Figure 24b). The reason for this is demonstrated in Figure 24c. However, LaGournerie discovered that such a perspectively correct representation strikes the observer not only as uncommon but also as patently wrong.

It should be noted that what applies to the spheres also applies to the other objects, and especially to the people depicted as if they were in the centre of the frontal-parallel plane. Thus, the seemingly correct global pictorial representation is in fact an assembly of many separate representations. Pirenne comments[63] that this result contradicts an empiricist view of perception because such an assembly does not correspond to any possible real scene.



a

b

c

Figure 24. (a) 'The School of Athens' by Raphael (in the Stanze della Seguatura 1508–1511). (b) The correct perspective distortions of the sphere held by Euclid and an accompanying man. (c) The constructive principle for the ellipses in (b)

Thus it appears that the spectator looking at Raphael's picture of the spheres must make a complicated intuitive compensation. On account of natural perspective, the circles appear foreshortened to him. They do not form in his eyes the retinal images which would be formed by actual spheres. But, on the basis of his knowledge of the shape and position of the surface of the painting, he recognizes them as circles drawn on a flat surface. Since real spheres always look circular, he concludes that these circles represent spheres. It will be noted that all this, which must somehow occur unconsciously, can be done as well when the spectator uses both eyes, and is in the wrong position. To most spectators, the *School of Athens*, in which the perspective is in parts inaccurate, appears as an outstanding example of the use of perspective.

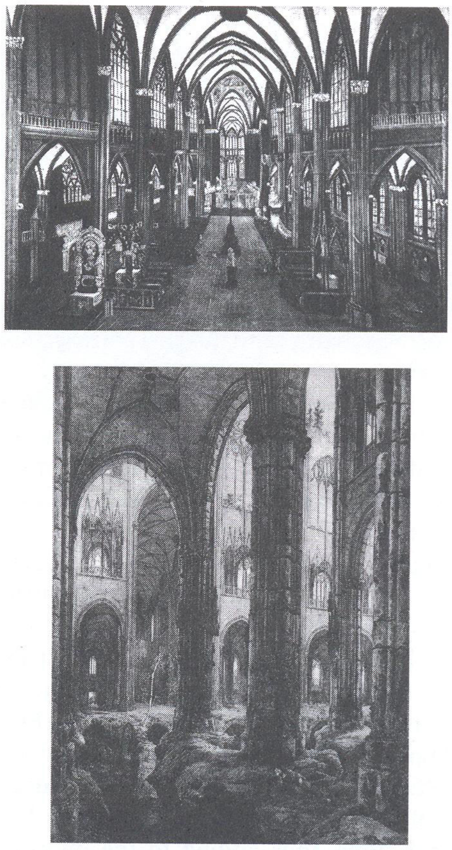
Contrary to Pirenne, I would argue that the 'complicated intuitive compensation' performed by the observer is not at all complicated. It is automatic and does not involve any information processing.[64] This result contradicts the modularity assumptions made by Fodor,[65] because the 'correct' ellipses must be processed *analytically* in order to prove that they represent reality correctly. By contrast, the 'distorted' views of circles immediately yield a unique spatial representation which portrays the world as it really seems to be. A parallel top-down and bottom-up processing of information takes place here. Another consequence is that, whereas the constituents of the complex scene are processed in parallel, so that each constituent can have its own stable frame of reference in which orthogonality and linearity are given, the total scene is processed serially, with the result that the incongruity between the different frames of reference and vanishing points does not become apparent. This corresponds to Hochberg's view[66] that the perception of a complex scene is the result of an integration of separate glances at the constituents of this scene. Global contradictions, as in Escher's paradoxical etchings, consequently do not 'pop out' but must be searched for analytically.

Comparing perspective pictures of Renaissance art with those of the Baroque period yields further insights into the mechanisms of spatial perception. Symmetry prevails in Renaissance art, resulting in the preference for a central perspective, whereas in Baroque and later art these symmetries are intentionally broken. The view of the interior of Regensburg cathedral and Blechen's romanticist rendering of a ruined church (see Figures 25a and b) may serve as examples. Apparently, the importance of breaking symmetries in order to induce a spatial effect is not confined to simple situations like the Necker cube. In more complex scenes, symmetry breaking has the side-effect of resulting in more occlusions, thus adding to the impression of a 3-D space because of their importance for 'direct information pick-up'.[67]

According to Gibson,[68] spatial orientation relies on the perception of the flow field and on identification of the invariants within it – this cannot be mimicked in 2-D paintings – although equally important is the structure of perceived occlusions. It has been demonstrated experimentally that these alone, under appropriate conditions, can result in a unique spatial specification.[69]

If one tries to condense the message from the techniques of spatial representations in visual art, the main result is that the pictures are neither constructed nor perceived analytically in a sequence of hierarchical levels. They are instead perceived in a parallel fashion where not the global consistency of projective geometry but the saliency of local processes in interaction is decisive for the subjective impression of a 3-D scene represented in a 2-D picture.

Besides this general result for the architecture of perception, also of importance is a specific finding concerning the role of symmetry. Symmetries play a major role in science: not only are the laws of Newtonian physics characterized by symmetries but so too are those of modern physics. Chemistry



a

b

Figure 25. (a) Interior of the cathedral of Regensburg (about 1600). (b) Ruin of a gothic church by Blechen (about 1840)

and a special christallography abound in intricate symmetries (at least seventeen can be described analytically). Finally, practically all animate objects display one or more axes of symmetry or hierarchies of bilateral symmetries.[70] In the light of the pervasiveness of symmetries in the world around us, it is surprising that on one hand Bower was unable to find a symmetry effect in the object perception of neonates,[71] while on the other hand the perception of 3-D bodies as spatial in 2-D pictures relies on the breaking of symmetry. My conjecture is that because the *forms of the objects* are symmetric, they have to be *represented* from an angle which allows them to separate 'change' and 'no change'. It is precisely the tension produced by this separation that gives rise to a perception which is realistic in the sense of Scholastic philosophy (*res* = objects as seen) as well as in that of everyday language, because it results in an optimal fit between the world as it is and the world as perceived.

In order to demonstrate that this role of symmetry is not confined to the inspection of simple forms or the pictorial representations of spatial scenes, I will use a case study from the history of science to show how important the symmetry breaking is for any veridical representation of the world of objects.

5. SCIENTIFIC MODELS AS REPRESENTATIONS OF THE WORLD OF OBJECTS: THE CRUCIAL IMPORTANCE OF SYMMETRY BREAKING

The extent to which the Aristotelian notion of an invariant form underlying variable and transient phenomena has determined the formation of theories about the world becomes especially apparent in theories about the solar system. In 1660, Athanasius Kirchner classified and systematized different theories from Ptolemy to Copernicus (see Figure 26). However, he excluded the then already accepted Keplerian system, because this ellipsoidal system was at odds with the presupposition that invariant forms ought to be simple (see Figure 27).

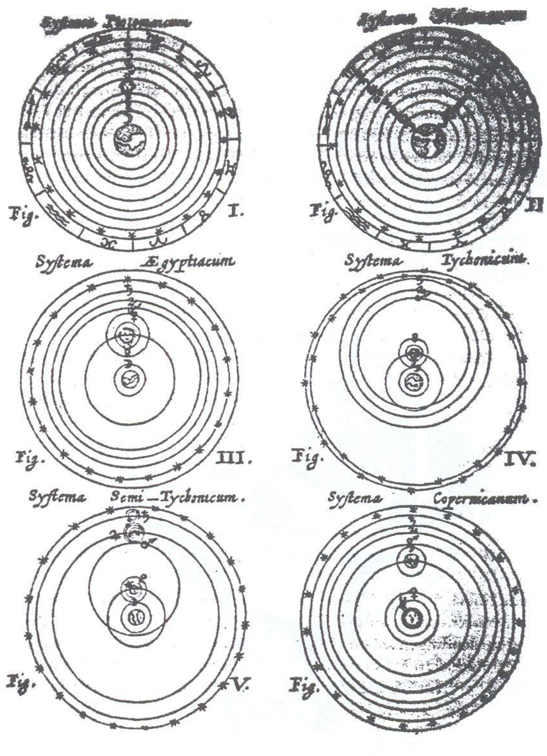


Figure 26. Six different models of the world according to Kircher (1660)

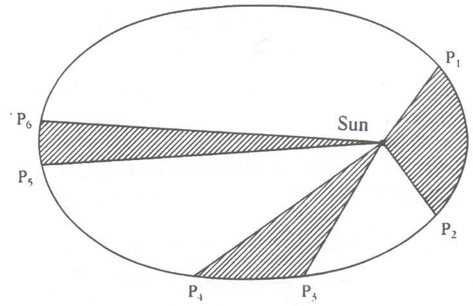


Figure 27. Illustration of Kepler's second law

Kepler himself was impeded in his scientific progress by this presupposition (see for instance his *Mysterium Cosmographicum* of 1597; Figure 28). Here he postulates that the diameters of the circular trajectories of the planets are determined by the sequence of Platonic bodies. This theory was so attractive because, in essence, it constituted a 'theory of everything' which tied the Platonic theory of elements (Figures 29a and b) to the Copernican model of the solar system.

However, its especial attractiveness lay in the perfect symmetry of the macrocosmic and microcosmic world thus envisaged. Plato's identification of the elements with what came to be known as the Platonic solids (*Timaeus* 53–56) and Kepler's celestial model were regarded as a proof that the world had been by a rational being.

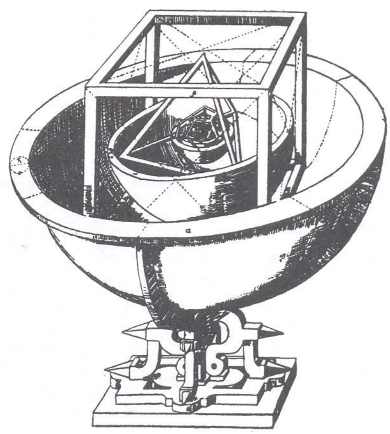
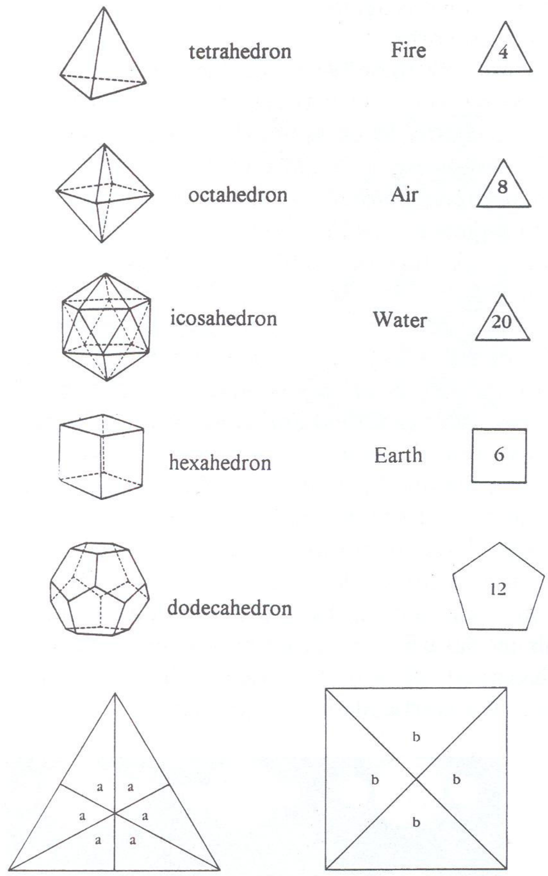


Figure 28. Kepler's model of the world as consisting of involuted Platonic bodies



Fire	:	Tetraeder	=	4 · 6 a	=	24 a	(1)
Air	:	Oktaeder	=	8 · 6 a	=	48 a	(2)
Water	:	Ikosaeder	=	20 · 6 a	=	120 a	(5)
Earth	:	Hexaeder	=	6 · 4 b	=	24 b	

Figure 29. Plato's correspondence of ideal bodies and elements

It is my intention, Reader, to demonstrate that the Highest and Most Good Creator in the creation of this mobile world and the arrangement of the heavens had his eye on those five regular bodies which have been celebrated from the time of Pythagoras and Plato down to our own day; and that to their nature He accommodated the number of the heavenly spheres, their proportions, and the system of their motions.[72]

Although this approach hampered the advent of the correct elliptoidal model for some years, it opened the way for the physical analysis of celestial phenomena and resulted in Newton's model. This in turn motivated Bohr's model of the atom, which is again a seemingly perfect correspondence between macrocosm and microcosm.

From my point of view, these two tendencies – towards the most perfect form and towards a correspondence between different levels of analysis – do not reflect primarily cognitive processes, which are biased by conservatism or cognitive heuristics. Rather, they indicate perceptual processes which involve the bifurcation between perceiving an object as it is and as it is seen in perspective from a specific point of view.

Kepler's work shows that he emancipated himself from the error-inducing influence of 'ideal forms' by developing the elliptoidal model of planetary motion. However, he himself did not reflect on the psychological processes which first led him astray but finally allowed him to find the correct solution. As far as I know, Christiaan Huygens was the first to conduct a phenomenological analysis of the perceptual and conceptual influences on the scientific process of representing the world of objects in analytical terms. He chose a problem which had haunted astronomy since the invention of the telescope: what is the real form of the planet Saturn?

Huygens analysed the relationship between observation and conceptual structure in science in his book *Systema saturnium* (1659, here quoted after volume XV of his *opera omnia*). In one illustration he shows thirteen different views of the planet Saturn as reported by astronomers since the beginning of the seventeenth century. Most of these views exhibit perfect bilateral symmetry, horizontally as well as vertically (see Figure 30).

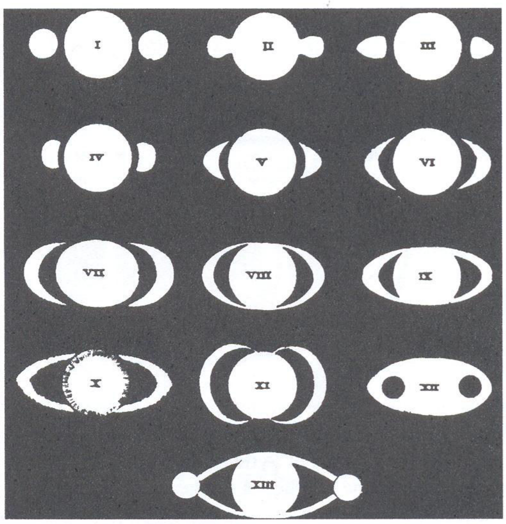


Figure 30. 13 different views of Saturn as reported up to Huygens' analysis

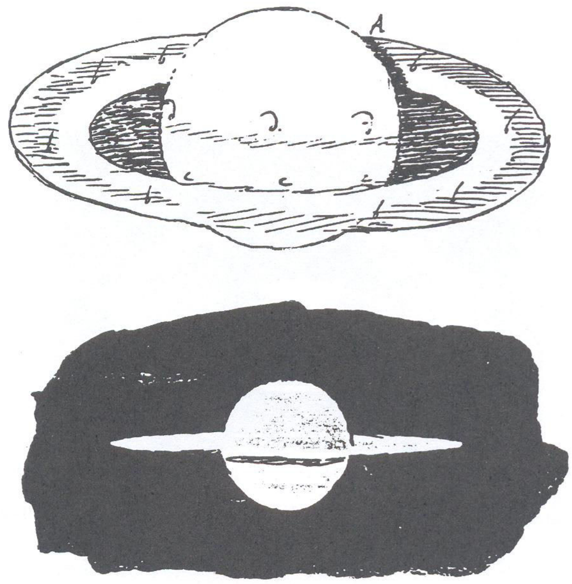
If one compares these images with a photograph taken on 24 November 1943 using a 100-inch telescope, one sees that they are not Figments of the imagination but instead represent observations (see Figure 31).

Figure 31. A photograph of Saturn taken with a 100-inch telescope on 11/24/1943

The saliency of symmetry in these views gave rise to two alternative interpretations. According to one, the planet was assumed to vary in form (see e.g., view XII in Figure 30, attributed to Gassendi, 1646); according to the other, the planet was a perfect sphere surrounded by a variable environment. The latter is especially apparent in view X observed by Divini. Huygens comments on views VIII und IX by Riccioli (1648–1650) as follows: "After somebody has elaborated a hypothesis which leads him to such a consequence, then he deludes himself and believes in the reality of what he hopes to see".[73] Huygens himself arrived at the representation of the planet Saturn, which today we know to be correct (Figure 32 a and b), by combining an invariant form (a spherical planet with a flat ring) with an elliptical orbit; together they produce all the views of Figure 30 as possible observations of the planet Saturn from the Earth (see Figure 33).

Huygens' analysis shows how immediately perceived symmetries may prevent the detection of more basic underlying symmetries and regularities. His solution required that these supervisual symmetries be broken; the resulting model exhibited the invariant features only because the stability of form in the object and its apparent variability, due to its perspective projections, were taken simultaneously and equally into account. Again, as in previous sections, a realistic representation is only achieved by integrating the 'change' and 'no change' aspects.

The essence of the last examples regarding the question of 'form versus matter' or 'perception versus conception' is best summed up by Albert Einstein (1950):



a

b

Figure 32. Huygens' model of Saturn

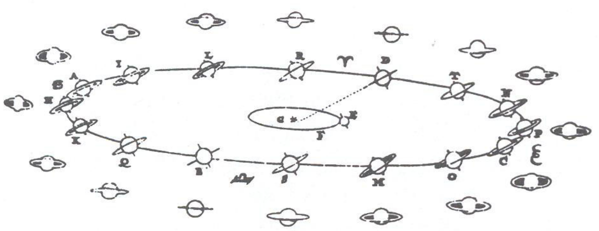


Figure 33. Huygens' integration of the different depictions of Saturn in Figure 30 as specific views due to different constellations of Saturn and Earth

I believe that every true theorist is a kind of tamed metaphysicist, [] The tamed metaphysicist believes that not all that is logically simple is embodied in experienced reality, but that the totality of all sensory experience can be 'comprehended' on the basis of a conceptual system built on premises of great simplicity. The skeptic will say that this is a 'miracle creed'. Admittedly so, but it is a miracle creed which has been borne out to an amazing extent by the development of science.

6. A LESSON LEARNED?

In my opinion, the examples from perception and from the history of science compellingly indicate that the classical juxtaposition of 'form versus matter' and 'perception versus cognition' obscures the real underlying question of whether the relation between these terms is complementary in nature. The first attempt to formalize this relationship can be found in the appendix to Euler's *Methodus Inveniendi Lineas Curvas Maximi Minimive Proprietate Gaudentes* (A method to find curved lines that maintain a maximum or minimun property), in which he proves that the principle of least action can be used to describe the motion of a point mass in a field of forces, an example being the motion of planets around the sun. Euler thus already put forward the conjecture that this principle might be a pervasive underlying principle of nature, provided one is interested in the analysis of action and not in the description of stationary states. This idea was expanded by Max Planck in his lecture delivered at the Prussian Academy of Sciences on 29 June 1920 to celebrate Leibniz's anniversary:

> Present-day physics, as far as it is theoretically organized, is completely governed by a system of space-time differential equations which state that each process in nature is totally determined by the events which occur in its immediate temporal and spatial neighborhood. This entire rich system of differential equations, though they differ in detail since they refer to mechanical, electric, magnetic, and thermal processes, is now completely contained in a single theorem, in the principle of least action. This, in short, states that, of all possible processes, the only ones that actually occur are those that involve minimum expenditure of action.[74]

Gestalt theorists, especially Wolfgang Köhler,[75] have regarded this principle of least action to be at work in the 'world out there' as well as in the world represented in the perceiver; for the Gestaltists this was the reason why there is an order in things (*res*) as well as in their percepts – and they believed that these orders correspond. In my opinion, modern theories of perception should follow this research programme.

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NOTES

[1] E.g. Gibson 1951, 1973. Cf. Schmidt 1992.
[2] For these see Zimmer 1995.
[3] Roth 1992.
[4] For further details see Shepard 1994.
[5] D'Arcy Thompson 1917.
[6] Richter and Scholz 1987.
[7] D'Arcy Thompson 1917.
[8] Köhler 1920.
[9] Koffka 1935, 175.
[10] Gibson 1979.
[11] Bower 1972.
[12] Zimmer 1984.

[13] Julesz 1971; Navon 1977.
[14] Marr 1982.
[15] Köhler 1920.
[16] See McClelland and Rumelhart 1986 for the notion of parallel and distributed processing in perception.
[17] Feldman 1982.
[18] Zimmer 1989.
[19] Guzman 1969.
[20] Perkins 1968, 1972, 1973.
[21] Ditzinger and Haken 1989.
[22] Marr 1982.
23Ibid.
[24] Hubel, Wiesel 1962.
[25] Konorski 1967; sometimes jokingly termed 'grandmother cells'.
[26] Gross 1973.
[27] Maturana, Varela, 1975; Schmidt 1987.
[28] Lettvin, Maturana, McCollouch, Pitts 1959.
[29] Simon 1969.
[30] Trevarthen 1968.
[31] Mishkin, Ungerleider 1982.
[32] Milner, Goodale 1992.
[33] Fodor 1968; Pylyshyn 1981.
[34] Julesz 1971.
[35] Panofsky 1980.
[36] Cf. Deregowski 1980.
[37] Cf. Ivins 1946.
[38] Cf. Goldmeier 1982.
[39] Ramachandran 1988.
[40] Koffka 1935.
[41] Cf. Wilzcek, Devine 1987.
[42] Cf. Zimmer 1986.
[43] Köhler 1920.
[44] Koffka 1935.
[45] Wulff 1922.
[46] Perkins 1968.
[47] Hochberg, Brooks 1960.
[48] Ibid., 354.
[49] Attneave 1981.
[50] Haken 1990.
[51] Gibson 1971.
[52] Gregory 1980.
[53] Panofsky 1976.
[54] Panofsky 1980, 102.
[55] Penrose 1973.
[56] E.g. Stadler 1929.
[57] Leber 1988.
[58] Cf. Panofsky 1976.
[59] Zimmer, in press.
[60] Stadler 1929, § 8.
[61] LaGournerie 1859, 170.
[62] Pirenne 1970, 122-33.
[63] See Zimmer 1986a.
[64] Fodor 1975.
[65] Hochberg 1962.
[66] Gibson 1979.
[67] Ibid.
[68] Zimmer 1986b.
[69] For examples see Weyl 1952.
[70] Bower 1972.

[71] From Kepler's introduction to the *Mysterium Cosmographicum.*
[72] Here Huygens alludes to Francis Bacon "Quod enim mavult homo verum esse, id potius credit" (*Novum Organon* I, aph. 49).
[73] Quoted in Hildebrandt, Tromba 1985, 192.
[74] Köhler 1920.

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