Urban Political Economy according to the Greatest Cumulative Ring Difference

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Abstract: City residents own housing that on average either is close to the urban center (residents who we label centrists) or far from it (decentrists). Centrists and decentrists do not feature in the literature, even as they defy one another along many contentious, and not just urban, policies – such as the decentralization of retail, carbon taxes, toll roads, building height controls or urban redevelopment. And so we should be interested in them. While we do not directly observe them, here we are able to derive lower bounds on them, by solving the linear programs that underlie them. These bounds are new to the literature, general, easily applicable to observable city aggregates, and efficient. We illustrate one of their many potential uses within the context of the 2016 US presidential election, where we provide some early evidence that Donald Trump’s shifting away from the consensus on global warming has benefited him in those metro areas where decentrists were stronger.

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1 Introduction

A large literature points to the many insights into city aggregates that become possible once one is given information about the city’s residents (e.g., Brueckner (1987)). This paper’s interest is with the reverse idea. We ask, what can be learned about the city’s residents once one is provided information about the aggregate city? We will show that at least one relevant aspect of residents’ preferences – labeled “urban centrism”, and applicable to much more than just the urban political economy of the center – can be gleaned from aggregate city data. We will use linear programming techniques to extract bounds on urban centrism from the observable distribution of the city’s housing across rings around that center. One of these bounds is the “greatest cumulative ring difference”, as in the paper’s title. While these bounds are surprisingly general, they are easily computable, too. They sum over suitable differences in rings’ housing stocks.

Except for also featuring in a companion paper (if from a different angle, Dascher (2017)), these bounds are new to the literature. We justify our interest in deriving them by emphasizing their uses as potential tools in urban political economy. Let us juxtapose residents with their average property close to the city center (centrists) with those with their average property near the periphery (decentrists). Bounding both groups’ strengths – inspecting cumulative ring differences! – will not just help us predict the outcome of a contest over whether to decentralize jobs and shops to the city periphery. Also, we may address the urban political economy of: limiting maximum building height, raising the cost of urban travel, introducing minimum lot size, or combatting urban blight. In short, bounds on centrists and decentrists must be useful because these two groups face off over all sorts of policies that affect them unevenly – policies of which there are many.

Consider limiting building height, for example. Centrists must be opposed, anticipating that it will depress their incomes. Decentrists should welcome it. Less obviously, consider taxing carbon emissions, or introducing a toll. Centrists rightfully anticipate a rise in the value of their real estate that more than offsets the increase in their commuting cost. Decentrists may justifiably expect the opposite. And so centrists will vote for, while decentrists will vote against, the carbon tax proposal, or the toll. Rationing central city land provides yet another, slightly different example. Developing vacant, or even blighted, plots near the center of the city may raise the number of centrists. Decentrists may attempt to block such development, fearing it to strengthen centrists’ ranks (prior to deciding on decentralization of retail, a carbon tax, a height limit, or a road toll).

Retail location policy, urban zoning, road tolls, urban blight or the adequate response to a warming climate are as important as they are contentious. And still we might opt for going on to ignore their centrist-decentrist dimension. We could point to the fact that centrists and decentrists do not easily reveal themselves to the city analyst. Centrists and decentrists reveal themselves by where they own, not by where they live. Public data on landlords’ individual housing portfolios are rarely ever available. Political preferences of owners of multiple properties do not easily map into geographic space. And so we might concede defeat, admitting urban centrism to be vacuous empirically.

This, however, would be an unfortunate route to take. As this paper and Dascher (2017)
argue, we should exploit the information embodied in the city’s observable spatial structure instead. If we look carefully, the distribution of population across city rings will tell us something about urban centrism. This is because centrist and decentrist numbers must be consistent with both aggregate urban housing and the distribution of that housing across rings. In a city with a relatively large share of housing near the periphery, say, there cannot be many centrists; and vice versa. This simple and intuitive idea informs the entire paper. We may never be able to compute centrist or decentrist numbers. But we may be able to bound them.

In fact, we will be able to derive general formulas for best lower bounds on centrists and decentrists, based on the spatial distribution of the city’s housing. Once derived, these formulas can immediately be taken to any city. To compute the smallest number of centrists that could conceivably be hidden away in the city’s fabric, say, follow a simple procedure. First, finely divide the city into \( n \) rings of equal width around the center. Second, compute the difference between housing or population in ring \( i \) and that in ring \( n + 1 - i \), producing “ring difference \( i \)”. Third, sum over the first \( j \) ring differences to obtain the “\( j \)-th cumulative ring difference”. Finally, pick the greatest of all cumulative ring differences. It is this maximum that identifies the minimum conceivable number, and hence a conservative estimate, of centrists.

From this paper’s perspective, at least, this procedure suggests that 68 or more percent of all landlords in the New York metropolitan area were centrists in 2000 (more on this later, in section 6). Our theoretical result not just extends, but also derives independently of (and in that sense reaffirms), Dascher (2017). There the greatest cumulative ring difference is a mere lower bound on centrists. Here it is found to be the best lower bound on centrists. And where there the greatest cumulative ring difference is derived heuristically, here it obtains as the minimum value of the appropriate underlying linear program. We exploit the fact that minimum centrists can be obtained by minimizing centrists (i) given the need to fit all landlords (and their tenants) into the city’s exogenous ring housing stocks while (ii) observing the usual non-negativity requirements.

We also will identify the minimum number of decentrists. The minimum conceivable number of decentrists will be shown to be minus the least cumulative ring difference. Our model suggests, for example, that in 2000 at least 26% of all landlords in the Albuquerque metropolitan area were decentrists (again, more on this in section 6). Bounds on centrists and decentrists may be helpful in the following sense: Should the share of minimum centrists exceed one half of the electorate, then centrists (and their centrist agenda) can with certainty be inferred to prevail. Vice versa, if the share of minimum decentrists exceeds one half of the electorate, then decentrists’ cause wins. And even if neither share attains 0.5, our concepts need not be mute. A given interest group’s likelihood to prevail should still be increasing in its, and decreasing in the opposing group’s, minimum share.

We build on the traditional model of the monocentric city, but in one important respect do we part with it. In the traditional literature, urban housing ownership is modeled in one of two ways. Under “common ownership”, all citizens receive an equal share of overall rental income (e.g. Pines/Sadka (1986)). Or landlords are “absentee” (e.g. Wildasin (1986), Kanemoto (1980)). Somewhat paradoxically, the first regime amounts to assuming that
every resident is a landlord; while the second goes as far as to assume that no resident is one. Of course, these regimes are meant to keep urban political economy tractable (for an overview see Helsley (2004)). Yet it might also be fair to say that they do not model housing ownership at its most consistent. And either regime defines away the large class of urban conflicts in which it is landlords – rather than landlords vs. their tenants – who oppose, and defy, one another.²

One of the theory’s predictions is that regions with a larger minimum share of centrists are more likely to embrace a higher cost of carbon consumption. Urban centrists are no more concerned about global warming than decentrists are. But they are more likely to see their property values rise sufficiently to offset the increased cost of commuting. When translated into the context of the 2016 US presidential election, this suggests that US metropolitan areas with a lesser minimum decentrist share – and a greater minimum centrist share – are more likely to favor Hillary Clinton over Donald Trump. This expectation is borne out only in part in the data. We will find that a receding lower bound on decentrists – but not a growing lower bound on centrists – raises Clinton’s take of the vote.

The paper has eight sections. Section 2 sets out the basic framework. Section 3 lays out two examples. Section 4 provides the general treatment of any city’s urban centrum. This section provides a linear programming solution to minimizing centrists for given city spatial structure. To appreciate that solution’s generality, we briefly note that where the typical linear program operates on real numbers, the programs we will solve here involve parametric information on city aggregates. Section 5 adds an analysis of decentrists’ minimum share. Section 6 illustrates our centrist/decentrist distinction by making use of a uniquely suited data set on population by distance from the center across US metropolitan areas. (But to be sure, it is the theoretical results in sections 4 and 5 that are at the heart of the paper.) Section 7 provides a brief discussion of extensions. Section 8 concludes.

2 Model

Monocentric City. A closed and monocentric city (as pioneered by Wheaton (1973), Pines/Sadka (1986) and Brueckner (1987)) juts \( \tilde{r} \) units of distance out from the CBD (with \( \tilde{r} \) determined shortly). Commuting costs for a resident living at distance \( r \) from the CBD are \( tr \). Ricardian rent \( q \) follows \( q(r) = t(\tilde{r} - r) \). The city’s overall population is \( s \), and the urban wage is \( w \). Residents consume one unit of housing. Housing is built by profit maximizing investors. One unit of capital \( k \) combined with one unit of land yields \( h(k) \) units of housing, where \( h' > 0 \) and \( h'' < 0 \) (again, Brueckner (1987)).

Housing. If \( p \) is the price of capital, investors choose \( k \) so as to satisfy the \( q(r)h_k(k) = p \) necessary for maximum profit. The optimal capital depends on rent \( q \) and price \( p \), and so can be written as \( k(t(\tilde{r} - r),p) \). Let \( h(r) \) be shorthand for the building height obtained

²The literature also addresses those conflicts that frequently arise between owners of developed land and owners of developable land at the urban fringe (e.g. Engle/Navarro/Carson (1992)), or allows for owner-occupiers pitted against each other (as in the case of nimby goods, e.g. Fischel (2005)). In the first case, however, one of the two antagonist types is not a city resident, while in the second tenants are non-existent once more.
for this optimal capital choice. Then the city boundary \( \tilde{r} \) is determined by the condition that the housing market clear,

\[
\int_0^{\tilde{r}} a(r) h(r) \, dr = s,
\]

where \( a(r) \) is land available in a ring of unit width \( r \) units of distance away from the CBD. Ratio \( a(r) h(r)/s \), also written \( f(r) \), indicates the share of the population commuting from within that ring to the CBD. Correspondingly, \( F(r) \) denotes the share of households commuting \( r \) or less.\(^3\) Now divide the city into \( i = 1, \ldots, n \) concentric rings of equal width \( n \) even) around the CBD, with \( n \) large enough to justify treating rent, building height, commuting times etc. as identical across ring \( i \)'s plots. Housing in ring \( i \) is approximately \( f(r_i) s \). We set \( f(r_j) s = b_i \), to conform with the LP notation introduced shortly.

Ownership. Each landlord owns one unit of housing (an “apartment”) that he resides in himself as well as that other that he rents out. These two apartments, to be sure, do not need to locate in the same ring.\(^4\) Realistically, information on a landlord’s two individual properties must be treated as private. And so we cannot say whether the landlord is a centrist or a decentrist. An unknown match matrix \( X = (x_{ij}) \) collects the frequencies with which the various possible matches between landlords and tenants occur, with row \( i \) (column \( j \)) indicating the landlord’s (tenant’s) location. Centrists (decentrists) are those landlords whose average property is closer to (further away from) the center than half the distance from the CBD to the city boundary, \( \tilde{r}/2 \). Hence centrist are those for whom

\[
(r_i + r_j)/2 < \tilde{r}/2
\]

or, equivalently, \( i + j - 1 < n \).\(^5\) An analogous condition applies to decentrists.\(^6\)

Matching. The previous inequality suggests that centrist (decentrists) are to be associated with entries of \( X \) that are located strictly above (below) its counter diagonal, i.e. the diagonal that stretches from \( X \)’s bottom left corner to its top right one. Moreover, being a centrist (or decentrist) does not depend on which apartment is owner-occupied, \( i \) or \( j \). We may conveniently suggest that landlords always occupy the ring that is closer to the city center. With \( i \leq j \), \( X \) becomes upper triangular (see (4) or (5) below). Now, to capture the overall number of households inhabiting ring \( i \) we need to sum over all of \( X \)'s entries in both, row \( i \) and column \( i \). The resulting sum must equal ring \( i \)'s available stock of apartments, \( b_i \). And so ring \( i \)'s housing constraint reads \( \sum_{j=1}^{n} (x_{ij} + x_{ji}) = b_i \).

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\(^{3}\)We assume \( a \) is continuous in \( r \). As \( h \) is (differentiable and hence) continuous in \( r \), so is \( f \).

\(^{4}\)Surely there are many other, often more complex, ways to introduce (i) resident landlords with their (ii) tenants into the city. As emphasized in the introduction, we certainly want to avoid the traditional “common ownership” setup, lest we assume away the class of centrist/decentrist-contests that is the interest of this paper.

\(^{5}\)This follows from assuming that residents in ring \( i \) commute distance \((i - 0.5)\tilde{r}/n \).

\(^{6}\)Note that even as decentrists have properties closer to the city extremes, “extremists” probably is not a better term. – Jacobs (1961) and Breheny (2007) also use the term “decentrists”, though with a very different meaning. For Jacobs, decentrists are those early 20th century urban and regional planners such as Lewis Mumford, Clarence Stein, Henry Wright and Catherine Bauer, who advocated “thinning out large cities” by dispersing their “enterprises and populations into smaller, separated cities or, better yet, towns” (p. 19).
**Linear Program.** Summing over all centrist-related entries in $X$ gives $\sum_{i=1}^{n-1} \sum_{j=1}^{n-i} x_{ij}$, the true, yet unknown, number of centrists, $l^c$. Contrast this with the smallest number of centrists conceivable, $l^c$. That latter number bounds the true number of centrists $l^c$ from below. To identify $l^c$, we minimize the number of centrists given ring housing constraints and the non-negativity requirements $x_{ij} \geq 0$. This translates into the following linear program

$$\min x_{ij} \sum_{i=1}^{n-1} \sum_{j=1}^{n-i} x_{ij} \quad \text{s.t.} \quad \sum_{j=1}^{n} (x_{ij} + x_{ji}) = b_i \quad (i = 1, \ldots, n)$$

$$x_{ij} \geq 0 \quad (i, j = 1, \ldots, n),$$

(3)

analysis of which is the focus of the next two sections.

### 3 The Minimum Share of Centrists, in Two Specific Cities

We run two eight-ring city examples on how to solve the linear program (3) next. These are examples to offer some intuition on how a feasible, and even optimal, solution to linear program (3) plays out. But in fact they are much more than just examples. They motivate a trial solution that later will generalize to any given city.

**Example City 1.** Our first city has “commuting density” $b = (38, 36, 30, 10, 12, 8, 4, 2)$. To this city, matrix $X_1$ in (4), in highlighting eight non-zero entries, suggests one basic feasible solution.\(^7\) We briefly illustrate feasibility. Adding up all entries in row 1 and column 1, for instance, gives $20 + 18 = 38$ or $b_1$, while adding up all entries in row 7 (consisting of zeros only) and column 7 gives just $0 + 4$ or $b_7$. Our feasible solution here displays one feature that we might expect of an optimal solution, notably that (4) assigns the maximum possible weight to entries on the counterdiagonal (in red on screen). This forces centrists’ numbers down as best as we can. We get $x_{18} = \min\{b_1, b_8\} = 2$. Similarly, $x_{27} = 4$, $x_{36} = 8$ and $x_{45} = 10$.

$$X_1 = \begin{pmatrix}
18 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
16 & 0 & 0 & 0 & 0 & 4 & 0 \\
11 & 0 & 0 & 8 & 0 & 0 \\
0 & 10 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

(4)

Put differently, whenever possible we allocate a peripheral apartment in some given outer ring $j, 5 \leq j \leq 8$, to a proprietor who owns her other, second apartment in corresponding inner ring $9 - j$. This must be a necessary property of a centrist-minimizing allocation. (Suppose that $X_1$ violated this property, i.e. suppose $x_{18} = 1 < 2 = \min\{38, 2\}$. Since

\(^7\)Here, as well as in all other match matrices below, entries with no explicit number attached equal zero.
there are no apartments, anywhere, capable of successfully turning a landlord in ring 1 – someone who would otherwise be a centrist – into a decentrist, an opportunity to reduce centrists has irrevocably been wasted.) At the same time, of course, not all apartments in a given peripheral ring \( j \) may be assignable to a landlord in corresponding ring \( n - j + 1 \).

In ring \( j = 5 \), for example, only 10 out of 12 apartments are.

There are \((b_1 - b_8) = 36\) apartments in ring 1 still waiting to be allocated, as are \((b_2 - b_7) = 32\) apartments in ring 2 and \((b_3 - b_6) = 22\) apartments in ring 3. We apportion these remainders to landlords owning both their properties within the same ring. Since any match on the main diagonal accounts for two apartments, we set \(x_{11} = (b_1 - b_8)/2 = 18\), \(x_{22} = (b_2 - b_7)/2 = 16\) and \(x_{33} = (b_3 - b_6)/2 = 11\) (all blue on screen). Note that \(x_{44} = 0\), given that \(x_{45} = 10\) already and that row 4 and column 4 must add up to \(b_4 = 10\). It remains to balance housing in ring 5, by setting \(x_{55}\) to 1 (brown on screen). – Now, invoking the simplex algorithm would reveal that the solution set out in (4) above not just is feasible but also: optimal. Instead of going through these details here, we offer a systematic treatment below (in the following section). We conclude that the trial number of centrists suggested by (4) also is the minimum number of centrists given the specific distribution of housing \(b\) in hand.

Adding up these centrists is simple enough. We merely need to collect the few non-zero entries found above the counterdiagonal. These are conveniently located on the upper half of the main diagonal (blue on screen). This gives \(\sum_{i=1}^{3}(b_1 - b_{n-i})/2 = 45\) minimum centrists. Minimum centrists’ share in city population becomes \(45/140\). Computing minimum centrists provides valuable information here. It is not possible for the true number of centrists to fall short of 45. But it is quite possible – if not utterly likely – for the true number of centrists to surpass 45. Of course, the latter likely occurs should true matches deviate from one of the optimal allocations.

**Example City 2.** Our second example city exhibits housing stocks described by \(b = (38, 14, 30, 10, 12, 8, 26, 2)\). We take an important step towards generalization by introducing the concept of ring difference \(\delta_i\), where \(\delta_i = b_i - b_{n+1-i}\) is the number of apartments in “leading” ring \(i\) minus that in “lagging” or “antagonist” ring \(n + 1 - i\). It is defined for \(1 \leq i \leq 4\). In our second example city, \(\delta_i\) is positive for \(i\) equal to 1 or 3 (since there we have a “surplus”) and it is negative if \(i\) equals 2 or 4 (because then there is a “deficit”).

Contrast this with our first example city, where all first three ring differences are positive.

\[
X_2 = \begin{pmatrix}
18 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 14 & 0 \\
5 & 0 & 0 & 8 & 12 & 0 \\
0 & 10 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(5)

\[\text{It is not, however, a unique optimal solution. For example, letting any landlord trade apartments with her or his tenant would generate another optimal solution.}\]
True to our strategy of emphasizing the counterdiagonal, feasible solution $X_2$ in (5) assigns as many apartments as possible in lagging rings to owners in corresponding leading rings. And because we have a surplus in rings 1 and 3, for these rings this works just fine. All apartments in rings 8 and 6 can be assigned to landlords living in rings 1 and 3, respectively. And while this works less well for apartments in lagging rings 5 and 7, remaining apartments are not always lost on us. Ring 2’s deficit (of $-(b_2 - b_7) = 12$), for instance, we may “save up for”, or “post to”, the next best successive ring boasting a surplus. In our example, this is ring 3 (where $b_3 - b_6 = 22$). The 12 apartments reflecting ring 2’s deficit can valuably be employed to offset the better part of ring 3’s surplus.

And so we set entry $x_{37}$ in $X_2$ to $b_7 - b_2$, or 12 (green). Intuitively, the 12 ring 7-apartments not assignable to ring 2-landlords now are assigned to landlords in ring 3, to at least turn those off centrism. Note that the same is not possible to do with the ring deficit arising in ring 4. There simply are no later rings. – Everything else parallels our discussion of the first example. We balance the first three rings’ housing constraints by setting $x_{11} = (b_1 - b_8)/2 = 18$, $x_{22} = 0$ and $x_{33} = (b_3 - (b_6 + (b_7 - b_2)))/2 = 5$. Again, moreover, the basic feasible solution, set out in (5), also is the optimal one. Minimum centrists are found to sum to 23, if only to see their share in the overall total attain a mere 23/140.

**Review.** What can be learned from these two examples? We have seen that in both cities minimum centrists may be written as the cumulative sum of the first three ring differences, $\sum_{i=1}^{3} (b_1 - b_{9-i})/2$. This is true even as $\delta_2$ is positive in the first example city while negative in the second. But why does it make sense to include $\delta_2$ in either example? The answer is this: On the one hand, including $\delta_2/2$ in the cumulative sum when positive acknowledges the fact that $(b_2 - b_7)/2$ landlords in ring 2 cannot be turned away from centrism. On the other hand, including $\delta_2/2$ in the cumulative sum when negative acknowledges the fact that $(b_7 - b_2)/2$ landlords in ring 3 can (be turned off centrism).

We must also wonder about why $\sum_{i=1}^{4} (b_1 - b_{9-i})/2$ excludes $\delta_4/2$. In particular, why is negative $\delta_4/2$ not included in the second city’s cumulative sum when negative $\delta_2/2$ is? Following our previous intuition, there is no need to “save” ring 5 apartments for later because there are no later surpluses to “swipe away”. The only remaining ring that could possibly feature a centrist landlord is ring 4. Yet here $\delta_4$’s negative sign indicates that the planner can already afford each landlord in ring 4 a ring 5-apartment that successfully counters that landlord’s initial impulse to “go centrist”. And with no further centrists to collect in the fourth ring, our cumulative sum should: stop short of it.

**Tentative Ideas.** Two ideas emerge from this: (i) Minimum centrists can be represented as a cumulative sum of successive ring differences. (ii) Successive ring differences should enter that cumulative sum if they are positive. And they should even enter the cumulative sum if they are negative, as long as they can help “wipe out” subsequent positive ones. Negative ring differences should be included if and only if they are followed by positive ones at least equal in size. I.e., the cumulative sum should include successive ring differences as long as this helps raise the cumulative sum. Equivalently, to identify the minimum number of centrists we must maximize the cumulative sum of ring differences.
4 The Minimum Share of Centrists, Anywhere

Primal vs. Dual Program. We allow for any \( n \times 1 \) vector of ring housing stocks \( b = (b_1, \ldots, b_n) \) now, except for ruling out any \( b_i \) to equal zero. We then put the corresponding linear program (3) into standard form. We first stack all \( n \) columns of \( X \) into one long \( (n^2 \times 1) \) vector \( x \). This gives \( x' = (x_{11}, \ldots, x_{1n}, \ldots, x_{n1}, \ldots, x_{nn}) \). To address the objective function in (3) in matrix notation, let \( c_i \) equal an \( n \times 1 \) vector consisting of ones only except for the last \( i \) entries, which are zero instead. For example, \( c_3 \) is a list of \( n - 3 \) ones followed by three zeros, i.e. \( c_3' = (1, \ldots, 1, 0, 0, 0) \). Define an \( n^2 \times 1 \) list of weights \( c \) by setting \( c' = (c_1', \ldots, c_n') \). Then our objective function \( \sum_{i=1}^{n-1} \sum_{j=1}^{n-i} x_{ij} \) can be cast as the product \( c'x \).

Next, let \( \tau_i \) denote an \( n \times 1 \) vector featuring 2 in its \( i \)-th row and 1 in all other rows. For example, \( \tau_2' = (1, 2, 1, \ldots, 1) \). Moreover, let \( J_i \) denote what becomes of the \( n \times n \) identity matrix once row \( i \) has been replaced with \( \tau_i' \). Then the coefficient matrix \( A \) is \( A = (J_1, \ldots, J_n) \); it is of dimensions \( n \times n^2 \). The tableau in Table (1) illustrates \( A \) in its bottom left part. This table also indicates our specific vector of objective function weights \( c \) (in its first row) as well as the vector of ring housing stocks \( b \) (last column).

\[
\begin{array}{ccccccccccc}
1 & 1 & 1 & 1 & \ldots & 1 & 0 & \ldots & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & b_1 \\
2 & 1 & 1 & 1 & \ldots & 1 & 0 & \ldots & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & b_2 \\
0 & 1 & 0 & 0 & \ldots & 0 & 0 & \ldots & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & b_3 \\
0 & 0 & 1 & 0 & \ldots & 0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 & 0 & b_4 \\
\vdots & & & & \ddots & & \ddots & & \vdots & \ddots & \vdots & & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & 1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & \cdots & 0 & b_{n-1} \\
0 & 0 & 0 & 0 & \ldots & 0 & 1 & \ldots & 1 & 1 & 1 & \ldots & 1 & 2 & b_n \\
\end{array}
\]

Table 1: Matrix \( A \), objective function weights \( c \) and housing stocks \( b \)

With this extra notation in hand, linear program (3) may equivalently be stated as \( \min_x c'x \) subject to \( Ax = b \) and \( x \geq 0 \), where the equality constraints may also be read off Table (1)’s rows. This program’s dual is \( \max_y y'b \) such that \( y'A \leq c' \), where \( y \) is the dual’s \( (n \times 1) \) vector of unknowns, \( y' = (y_1, \ldots, y_n) \). Table (1) also indicates the dual’s constraints; these can be read off its columns. For instance, the constraint complementary to \( x_{11} \) being strictly positive simply is \( 2y_{11} \leq c_{11} = 1 \) (see first column in Table (1)).

Rather than immediately analyze the general case, we focus on a seemingly special case first. This case also allows us to best connect with the principles that emerge from our discussion of the two example cities (section 3). To address this special case, let us introduce the partial cumulative sum \( \Delta(i) = \sum_{j=1}^{i} \delta_j/2 \). This sum cumulates successive ring differences \( \delta_j \) up to ring \( i \), where of course \( i \leq n/2 \). And let index \( i^* \) be the index that maximizes this cumulative sum, i.e.

\[
i^* = \arg \max_i \sum_{j=1}^{i} (b_j - b_{n+1-j})/2.
\]

8
Our point of departure on the way to the fully general solution is a city for which (i) \( \Delta(i^*) > 0 \) and (ii) all ring differences \( \delta_i \) are negative except when \( i = i^* \), when \( \delta_{i^*} > 0 \).

**Trial Solution.** We set out a basic feasible solution to the primal problem next. Table (2) shows \( X \) in tabular form and may be a useful reference as we go along. Again, entries of \( X \) never addressed are zero. Moreover, also note the formal resemblance between Table (2) on the one hand and matrices \( X_1 \) and \( X_2 \) on the other. Now, we begin by considering the elements on the counterdiagonal of match matrix \( X \). Here we set (red on screen)

\[
x_{i,n+1-i} = \min \{b_i, b_{n+1-i}\} \quad (i = 1, \ldots, n/2).
\]

(7)

Given our sign assumptions regarding the \( \delta_i \), this entails setting all entries \( x_{1,n} \) “up” to \( x_{i^*-1,n+2-i^*} \), and again from \( x_{i^*+1,n-i^*} \) to \( x_{n/2,n/2+1} \), equal to the leading ring’s stock, \( b_i \). Only \( x_{i^*,n+1-i^*} \) becomes the lagging ring’s stock, \( b_{n+1-i^*} \). Note how this assignment makes as many owners of property in leading rings (voters who otherwise likely are centrists) as possible disavow centrisn.

Moreover, set (green on screen)

\[
x_{i^*,n+1-i} = (b_{n+1-i} - b_i) \quad (i = 1, \ldots, i^* - 1).
\]

(8)

Note that the expressions on the r.h.s. represent ring deficits. Deficits originating in rings prior to \( i^* \) are posted to leading ring \( i^* \), as the earliest next ring offering up an excess. “Apartment savings” originating in rings up to \( i^* \) then are matched up with apartments in ring \( i^* \). This generalizes how we proceeded earlier when setting \( x_{37} \) equal to 12 in example city 2.

Next, let (blue on screen)

\[
x_{i^*i^*} = \left( b_{i^*} - (b_{n+1-i^*} + \sum_{k=1}^{i^*-1} (b_{n+1-k} - b_k)) \right)/2,
\]

(9)

or \( \Delta(i^*) \). At first sight nothing seems to preclude \( x_{i^*i^*} \) from being strictly negative, in contradiction to primal variables’ non-negativity constraints. However, recall that \( i^* \) maximizes the cumulative sum of ring differences. And so \( \sum_{j=1}^{i^*} \delta_j /2 \geq 0 \), i.e. a non-negative number. And note that this latter number just coincides with the r.h.s. of (9). Put yet differently, ring excess \( \delta_i \) is more than sufficient to offset the ring deficits \( \delta_k \) associated with, and inherited from, all the rings prior to \( i^* \). And so \( x_{i^*i^*} \) really is non-negative.

At last we set (brown on screen)

\[
x_{n+1-i, n+1-i} = (b_{n+1-i} - b_i)/2 \quad (i = i^* + 1, \ldots, n/2).
\]

(10)

Ring deficits originating in rings following \( i^* \) are relegated to main diagonal elements below the counterdiagonal, to the desirable effect of contributing nothing to the number of centrists. Now, we claim that equations (7), (8), (9) and (10) set out an optimal solution of the primal.
**Trial Solution is Feasible.** To confirm feasibility, we take a brief (nonetheless exhaustive) tour through the city’s rings, i.e. through all row/column pairs of $X$. (Here again, consulting Table (2) may be helpful.) Consider any row/column pair (i.e. ring) with $i = 1, \ldots, i^* - 1, i^* + 1, \ldots, n/2$ first. The single positive entry here is the counterdiagonal entry, set equal to $b_i$ (see (7)). Turn to row and column $i^*$ next. Non-zero entries can be found on the main diagonal (see (9)), on the counterdiagonal (see (7)) and “towards the end of” row $i^*$ (see (8)). Summing over them, by the very design of (9), just yields $b_{i^*}$. This completes our discussion of housing constraints in leading rings $i = 1, \ldots, n/2$.

We cover lagging rings $i = n/2 + 1, \ldots, n$ next. First consider some lagging ring $i = n/2 + 1, \ldots, n - i^*$. Here we identify two positive entries, one located on the main diagonal and the other located on the counterdiagonal (see (10) and (7), respectively). Adding them gives $b_i$. Next consider both row and column $n - i^*+1$. Here the single strictly positive entry is $x_{i^*,n-i^*+1} = b_{n-i^*+1}$, on the counter diagonal by (7). At last, address rows and columns $i = n - i^* + 2, \ldots, n$. There is nothing to consider in those remaining rows. But in each of these columns, strictly positive entries feature twice, i.e. towards “the end of” row $i^*$ and on the counterdiagonal (see (8) and (7)). These two entries also sum to $b_i$. We conclude that all housing constraints are met, and that the landlord-tenant matching set out by equations (7), (8), (9) and (10) is feasible. Let $\bar{x}$ denote this feasible vector.

**Complementary Slackness.** We invoke complementary slackness between the primal and the dual. For $i = 1, \ldots, n/2$, entries on the counterdiagonal $x_{i,n+1-i}$ are strictly positive (see (7)), as is the main diagonal element $x_{i^*,i^*}$ (see (9)). By complementary slackness, the corresponding constraints of the dual – read off the corresponding columns

<table>
<thead>
<tr>
<th>Ro./Co.</th>
<th>1</th>
<th>$i^*$</th>
<th>$n/2 + 1$</th>
<th>$n - i^*$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\ldots$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td>$x_{i^<em>,i^</em>}$</td>
<td>0</td>
</tr>
<tr>
<td>$i^*$</td>
<td>$x_{i^<em>,i^</em>}$</td>
<td></td>
<td>$x_{i^<em>,n+1-i^</em>}$</td>
<td>$\ldots$</td>
<td>$x_{i^<em>,n^</em>}$</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>$x_{i^<em>,i^</em>+1}$</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\ldots$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>$x_{n/2+1,n/2+1}$</td>
<td></td>
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<tr>
<td>$n - i^*$</td>
<td></td>
<td></td>
<td></td>
<td>$x_{n-i^<em>,n-i^</em>}$</td>
<td>0</td>
</tr>
<tr>
<td>$\ldots$</td>
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<td></td>
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<tr>
<td>$n$</td>
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<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Non-Zero Elements in Basic Feasible Solution
of Table (1) – must be met with equality, and so
\[ y_i = -y_{n+1-i} \quad (i = 1, \ldots, n/2) \quad \text{and} \quad y_{i^*} = 1/2. \] (11)

These equations specify the weights on ring housing stocks \( b_i \) in the dual’s objective.

For \( i = 1, \ldots, i^*-1 \), entries \( x_{i^*,n+1-i} \) are strictly positive, too (see (8)). Again, by complementary slackness, corresponding constraint inequalities in the dual become binding. And so, according to Table (1), \( y_{i^*} = -y_{n+1-i} \). Combining this with \( y_{n+1-i} = -y_i \) and the fact that \( y_{i^*} = 1/2 \) (see (11)) gives the first set of equations in (12). At last we make use of equations (10). For \( i = i^* + 1, \ldots, n/2 \), constraint (in)equalities translate into \( y_i = 0 \).

Joint with the first set of equations in (11), this in turn implies the second set of equations in (12):
\[ y_i = 1/2 \quad (i = 1, \ldots, i^*-1) \quad \text{and} \quad y_i = 0 \quad (i = i^* + 1, \ldots, n - i^*). \] (12)

Table (3) collects the full solution to equations (11) and (12), denoted \( \bar{y} \).

| \( i \) | 1 | \ldots | \( i^* \) | \( i^* + 1 \) | \ldots | \( n - i^* \) | \( n - i^* + 1 \) | \ldots | \( n \) |
|---|---|---|---|---|---|---|---|---|
| \( \bar{y}_i \) | 1/2 | 1/2 | 1/2 | 0 | 0 | 0 | \(-1/2\) | \(-1/2\) | \(-1/2\) |

Table 3: The dual’s optimal solution

Next we show that \( \bar{y} \) is feasible. Each constraint at most involves two elements of \( y \). So consider constraints of the type \( \bar{y}_i + \bar{y}_j \leq 1 \). Since either \( \bar{y}_i \) and \( \bar{y}_j \) at best equal \( 1/2 \), their sum cannot exceed 1. Next, consider constraints of the type \( \bar{y}_i + \bar{y}_j \leq 0 \). Note first that whenever \( c_{ij} = 0 \), corresponding entries \( x_{ij} \) of \( \bar{X} \) are on or below its counter diagonal and so \( i + j \geq n + 1 \). By Table (3), if \( y_i = 1/2 \) then \( i \leq i^* \). But then \( j \geq n + 1 - i^* \) and hence, again by Table (3), \( \bar{y}_j = -1/2 \). And so \( \bar{y}_i + \bar{y}_j = 0 \). Finally, by Table (3), whenever \( \bar{y}_i = 0 \) then \( i^* + 1 \leq i \) and so we must have \( j \geq n - i^* \) one last time by Table (3). But then \( \bar{y}_i + \bar{y}_j \) equals zero or \(-1/2\). We conclude that \( \bar{y} \) is feasible. That is, \( \bar{y}'A \leq c' \).

**Basic Feasible Solution is Optimal.** Let us now put together feasibility and complementary slackness, using a standard argument in linear programming (Chvatal (1980), Luenberger/Ye (2016), Hadley (1963)). First, feasibility of \( \bar{x} \) and \( \bar{y} \) implies \( b = A\bar{x} \) and \( \bar{y}'A \leq c' \), respectively, and hence \( \bar{y}'b = \bar{y}'(A\bar{x}) = (\bar{y}'A)\bar{x} \leq c'\bar{x} \). Second, complementary slackness implies \( (\bar{y}'A - c')\bar{x} = 0 \) or \( (\bar{y}'A)\bar{x} = c'\bar{x} \). And so we may conclude that \( \bar{y}'b = c'\bar{x} \). This in turn implies that \( c'\bar{x} \) equals minimum centrists, and hence that \( \bar{x} \) solves (3). Of course, if \( \bar{x} \) is optimal, then so is \( \bar{y} \), justifying Table (3)’s title.

We compute the objective function values for primal and dual, providing a check on optimality of \( \bar{x} \) and \( \bar{y} \) as well as, of course, the desired minimum number of centrists itself. On the one hand, summing over all entries above the counter diagonal the objective function value in the primal gives \( x_{i^*i^*} \) as on the r.h.s. of equation (9). But then:
\[ l^c = \Delta(i^*) = \max \sum_{i=1}^{i^*} \left( b_j - b_{n+1-j} \right)/2. \] (13)
On the other hand, computing the sum of ring stocks using the optimal weights in (11) and (12) yields the very same formula, i.e. $\sum_{j=1}^{i^*} (b_j - b_{n+1-j})/2$. This formula represents the optimal value of both primal and dual. And so it also represents the minimum conceivable number of centrists. We briefly pause to appreciate its generality: the greatest cumulative ring difference gives a universal closed form solution for minimum centrists. It provides an observer of an arbitrary given city with a prediction of centrists’ minimum.

Our proof is for a city whose ring differences, with the exception of $\delta_i$, are all negative (also see the first two rows in Table (5) in the Appendix). The Appendix shows how the proof quickly generalizes. Subsections 10.1 through 10.3 show that our results in essence remain unchanged as some, or even all, ring differences exhibit an arbitrary sign. Formula (13) remains valid throughout. This is quite straightforward since also accounting for positive ring differences (Appendix) is simpler than accounting for negative ones (this section): witness solution $X_1$ as opposed to $X_2$ (in section 3). Now, translating minimum centrist numbers in formula (13) into minimum centrists’ share in all landlords, by dividing $\Delta(i^*)$ by $s/2$, gives the following variant of this result:

**Proposition 1: (Greatest Cumulative Ring Difference and Centrists)**

Centrists’ minimum conceivable share of the landlord population, $\lambda_c$, is given by the greatest cumulative ring difference, $\lambda_c = \max_i \sum_{j=1}^{i} (b_j/s - b_{n+1-j}/s)$.

Proposition 1 extends Dascher (2017), where $\lambda_c$ is introduced a mere lower bound to centrists’ true number. We here add that $\lambda_c$ even is the best lower bound (because it is the minimum). This makes us more confident to work with $\lambda_c$ empirically (section 6).

## 5 Centrists vs. Decentrists

**Minimum Decentrists.** We bring in decentrists now. Intuitively, where before we have used $b_{n+1-i}$ to “swipe away” or “swamp” potential centrists in $i$ (as best as we could), conversely we now use $b_i$ to “swamp” decentrists in $n+1-i$ (as best as we can). Applying a proof similar to that in section 4 (omitted for brevity), we find that minimum decentrists correspond to: minus the least cumulative ring difference. That is, if $i^{**} = \arg \max_i \sum_{j=1}^{i} (- (b_j - b_{n+1-j}))/2$, then minimum decentrists $\lambda_d$ are equal to

$$\lambda_d = -\Delta(i^{**}) = -\min_i \sum_{j=1}^{i} (b_j - b_{n+1-j})/2.$$  \hspace{1cm} (14)

Translating this number into a share gives

**Proposition 2: (Least Cumulative Ring Difference and Decentrists)**

Decentrists’ minimum conceivable share of the landlord population, $\lambda_d$, is given by minus the least cumulative ring difference, $\lambda_d = -\min_i \sum_{j=1}^{i} (b_j/s - b_{n+1-j}/s)$.
Upper Bounds. We quickly turn lower bounds in Propositions 1 and 2 into corresponding upper bounds. Subtracting centrists from overall landlord population $s/2$ gives the sum of decentrists and indifferent landlords. This in turn is the sum of all elements of $X$ strictly below or on the counter diagonal. The following linear program looks for the maximum sum of decentrists/indifferents:

$$\max_{x_{ij}} \left( s/2 - \sum_{i=1}^{n-1} \sum_{j=1}^{n-i} x_{ij} \right) \quad \text{s.t.} \quad \sum_{j=1}^{n} (x_{ij} + x_{ji}) = b_i \quad (i = 1, \ldots, n)$$

$$x_{ij} \geq 0 \quad (i, j = 1, \ldots, n). \quad (15)$$

Comparing linear programs, clearly the maximizer to (15) coincides with the minimizer to (3). But this implies that $s/2 - l^c$ is the maximum conceivable number of decentrists/indifferents. And so $s/2 - l^c$ is an upper bound to decentrists only (Proposition, Part (ii)). A similar argument suggests that $s/2 - l^d$, where $l^d$ is the minimum number of decentrists, is an upper bound to centrists (Proposition 3, Part (i)).

Proposition 3: (Upper Bounds on Centrists and Decentrists)

(i) $\lambda^c$ is bounded from above by $1 - \lambda^d$. (ii) $\lambda^d$ is bounded from above by $1 - \lambda^c$.

Disagreeing on Carbon Tax. One of the many dimensions along which centrists and decentrists differ (section 1) is the appropriate response to global warming. Should a carbon tax be introduced? Such a tax would also raise the cost of urban commutes, from $t$ to $t' = t + t'$. Consider a landlord who lives in a property in ring $i$ (and hence has commuting cost $t r_i$), and rents out another to a tenant in ring $j$ (and so receives $t (\hat{r} - r_i)$). Her (net) income becomes $w + 2t (\hat{r}/2 - (r_i + r_j)/2)$. Whether or not she will welcome the increase in $t$ will clearly depend on whether or not her average property distance from the center $(r_i + r_j)/2$ is less than $\hat{r}/2$, i.e. on whether or not she is a centrist.

Centrists and decentrists constitute only half of the electorate. Tenants, as the other half, see their real income dwindle as the urban cost of living $t\hat{r}$ rises. At the same time, there are extra benefits to taking $t$ to $t'$. Taxing urban commutes helps fight global warming (as the city structure defined in (1) gradually adapts over time) or at the very least provides a psychological benefit. Taxing urban commutes also generates tax revenue, part of which might be redistributed to the electorate.

We adopt a random voter turnout perspective, as in Brueckner/Glazer (2008). Let the (carbon tax) policy’s probability of electoral success $\pi$ be an increasing (decreasing) function of the centrist share $\lambda^c$ (decentrist share $\lambda^d$). More specifically, $\pi = G(\lambda^c, \lambda^d)$ with partial derivatives $G_1 > 0$ and $G_2 < 0$. This we combine with the additional (plausible yet by no means forceful) assumption that $\lambda^c$ is increasing in $\hat{\lambda}^c$, and that $\lambda^d$ is increasing in $\hat{\lambda}^d$. This implies $\pi = G(\hat{\lambda}^c, \hat{\lambda}^d)$, again with partial derivatives $G_1 > 0$ and $G_2 < 0$. We then consider the linearized version of

Empirical Model of Voting on a Carbon Tax:

The probability of the carbon tax proposal’s electoral success $\pi$ is increasing (decreasing) in centrists’ (decentrists’) minimum share in the electorate $\hat{\lambda}^c, \hat{\lambda}^d$. 

13
During the campaign for the 2016 U.S. presidential election, Donald Trump certainly was the candidate more prone to side with those who reject a carbon tax. At the same time, Hillary Clinton was the one more likely to raise the cost of carbon consumption. Clinton appears to have been centrist’s favorite candidate, Trump must have been decentrist’s favorite. We compute Clinton’s share in all votes cast in support of either Clinton or Trump using Dave Leip’s data set on the 2016 U.S. presidential election. Table (8) illustrates Clinton’s share in the overall vote. For instance, in half of all metro areas Clinton captured no more than 43% of votes cast for either herself or Trump. Of course, the metro areas in which Clinton did not do well also are the metro areas that are populated less, and so this observation is consistent with Clinton winning the overall popular vote.

Data on population, as well as population-weighted densities, by distance (in miles) from the city center are provided by the U.S. Census Bureau for all U.S. metropolitan areas and years 2000 and 2010 (also see Wilson (2012)). We focus on 2000 below but results do not change much if 2010 is used.) This data set is ideally suited to the purposes of this paper. We define the city boundary as the index of the last ring exhibiting a population weighted density of greater than 500. Table (8) also sketches out the distribution of metropolitan area sizes in our sample of 336 metro areas, i.e. of the number of rings the agglomeration area commands up to its boundary. On the one hand, half of all metro areas in the sample extend six miles out from the CBD – or less. On the other hand, some metro areas are quite large in terms of their area. For example, the largest metro area in the sample (Miami – Fort Lauderdale – Pompano Beach) commands $\tilde{r} = 80$.

We next compute both bounds $\lambda^c$ and $\lambda^d$ for every metropolitan area (Propositions 1 and 2). Table (8) sketches our bounds’ distributions. In more than three fourths of all metro areas does the minimum share of centrists fail to cross the 0.5-threshold. Nor can decentrists claim to be decisive often. Three fourths of metro areas exhibit a minimum decentrist share of 10% or less. The fourth row in Table (8) provides some information on metros’ “bands of confidence”. The true number of centrists is owned by $[\lambda^c, 1 - \lambda^d]$ (Proposition 3, Part (i)). The larger the interval in absolute size, the less precise our estimate of centrists’ true number. According to Table (8), the smallest interval size is 0.21, while the largest is 1. For half of all metro areas, interval size is 0.81 or more, i.e. certainly for these metro areas we do not pin down $\lambda^c$ very well.

Figure (1) provides some extra illustration, in mapping the distance of four large metro areas’ rings from the metro area center into rings’ housing shares $b/s$. This figure also

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9Following the New York Times during the campaign suggests as much. For example, see “Climate Change Divide Bursts to the Forefront in Presidential Campaign”, New York Times August 1st, 2016.

10This dataset provides votes at the county level. We aggregate these data for metropolitan areas.

11Model and data set properties certainly do not even agree roughly. Metro areas clearly are far from monocentric (Glaeser/Kahn (2004)). Moreover, neither will tenants constitute exactly one half of the population, nor will housing ownership be evenly distributed across landlords. (Yet again, considering everyone a homevoter (as is sometimes done) seems even less appropriate.)

12However, the first ring was included even if that ring’s weighted density fell below 501. Also, note that our cutoff is not innocuous, see below.

13For some 30 metro areas, one or more variables are missing, and so these observations were dropped.
Figure 1: Population by Distance from Center, and Bounding Centrists

shows corresponding lower and upper bounds on centrist shares, i.e. \( \lambda^c \) and \( 1 - \lambda^d \). Not all of these barcharts conform with intuition. Phoenix may be sprawling, and Boston may be compact at the center, yet corresponding metro areas exhibit comparable minimum centrist shares, of 45% and 44%, respectively. As the barcharts show, very different city shapes can conceal, or give rise to, very similar minimum centrist shares. We also see that commuting densities for Houston and Detroit are much less amenable to centrism, and for Detroit we may even state that at best 99% of all landlords could be centrist. Dascher (2017) highlights a relationship between a city’s physical shape and its urban political economy that seems quite apparent from these barcharts: metropolitan areas whose commuting densities are more skewed to the right tend to house more centrists.

With the simplest possible specification (the linear probability model) in mind, we next regress Clinton’s share of votes on both our bounds as well as (i) the share of whites in metro population, (ii) average income, (iii) the share of those who completed a bachelor’s degree in metro population, (iv) metro area size population (for 2000) and even (v) metro area average density (for 2000).\(^{14}\) We use the share of whites to capture Trump’s resonance among white voters, average income to address a lack of taste for redistribution, we use

\(^{14}\)These five covariates closely follow the early discussion in Florida (2016). Data on average income by metro area are from 2015, and were retrieved from the Bureau of Economic Analysis. Data on bachelor degrees in metro areas relate to those who are 25 years or older and are from the American Community Survey’s “Educational Attainment Package” for 2015. Metro size is computed from the same U.S. census data set underlying our computation of the city boundary \( \tilde{r} \). Data on metro average density (for 2000) and whites in total population are also provided by U.S. Census Bureau.
Table 4: OLS Regressions of ShareClinton on minimum centrists and decentrists

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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</table>

bachelor degrees to proxy for voters’ resilience against populist slogans as well as metro size and urban density to capture minorities’ greater attraction to larger, and denser, urban areas.

OLS regressions (1) through (6) in Table (4) explore the role of minimum centrists and decentrists for Clinton’s tally. Column (1) shows that a greater minimum share of centrists increases Clinton’s share of votes, while a greater minimum share of decentrists decreases it, and both estimates are significant (certainly at the 10 percent level; $t$-statistic in parentheses). For instance, observing minimum decentrists to go up by ten percentage points permits us to roughly predict a 1.16 percentage point drop in Clinton’s vote share. At the same time it is true that our bounds shed light on a tiny fraction of the overall variation of Clinton’s performance across metropolitan areas only. We explain more of this variation when adding the share of whites (see column (2)). This also leaves our coefficient estimates for $\lambda^c$ and $\lambda^d$ largely unchanged. The same is true after also including average income as an additional regressor, in column (3). Column (4) includes the share of bachelor degree recipients among those who are 25 years or older. Adjusted R-squared virtually jumps from 21% to 50%, indicating the contribution of education to the election outcome. Our coefficient for $\lambda^c$ turns insignificant (even at the 10% level). Intuitively, the fact that $\lambda^c$ positively correlates with density, income, city size and education contributes to our overestimating its influence in regressions (1) through (3). At the same time, our coefficient for $\lambda^d$ never turns insignificant (at the 10% level). These observations remain true as we add further controls. The minimum share of decentrists continues to be significant, and the minimum share of centrists continues to be insignificant, as we include our two agglomeration-related controls, i.e. metro size.
(column (5)) and average metropolitan population-weighted density (column (6)).\textsuperscript{15} It is in that sense that we tentatively suggest that the US urban landscape – as reflected by its minimum share of decentrists – may have contributed to Hillary Clinton’s defeat.\textsuperscript{16}

Propositions 1 and 2 are the paper’s core. This section is incomplete in too many ways to offer definite results, and is meant to offer a first test only. A number of issues, best addressed in a separate paper, deserve more attention. First, we want to replace our linear probability model in Table (4) here with a probit approach. Second, bounds $\lambda_c$ and $\lambda_d$ call for techniques that allow regressors to be intervals. Third, results in Table (4) are not robust with respect to the urban boundary’s definition. Replacing the current cutoff of “ring density greater than 500” with “ring density greater than 1,000” inhabitants per square mile renders the coefficients of both $\lambda_c$ and $\lambda_d$ insignificant in columns (4) through (6).\textsuperscript{17} Fourth, other specifications might be tried lest omitted variables bias our results. E.g., one might argue for a difference-in-differences approach, by tracing the difference between Democratic and Republican votes across 2012 and 2016 and attributing the inevitable “difference in differences” to bounds $\lambda_c$ and $\lambda_d$.\textsuperscript{18}

7 Discussion

Formulas (13) and (14) lend themselves to straightforward sensitivity analysis. For example, developing a small amount of extra housing $db_i$ in any of the rings $1, \ldots, i^*$ amounts to augmenting the stock $b_i$ in these rings. As formula (13) reveals, this raises minimum centrists by $db_i/2$. Centrists’ minimum number goes up by the very same amount the number of landlords does. And so minimum centrists’ share in the landlord population $\Lambda_c$ goes up, too. Of course, housing construction in lagging rings $n-i^*+1, \ldots, n$ has just the opposite effect. A similar argument, of course, applies from the decentrist perspective.

All of this suggests that, prior to deciding on policy, centrists should seek to strengthen their ranks first, e.g. by encouraging housing construction near the city center (where likely $i<i^*$ holds). This not necessarily raises centrist; but it does raise minimum centrists, i.e. members that centrists as an interest group can be sure of. The following brief example from Berlin, Germany, may be instructive. In 2014, a majority of Berlin voters struck down the proposal to develop the former Tempelhof airfield (e.g. Nitsch (2009)). Elder

\textsuperscript{15}Coefficients for our controls are not our focus, but we may briefly note that our estimates of almost all of them (the exception being metro size) conform with what we would expect: A metro area tends to vote more strongly for Clinton if it is (i) less white, (ii) poorer (less rich), (iii) more educated and (iv) denser.

\textsuperscript{16}Florida (2016) appears to suggest as much when arguing that the share of those “who drive alone”, as an indicator of sprawl, should reduce Clinton’s share of the vote. – Somewhat more explicitly here, where in the previous presidential election Mitt Romney and Barack Obama appeared to agree on the necessity of combatting climate change, Trump’s strategic shifting away from this consensus and embracing the climate sceptic’s position may have secured him the crucial extra vote that put him into office.

\textsuperscript{17}Not disclosing this would make us guilty of “fishing”, according to Goldberger (1991).

\textsuperscript{18}These bounds should play no role as long as both candidates agree on their stance towards climate change, i.e. should have played no role in the 2012 or 2008 election. But they must have played a role in 2016. To quote from the aforementioned New York Times article, “The divide between the two parties over the issue [in the fall of 2016, the author] is the widest it has been in the decades since it emerged as a public policy matter. That is all the more remarkable given that during the 2008 election, the Democratic and Republican positions on climate change were almost identical.”
citizens pointed to the airfield’s importance for the Berlin Airlift 1948; environmentalists highlighted the airfield’s role as a park or “urban cold island”.

At the same time, the former airfield is large, very near the city’s center, and endowed with an abundance of infrastructure. Two subway lines, a commuter railway line, and a highway run along its edges. Nowhere else would housing make more sense than here, in the immediate vicinity of a transportation hub that connects residents with many of the city’s jobs. Nor are alternative public spaces/parks particularly scarce. From a traditional urban political economy perspective (tenants vs. landlords), voters’ decision not to develop the airfield seems difficult to explain. Not so from this paper’s perspective:

Centrists’ minimum share was 47% in 2002 (Daminger (2017)). This at least raises the possibility that a narrow majority of decentrists decided to prevent the airfield’s development, anticipating that such construction would push centrists’ share in the total vote over the 50% threshold and so put an end to the decentralization underway. (Tenants can be expected to be indifferent.) This connects with recent work on how an urban majority may attempt to drive away its minority complement by *choosing inefficient policies, only to cement its hold on the city*. In Brueckner/Glazer (2008), an incumbent public-good-savvy majority picks a level of the local public good that is excessive even by its own preferences, only to induce members of the less-savvy minority to leave. In Kahn (2011), liberal cities on the American West coast choose excessive zoning, only to keep out those who might be slightly less liberal.

8 Conclusions

Introducing a centrist/decentrist-distinction into the urban model may contribute to understanding better diverse urban policies such as rationing central city land, decentralizing retail, tightening building height limits, implementing minimum lot size, or introducing road tolls or carbon taxes. In an effort to assess centrists/decentrists’ relative strengths, this paper derives formulas for the minimum shares of centrists and decentrists from aggregate data on population by distance from the city center. These shares turn out to be efficient lower bounds, and are simple to apply to any arbitrary city. The paper’s empirical section, drawn from observations on the 2016 presidential election and the role climate change plays in it, provides a first and early test of these bounds’ usefulness.

We add that both bounds have some merit in also capturing urban physical form (see Dascher (2017)) or reflecting urban compactness (e.g. Dantzig/Saaty (1974)). Future research might address the mutual causality between urban political economy and urban centrism. Centrists and decentrists are not just likely to disagree on whether to introduce building height restrictions, say. They are just as likely to be shaped by them (as appears to be implied by the analysis of height restrictions in Brueckner/Bertaud (2005) and Borck (2016)). Ultimately some cities may suffer from cumulative causation. A slightly greater share of decentrists initially may imply building height restrictions that feed back into ever smaller numbers of city center advocates, high rises, residents, jobs, and shops.
9 Literature


10 Appendix

10.1 Cumulative Ring Difference

We introduce some extra notation first. Consider the cumulative sum $\Delta(h)$, for some $h$ between 1 and $n/2$. Suppose $\Delta(h)$ is preceded by some other cumulative sum $\Delta(g)$ that is greater than it, i.e. $\Delta(h) < \Delta(g)$ for $g < h$.\footnote{We define $\Delta(0) \equiv 0$. Even the first ring may be overshadowed, by $\Delta(0)$, if $\delta_1 < 0$.} Borrowing terminology established in the context of the “Rising Sun Lemma” (Spivak (1994)), then we will say that the cumulative sum up to, and including, ring difference $h$ is “in the shadow of” the cumulative sum up to, and including, ring difference $g$.\footnote{In our first example city, the cumulative ring difference at 4 is overshadowed (by the cumulative ring difference at 3, say), while in the second example city cumulative ring differences at 2 and 4 are (by}

Of course, there will be rings that are never overshadowed. Among those, ring $i^*$, defined in equation (6), is the one exhibiting the greatest cumulative sum, $\Delta(i^*)$. For the two numerical cities in section 3, to give an example, $i^* = 3$.

10.2 Not All Ring Differences Negative

In the main text we took the first step towards a fully general analysis. Our point of departure was the city of the type spelt out in Table (5). The table header has the ring difference index $i$, the second row provides ring difference $\delta_i$’s sign, and the third row indicates whether or not the corresponding cumulative ring difference $\Delta(i)$ is overshadowed (● is a suggestive shorthand) or not (○). As mentioned above, in this city all ring differences both prior to $i^*$ and beyond $i^* + 1$ are negative and overshadowed.

Nothing of substance changes if one (or more) of those shadow differences is (are) positive, rather than negative. To see this we turn to the city set out in Table (6) below, with the second ring the one ring to have flipped its sign. We assume that everything else remains the same, and so $\Delta(2) < \Delta(0)$ while $i^*$ keeps maximizing $\Delta(i)$.


\cite{Wildasin1986} Urban Public Finance, Chur: Harwood.

Table 5: A Parametric City

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>$i^{*} − 1$</th>
<th>$i^{*}$</th>
<th>$i^{*} + 1$</th>
<th>$i^{*} + 2$</th>
<th>$\ldots$</th>
<th>$n/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_i$</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$\Delta(i)$</td>
<td>§</td>
<td>§</td>
<td>§</td>
<td>§</td>
<td>○</td>
<td>§</td>
<td>§</td>
<td>§</td>
<td>§</td>
</tr>
</tbody>
</table>

Table 6: Negative and Positive Ring Differences

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>$i^{*} − 1$</th>
<th>$i^{*}$</th>
<th>$i^{*} + 1$</th>
<th>$i^{*} + 2$</th>
<th>$\ldots$</th>
<th>$n/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_i$</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$\Delta(i)$</td>
<td>§</td>
<td>§</td>
<td>§</td>
<td>§</td>
<td>○</td>
<td>§</td>
<td>§</td>
<td>§</td>
<td>§</td>
</tr>
</tbody>
</table>

We introduce the following three, i.e. not numerous, changes to the primal’s solution: (i) Entry $x_{2,n−1}$ ceases to be $b_2$ and turns into $b_{n−1}$ instead. (ii) Entry $x_{2n}$ becomes $(b_2 − b_{n−1})$, replacing the zero it was before. (iii) Entry $x_{i^{*},n}$ drops from ring difference $(b_n − b_1)$ to the “difference of ring differences” $(b_n − b_1) − (b_2 − b_{n−1})$. These changes maintain feasibility, as is easily checked by consulting the housing constraints of the four rings affected.

Note that $x_{i^{*},i^{*}}$ is not among the entries changed. This particular entry continues to equal $\Delta(i^*)/2$. Since this entry is the only one to enter the primal objective’s optimal value, our formula does not change either. Note the role of ring 2 still being overshadowed here. While $\delta_2$ is positive, it is not sufficiently so to offset the negative $\delta_1$ that precedes it. And hence $(b_n − b_1) − (b_2 − b_{n−1})$ or $x_{i^{*},n}$ indeed is strictly positive. Now let us check the implied changes for the dual. Since $x_{2,n−1}$ and $x_{i^{*},n}$ continue to exceed zero, complementary constraints of the dual continue to be binding. And since $x_{2n}$ now also exceeds zero, the corresponding dual constraint becomes binding, so that $y_2 = −y_n$. This we knew before, and so this extra equation is redundant. We conclude that formula $\Delta(i^*)/2$ continues to apply. Of course, the objective’s numerical value changes.

Exploring a sign change for any other ring difference, or for additional ring differences, proceeds along similar lines. That is, formula $\Delta(i^*)/2$ continues to capture the minimum number of centrists whatever the signs of the ring differences in rings up to $i^*$, as long as these ring differences are overshadowed.

10.3 Not All Ring Differences Overshadowed

What (if anything) changes if one (or more) of the ring differences are not overshadowed? Let us allow for the possibility that not all ring differences prior to $i^*$ are overshadowed, as in Table (7). Let all ring differences from 1 up to $i' − 1$ be in the shadow of ring 0, and all ring differences between $i' + 1$ and $i^{*} − 1$ be overshadowed by $i'$, so that $i^*$ is not in the shadow. One optimal feasible solution assigns $\sum_{j=1}^{i'} δ_j/2$ to $x_{ii'}$, and $\sum_{j=i'+1}^{i^*} δ_j/2$ to $x_{i^{*}i^{*}}$, and zero to any other element above the counterdiagonal. The corresponding minimum number of centrists becomes the sum of these two (only non-zero) terms. But this is just cumulative ring differences 1 and 3, respectively, for example).

21 These assumptions are not restrictive. First, if $\Delta(0) < \Delta(2)$, we would have to consider alternating spells of ring differences in the shadow and not in the shadow. This case is considered shortly. And second, if $i^{*}$ shifted due to $\delta_2$ flipping its sign, nothing would change in the argument below as long as $2 < i^*$.
our familiar $\Delta(i^*)/2$. Adding extra spells of ring differences in the shadow adds nothing of substance here.

\[
\begin{array}{ccccccccc}
i & 1 & \ldots & i' - 1 & i' & i' + 1 & \ldots & i^* - 1 & i^* & i^* + 1 & \ldots & n/2 \\
\Delta(i) & \bullet & \bullet & \bullet & \circ & \bullet & \bullet & \bullet & \circ & \bullet & \bullet & \bullet \\
\end{array}
\]

Table 7: Alternating Spells of Shadow and Light

At last we turn to the question of what happens if any ring differences following $i^* + 1$ (rather than preceding $i^*$) exhibit a positive sign. Recall that, by definition of $i^*$, ring differences beyond $i^*$ must be overshadowed. Let one of these ring differences be positive, rather than negative, i.e. $i^* + 2$ say. Being in the shadow of $i^*$, the excess arising in ring difference $i^* + 2$ is “swamped” by the deficit in the previous ring difference at $i^* + 1$. Once more, there is no change in the number of minimum centrists.

10.4 Descriptive Statistics

\[
\begin{array}{cccccc}
\text{ShareClinton} & 0.19 & 0.35 & 0.43 & 0.53 & 0.81 \\
\bar{r} & 1 & 4 & 6 & 12 & 80 \\
\Delta^c & 0 & 0 & 0.10 & 0.25 & 0.76 \\
\Delta^d & 0 & 0 & 0.02 & 0.10 & 0.79 \\
(1 - \Delta^d - \Delta^c) & 0.21 & 0.71 & 0.81 & 0.88 & 1 \\
\text{ShareCollege} & 0.12 & 0.22 & 0.26 & 0.33 & 0.61 \\
\text{ShareWhite} & 0.07 & 0.61 & 0.75 & 0.84 & 0.96 \\
\text{SizeMetro (millions)} & 0.01 & 0.06 & 0.11 & 0.30 & 4.13 \\
\text{DensityMetro (1,000 per sq mile)} & 0.51 & 1.23 & 1.91 & 2.76 & 8.30 \\
\text{MeanIncome (1,000 $)} & 16.9 & 33.2 & 39.2 & 47.9 & 153.4 \\
\end{array}
\]

Table 8: Descriptive Statistics