Function Follows Form

Kristof Dascher

Abstract: Urban policy molds urban form. This paper suggests that urban form also molds urban policy. Urban form frames the political economy of whether to relocate the city center’s jobs and shops. In that sense, city functions follow urban form. This view rivals the form follows function that has become proverbial in architectural theory. Extensions to this analysis of the suburbanization of employment and shopping address the city’s topography, the convexity of its skyline, the possibility of sprawl and the effects on jurisdictional merger.

Keywords: City Shape, City Skyline, City Skewness, Sprawl, Suburban Shopping

JEL-Classifications: R12, D72, R52

Kristof Dascher
Regensburg University
Department of Business, Economics, Information Systems Management and Real Estate
93 040 Regensburg
Germany
Email: kristof.dascher@wiwi.uni-regensburg.de

1For valuable comments on an earlier version I am indebted to Rainald Borek, Alexander Haupt, Yarema Okhrin and Arthur Silve. I am also grateful for comments I enjoyed at the “Bavarian Micro Day”, the statistics seminar at Augsburg University, and the IIPF conference in Dublin. Any remaining errors are mine.
1 Introduction

There can be no doubt that urban policy molds urban form. When commenting on Paris’ urban
development, Alexandre Gady, a historian of that city, appears to suggest the reverse though:

“Paris – it’s beautiful. But it’s a doll’s house! And that’s one reason the Parisian élite
is so conservative. They live in the doll’s house. . . . The blindness of the élites is to
reproduce a model of returning to the center, always back to the center . . . ” (as quoted
by Gopnik (2014))

Gady argues that urban form molds urban policy. This paper’s interest is in the urban political
economy of precisely this form-to-policy connection. We will replace the example of Paris with a
monocentric city model, the notion of “doll’s house” with the city’s form, and the claim of “élites’
blindness” with an analysis of residents’ pursuit of their interests. We will then credit the above
quote with containing more than just a grain of truth. In fact, we will then credit urban form with
predicting (the urban political economy of the) decentralization of retail and employment.

A two-paragraph-summary of the paper reads as follows. Consider a city that decides on whether
to decentralize jobs and shops out to the city’s periphery (e.g. Lampugnani (1985), Garreau (1991),
Glaeser/Kahn (2010)). In principle, any resident’s preference regarding this decision might be read
off her properties’ “average location”. A resident with an average property near the city center will
not easily give up on that center; the opposite must be true for a resident whose average property
is close to the periphery; and a resident with no property at all might well be indifferent. If we
knew city residents’ average properties, then we could assign residents their policy preferences.
We could even go on to predict the city’s decentralization decision. Unfortunately, only, average
properties are not observable. And so neither are residents’ preferences.

But the city form is. The city’s form is reflected by the distribution of commuting distance, sample
data from which often are available. This form puts natural constraints on the properties residents
can possibly own. After all, resident properties must nest into the city’s given form. Detailed
“nesting constraints” emerge once we inspect the city’s form closer. The most binding of those
happens to vary with a simple index of the city’s physical form, i.e. the commuting distribution’s
skewness, irrespective of the property assignment. Urban skewness literally puts bounds around
resident interests. If urban skewness is positive enough then a majority holding on to the traditional
center is inevitable. Conversely, if this skew is sufficiently negative then decentralization sets in.
The city’s skew reveals urban form’s hidden grip on local politics. Or to provide yet another
window on this theme, a concept that is graphic predicts another that is political.

And so while urban form is an interesting field in its own right (Lynch (1960), Baranow (1980),
Roek et al. (2013)), this paper argues that city morphology has uses that go beyond the descriptive. Besides, reading restrictions on a city’s various political interests off its physical form also
complements a prominent view due to Louis Sullivan. According to Sullivan (1896), “...it is the
pervading law of all things organic and inorganic, ...that the life is recognizable in its expression”. Among architectural theorists Sullivan’s view has become the proverbial *form follows function.*

This paper’s suggesting that the built environment impact on the polity’s decision on where to locate a city’s commerce and business provides an explanation of how instead building contours (form) determine buildings’ uses (function), or more briefly of how ...function follows form. This reversal of ideas motivates the paper’s title.

And this reversal of ideas conflicts with the standard urban paradigm. Understanding the impact of policy on form is central to urban economics (e.g. Bertaud/ Brueckner (2005), Brueckner (2005), Bento/Franco/Kaffine (2006), Baum-Snow (2006), de Lara et al. (2013)). Yet the issue of repercussions of urban form on urban policy appears to have attracted much less attention – even as, say, the role of urban compactness for the viability of urban public transport (e.g. Bertaud (2003)) or the impact of existing infrastructure on subsequent zoning (Garcia-López et al. (2015)) have been of interest. In part this is for good reason. A young city’s policy is unlikely to be informed by its form. But for any mature city this seems more difficult to justify. It is hard to imagine that structures respond quicker to policy than policy does to structures.

We naturally extend our analysis to how the city’s form may follow the city’s topography. Consider a city stretched out on a peninsula or partly squeezed in by nearby mountains. Most of that city’s housing must be near the city’s center. This city’s form is skewed to the periphery. Fitting in our building block of function following form, voters will turn down any proposal to decentralize. Metaphorically, New York is compact because its geography discourages (political support for) the decentralization of its business and shopping. Alternatively, a city that is able to expand in any direction will naturally find much of its housing on its periphery, and so a majority of its voters will support decentralization. This even suggests that it might be the difference in topography (Europe as a whole being more mountainous than the US) that has European cities decentralize less.

We will also see that while it is city’s skew that helps us assess the urban political economy of decentralization, it is the city skyline’s convexity that assists in predicting changes in this political economy. The urban skyline is our second graphic tool for uncovering the city’s political economy. Since the urban skyline is a variant of the city’s population gradient, this also assigns additional weight to the literature on the population density gradient (e.g. McDonald (1989), Kim (2007)). Quite intuitively, building height restrictions (whether imposed by soil instability or by urban planning and zoning) take away from the city center’s “resilience”. If the city is not permitted to add population to its “leading rings”, then anything that adds extra layers of “lagging rings” at the periphery will always strengthen decentralization supporters’ ranks.

Our framework also indicates applications in the fields of jurisdictional fragmentation and urban sprawl. If policy makers are aware of urban form’s grip on local politics then urban form feeds
into decisions on jurisdictional merger. Central cities must fear annexing their suburbs if this, by reducing skew, will make decentralization more probable. For a similar reason central cities may refrain from imposing building height restrictions, may zone large tracts of the urban periphery for park use only, or may impose growth controls. Regarding sprawl, if cities differ in their form (which they do not in the present paper) any form-inspired decision to decentralize business and commerce in one city begins to attract migrants to that city’s periphery. Original city form becomes another determinant of urban sprawl, entering the list of determinants identified in the literature (e.g., Glaeser/Kahn (2010), Burchfield et al. (2006)).

The paper has seven sections. Section 2 outlines the basic model. Section 3 extracts the political economy of decentralization from the city’s form, by introducing computable and useful bounds on urban voter shares (Proposition 1). Section 4 puts these voter shares down to a graphic aspect of the city’s form. This section finds that we may estimate urban voter shares from the city’s skewness (Proposition 2). Section 5 offers a comparative statics treatment of the city’s form, analyzing the effects of variations in topography and technology on urban form, and hence on city functions. For instance, we find that being linear and exhibiting a convex skyline also makes a city less likely to decentralize (Proposition 3). Section 6 addresses a number of extensions (on concerns with city openness, housing ownership, and welfare), and section 7 concludes.

2 Model

Consider the following simple variant of the closed city model developed by Wheaton (1973), Pines/Sadka (1986), Brueckner (1987). A monocentric (though for much of the analysis not necessarily circular) city extends at most ̃r units of distance out from the center. Each resident occupies one unit of housing (an “apartment”) and commutes to the city center (CBD) to work and shop. Round trip commuting costs for a resident living at distance r are tr, so that Ricardian rent becomes q(r) = t(̃r − r), Landlords are resident, not absentee, because a closed city’s landlords can hardly be influential if they are absentee. City population equals 1. There is no agricultural hinterland.

Apartments are built by profit maximizing investors. One unit of capital k poured into a building site of unit area yields h(k) units of floor space, where h' > 0 and h'' < 0 (Brueckner (1987)). If p is the price of capital, investors choose k so as to satisfy the q(r)h(k) = p necessary for maximum profit. The optimal capital will clearly depend on rent q and capital price p, and so can be written as k(t(̃r − r), p). Let h(r) be shorthand for the corresponding optimal building height h(k(t(̃r − r), p)). In equilibrium the city boundary ̃r is determined by the requirement that the urban housing market clear:

\[ 1 = \int_0^{̃r} a(r)h(r) \, dr, \]

2Our analysis below could also be cast in an open city framework. We briefly explore the open city later (section 6). There we will indicate that an open city is naturally more inclined to decentralize.
where \( a(r) \) is land available at \( r \) units of distance away from the CBD.

Since we have set housing consumption equal to 1, building height or (residential) skyline \( h(r) \) also equal the city’s population density. At the same time \( f(r) \), as defined by the product of population density with land,

\[
f(r) = a(r)h(r),
\]

approximates the number of commuters populating the unit-width ring at distance \( r \). I.e., \( f(r) \) is the city’s commuting density function, with \( F(r) \) the corresponding cumulative distribution function.\(^3\) We follow Arnott/Stiglitz (1981) in referring to \( f \) as the city’s form or shape. To be sure, both city shape and city skyline are graphic.\(^4\) Sections 2 through 4 proceed with the Arnott/Stiglitz notion of shape; the city’s skyline makes its appearance in section 5.

Believing the impact of shape on policy to outpace the impact of policy on shape, we now freeze \( f(r) \) at what it is in equilibrium (1). Up until section 5, the city shape that the standard closed city model concludes with is the city shape that our analysis begins with.\(^5\) Now, while \( f(0) = f(\tilde{r}) = 0 \) (because at \( r = 0 \) there is no land while at \( r = \tilde{r} \) there is no housing), not much is known beyond this, especially if the city is fully circular. It is true that increasing distance from the CBD has rent, and hence building height \( h(r) \), fall. Yet it is also true that increasing distance from the CBD has circumference, and hence ring areas, rise. City shape \( f \) may be anything, i.e. increasing in \( r \) and/or decreasing in \( r \).\(^6\)

City apartments are owned by resident landlords. Each landlord owns (no housing or land other than) two apartments located anywhere in the city. One of these apartments he occupies himself, the other he rents out to his single tenant.\(^7\) Now, to best introduce the model’s various nesting constraints, we briefly rephrase the model’s key concepts within a discrete framework. We partition the city into \( n \) equidistant rings of width \( \tilde{r}/n \) each. Let ring \( i \) residents travel costlessly to commuting nodes at \( r_i = (i - 1/2)(\tilde{r}/n) \), from where they go on to the CBD at cost \( t r_i \). (Soon we will shrink ring width again so that the costless-within-ring-travel assumption becomes redundant.) Housing’s stock in ring \( i \), denoted \( s_i \), approximately equals \( f(r_i) \).

Now imagine a landlord who resides in ring \( i \) himself yet rents out his extra, second property in ring \( j \). Such an assignment implies a sum of commuting costs and rental income equal to \(-t r_i + q(r_j)\), where \( q(r_j) \) is rent at distance \( r_j \). Let \( \omega \) subsume any other benefit common to all landlords,

\(^{3}\) We assume \( a \) is continuous in \( r \). As \( h \) is (differentiable and hence) continuous in \( r \), so are \( f \) and \( F \).

\(^{4}\) At the same time, the skyline is more graphic or visual than shape is. In our model, the skyline of residential structures (i.e. excluding office buildings) reveals itself to the eye of the distant observer while the city’s shape only reveals itself to the statistician constructing histograms of commuting distance.

\(^{5}\) Section 6 then includes, and allows for variation in, exogenous (non-policy dependent) determinants of city shape \( f \) in the analysis.

\(^{6}\) Only in the two polar cases do we see clearer. In a city that is both linear (e.g. peninsular) and height-control free, \( f \) must be decreasing; while in a city that is circular and single story only, \( f \) should be increasing. This perspective we take up again in section 6.

\(^{7}\) Adding an extra group of owner-occupiers would add a group of voters whose incentives vis-à-vis the ring road proposal introduced below are obvious (those close to the center are against the ring road, those far from the center vote for it), while adding an extra group of landlords owning three properties each would add a group of voters whose voting behavior follows a pattern similar to the voting behavior of the two-property-landlords prominent below. In that sense, the property assignment chosen here spans the paper’s key idea. We briefly return to this issue later (section 6).
such as the wage or some local public good that does not distance-decay. Then landlord utility is \( \omega + t(\tilde{r} - r_i - r_j) \). Landlord utility is independent of whether the landlord resides in \( i \) and his tenant in \( j \), or vice versa, and so we always will conveniently put the landlord into that of his properties that suits our exposition best.

The paper’s central policy metaphor is the ring road. A ring road is a rival to the traditional Central Business District (CBD). Locations along it connect almost as well to one another as locations in the traditional core do.\(^8\) And so let a costless ring road be proposed to city residents, by some interested party (identified shortly). A ring road would shift the city’s center of attraction from its inherited position (the CBD) out to the urban boundary \( \tilde{r} \) (the ring road), in a single instant and with \( t \) unchanged.\(^9\) Instead of travelling \( r_i \) to the center of the city in order to work and shop, every resident in ring \( i \) now commutes \( \tilde{r} - r_i \) to the city’s periphery to work and shop, in those office parks and shopping malls strewn along the ring road that permits its users to circle the city on it at no cost. The ring road proposal is approved if it captures a majority of the vote. Implementing the ring road surely must be one of the most fundamental policy decisions a city can possibly ever take.\(^10\) It is important to see why tenants will be indifferent to this proposal, and hence may be ignored throughout. Tenants’ cost of living, or \( tr + q(r) \), equals \( t\tilde{r} \). This marginal commuting cost does not change as long as the distance between center and periphery does not. We conclude that it is only landlords who will vote. Landlord-voters are divided over which decision to take, depending on their specific properties’ locations (not known to us). Landlord utility becomes \( \omega - t(\tilde{r} - r_j - r_i) \) with the ring road instead of \( \omega + t(\tilde{r} - r_j - r_i) \) without it. The attendant change in utility is \( 2t(r_i + r_j - \tilde{r}) \). This change is strictly negative if \( r_i + r_j < \tilde{r} \), or if

\[ i + j \leq n. \tag{3} \]

Landlords whose property location indices satisfy inequality (3) will oppose the CBD’s displacement. These we label as (landlord) opponents. All the other landlords can be counted on to support it, and become the model’s (landlord) proponents as soon as indifferent landlords become negligible (which they do next).

3 Extracting Bounds from the City’s Shape

We cannot derive landlord opposition to the ring road for the true yet unknown landlord property portfolios nesting into the city’s shape. Yet we can derive fictitious landlord portfolios from the exogenous city shape that generate minimum landlord opposition to the ring road. The observable

---

8 Vienna provides an early prominent example of a ring road, so much so that its ring road is actually referred to as the “Ring”. Victor Gruen, inventor of the modern US style shopping mall, appears to have modeled his malls on Vienna’s ring (Gladwell (2014), Hardwick (2010)).

9 While a simultaneous shift of course is unlikely, any shift from the CBD to the city periphery might be helped along by coordination. Rauch (1993) points to the role of business park developers in coordinating industry relocation, while the shopping center industry attests to the importance of retail space developers in coordinating movements in retail (e.g. Brueckner (1993)). Sometimes it is even the city’s government that provides this coordination. For example, Vienna’s ring road was where suddenly one could find “the new exchange, the university, a civic and national government section around the new town hall and parliament house, a museums section, the opera house” (Girouard (1989)).

10 In turning the city’s spatial setup on its head this decision not just is a fundamental one; its underlying political economy in part also parallels that of introducing a toll or of neglecting radial roads (section 6).
minimum opposition to the ring road thus provided a conservative estimate of the unobservable true opposition. Let us briefly preview the steps this section takes. First we identify the portfolios that give rise to the weakest conceivable opposition for each nesting constraint. Next we select that nesting constraint that reveals the largest of these minimum opposition figures. Finally we study the effect changing city’s skewness has on this largest minimum resistance. For example, there we look for shapes robust enough to withstand the ring road proposal’s temptation.

Consider the stock of apartments in the first ring, $s_1$, first. All of these apartments are tied up in matches involving landlords who suffer from the ring road – with the exception of those matches involving a tenant in ring $n$. Matches involving apartments both in rings $i = 1$ and $j = n$ fail necessary condition (3). Apartment stock $s_1$ would be a good first estimate of landlord opponents were it not for the fact that every resident in ring $n$ could be tenant to a landlord in ring 1 (rather than to a landlord in any of the remaining rings). Making allowance for this observation, really only $(s_1 - s_n)$ apartments may safely be traced back to landlord opponents. Further, these latter $(s_1 - s_n)$ units might involve landlords and tenants only ever from the first ring, acting to depress further the number of landlord opponents we can be sure of. Hence our first lower bound becomes $(s_1 - s_n)/2$.

Put differently, $(s_1 - s_n)/2$ is a lower bound to the set of all conceivable landlord opponent figures that could involve the first ring. This latter set contains the true number of landlord opponents. Of course, if the city shape is such that $s_1 < s_n$, then $(s_1 - s_n)/2$ is negative. In this specific case $(s_1 - s_n)/2$ is not a very good lower bound. A lower bound of zero landlord opponents is an obviously better choice. Yet this need not bother us. There are many more lower bounds on offer. For example, apartments in the first two rings, $(s_1 + s_2)$, give another conservative estimate of landlord opponents if we allow for (i) all $(s_{n-1} + s_n)$ tenants being matched up with some landlord in the first two rings and (ii) all remaining apartments in the first two rings to be matched up with one another. Making these two adjustments points to $((s_1 + s_2) - (s_{n-1} + s_n))/2$ as lower bound to the set of all conceivable landlord opponent figures that could involve the first two rings.

Already we have identified two nesting constraints that offer some minimum opposition to the ring road consistent with the city’s shape. This idea can be generalized. Including all $j$ first, as well as last, rings, any partial sum $P'(j) = \sum_{i=1}^{j}(s_i - s_{i+1:n})/2$, with $j = 1, \ldots, n/2$, is a lower bound to the number of landlord opponents. Returning to our initial continuous setup, we refine the city’s partition into rings by both increasing $n$ and $j$ such that $j/n$ stays constant. The resulting

\[^{11}\text{The smallest number of landlord opponents conceivable obtains if landlords and tenants share equally in } (s_1 - s_n), \text{ yielding the } (s_1 - s_n)/2 \text{ mentioned above. On the one hand, if } (s_1 - s_n) < \sum_{j=2}^{n-1} s_j \text{ then each of the } (s_1 - s_n) \text{ remaining apartments in ring 1 could be occupied by a landlord who is successfully matched up with a tenant in rings } 2, \ldots, n - 1. \text{ The resulting number of landlord opponents would surely exceed our presumed minimum of } (s_1 - s_n)/2. \text{ On the other hand, if } \sum_{j=2}^{n-1} s_j < (s_1 - s_n) \text{ then only } \sum_{j=2}^{n-1} s_j \text{ apartments could be occupied by landlords in ring 1 matched up with some tenant from rings } 2, \ldots, n - 1. \text{ The remainder would have to be occupied by both landlords and their tenants. The resulting opponent figure, or } (\sum_{j=2}^{n-1} s_j + (s_1 - s_n))/2, \text{ would also exceed our } (s_1 - s_n)/2. \text{ So it is true that we can be confident of } (s_1 - s_n)/2 \text{ landlords to oppose the ring road. – As an aside, if the } n\text{-th ring was populated by owner-occupiers only then the minimum number of landlord opponents associated with the first ring would be } n/2, \text{ and hence larger always. We make use of this observation in section 6.}

\[^{12}\text{It may be helpful to briefly note that the simple (non-cumulative) ring difference } (s_2 - s_{n-1})/2 \text{ cannot be another lower bound. Apartments in the second ring may also house landlords who own their second property in the last, } n\text{-th, ring and who hence are strictly better off by adopting the ring road. This disqualifies } (s_2 - s_{n-1})/2 \text{ as lower bound.}
sequence of partial sums converges to

\[ l^o(b) = \left( \int_0^b f(r) dr - \int_{\tilde{r} - b}^{\tilde{r}} f(r) dr \right) / 2, \]

where we have adopted \( b = (j/n) \tilde{r} \).

Because integrating \( f(r) \) over \([\tilde{r} - b, \tilde{r}]\) is the same as integrating \( f(\tilde{r} - r) \) over \([0, b]\), we may usefully rewrite \( l^o(b) \) as

\[ l^o(b) = \int_0^b \left( f(r) - f(\tilde{r} - r) \right) dr / 2 = \int_0^b D(r) dr / 2, \quad (4) \]

with \( b \in [0, \tilde{r}/2] \). Here the second equation in (4) defines the ring difference at \( r \), \( D(r) = (f(r) - f(\tilde{r} - r)) \). Each such ring difference \( D(r) \) juxtaposes commuters living in the “leading ring”, at \( r \), with commuters living in its antagonist “lagging ring”, at \( \tilde{r} - r \). Equation (4) casts lower bound \( l^o(b) \) as a sum of those ring differences.\(^{13}\) It is clear that most we must be interested in the largest of all these lower bounds \( l^o(b) \). It is this bound that is the most successful at extracting political information from the given city shape. To identify it, it remains to maximize \( l^o(b) \) with respect to \( b \). This last step is taken shortly, when stating Proposition 1.

A nearly identical argument applies towards bounding from below the number of those landlords who are certain to strictly benefit from, and hence support, the project. To see this note that all \( s_n \) units are tied up in matches that incite their owners to support the ring road – with the exception of those involving a tenant in ring 1. Put differently, replacing the inequality in (3) by \( i + j \geq n + 2 \) and fixing \( i \) at \( n \) implies \( j \geq 2 \). To assess minimum conceivable support let every resident in ring 1 be linked to someone in ring \( n \) (rather than to someone in any of the other rings). This is another conservative scenario, and in it only \((s_n - s_1)\) apartments point to landlords who would benefit from the policy proposal. Further, suppose, pessimistically, that all of these \((s_n - s_1)\) apartments join landlords and tenants from ring \( n \) only. Then \((s_n - s_1)/2\) emerges as our first lower bound on the number of landlord proponents.

Here, too, there are many more lower bounds. For example, another lower bound derives from consulting both the two last and first rings, and comes to \((s_n + s_{n-1}) - (s_1 + s_2))/2\). Generally, if the last, as well as first, \( j \) rings are included, the lower bound on landlord proponents can be written as \( l^p(j) = \sum_{i=1}^j (s_{n+1-i} - s_i)/2 \), where \( j = 1, \ldots, n/2 \). Casting the partial sum of landlord proponents extracted from the first \( b \) ring differences in terms of arbitrarily small ring width, and recalling \( b = (j/n) \tilde{r} \) as well as the concept of ring difference \( D \), gives

\[ l^p(b) = \int_0^b \left( f(\tilde{r} - r) - f(r) \right) dr / 2 = \int_0^b \left( - D(r) \right) dr / 2, \quad (5) \]

again where \( b \in [0, \tilde{r}/2] \). Note that \( l^p(b) = -l^o(b) \). The largest of all these latter lower bounds is found by maximizing the integral with respect to \( b \). Equivalently we may minimize this integral’s negative, i.e. \( l^o(b) \), and proceed with the negative of the minimum value obtained. This final step also is taken when stating Proposition 1, i.e. now.

**Proposition 1: (Extracting Voter Share Estimates from the City’s Shape)**

\(^{13}\)Our notation for ring difference \( D \) surely will not be mistaken for population density, which is written \( h \). Note that ring differences will resurface when introducing city skewness below (in section 4).
(i) (Lower Bounds, and Existence): Lower bounds on the number of landlord opponents, \( l^o \), and on the number of landlord proponents, \( l^p \), are identified as
\[
l^o = \max_{b \in [0, f/2]} \left[ \int_0^b D(r) \, dr \right] / 2 \quad \text{and} \quad l^p = -\min_{b \in [0, f/2]} \left[ \int_0^b D(r) \, dr \right] / 2.
\]
respectively, and exist always.

(ii) (Positive Bounds): Both lower bounds \( l^o \) and \( l^p \) are nonnegative.

(iii) (Upper Bounds): Opponents’ number \( l^o \) is bounded as in \( l^o \leq 1/2 - l^p \); while proponents’ number, \( l^p \), is bounded via \( l^p \leq 1/2 - l^o \).

(iv) (Useful Bounds): Lower bounds \( l^o \) and \( l^p \) are zero both (i.e. useless) if and only if the density of commuting distance \( f \) is symmetric.

(v) (Sufficient Bounds): If \( l^o > 1/4 \) (alternatively if \( l^p > 1/4 \)) then a majority of landlord opponents (landlord proponents) reject (push through) the ring road.

(vi) (Special Bounds): If \( f \) is decreasing in \( r \) on \([0, \tilde{r}/2] \), then \( l^o = F(\tilde{r}/2) - 1/2 \) and \( l^p = 0 \), while if \( f \) is increasing in \( r \) on \([0, \tilde{r}/2] \) then \( l^o = 0 \) and \( l^p = 1/2 - F(\tilde{r}/2) \).

Part (i) provides estimates of the impact of the city’s “physical sphere” (differences between housing stocks in antagonist rings) on its “political sphere” (lower bounds on landlord opposition or landlord consent), a transmission that operates as silently as it is fundamental. These estimates can always be computed. Part (ii) notes that lower bounds must be non-negative since the maximizers involved in computing either bound equal 0 at worst, reducing the corresponding integral to zero then. Part (iii) adds that the lower bound on landlord proponents of the ring road also converts into an upper bound on landlord opponents, and vice versa.\(^{14}\)

Part (iv) emphasizes that, with the exception of the improbable case where \( f \) is symmetric, at least one of the two lower bounds must bind. For this reason alone our pair of lower bounds must be useful. Finally, and most importantly, the ring road proposal is rejected under majority rule with certainty as soon as \( l^o \) exceeds one fourth of the housing stock, irrespective of the city’s specific apartment portfolio assignment (Part (v)). Lower bounds become particularly useful once they exceed the one-fourth threshold. But they should also be useful even when this is not true. Note that if we eventually allowed for construction, Part (v) would also point to the possibility of a long run lock-in. A city with a shape such that \( l^o > 1/4 \) holds on to its center. Yet this in turn just affirms that shape.

Fig. (1) illustrates lower bounds \( l^o \) and \( l^p \). In each of the Figure’s panels the horizontal axis gives commuting distance \( r \) from the CBD, while the vertical axis gives commuting density \( f(r) \).\(^{15}\) (Axes are not scaled identically across panels.) The vertical line at each panel’s center rises up above “midtown” \( \tilde{r}/2 \), about which the graph of \( f(r) \) is “folded over” (reflected) in order to obtain, and illustrate, ring differences \( D(r) \) at all distances between 0 and \( \tilde{r}/2 \). I.e., ring differences are represented by the vertical distances between the two graphs (lines) shown left of \( \tilde{r}/2 \). Note how the commuting density is increasing over at least some subset of the support in most panels. In a

\(^{14}\)Since the interval \([l^o, 1/2 - l^p]\) contains the true \( l^o \) for any given city, this interval’s size effectively indicates the precision of our lower bound \( l^o \). (A similar point applies to \( l^p \).) For city shapes for which this interval is small our lower bound supplies a more precise estimate than for city shapes for which this interval is large.

\(^{15}\)These commuting densities merely serve to illustrate our lower bounds, rather than simulate equilibrium commuting densities obtained from the housing market equilibrium set out in eq. (1).
circular city this easily arises whenever the increase in built-up area from adding yet another ring outweighs the diminishing population density that is typical of many (though not all) cities.

We turn to the stylized city shapes (a) through (c) first. Panel (a) shows what we might dub a “classical city”. With its true distribution documented by de Lara et al. (2013, cf. Fig. 3), Paris quite closely (though not perfectly) resembles this “classical city”. Panel (b) depicts a “hat city” that reflects the fact that the CBD needs land, too. Panel (c) illustrates an “edge city” (if a monocentric one), across which most commuters travel long distances. We note that panels (a) and (c) illustrate the special cases set out in Proposition 1’s Part (vi). – Lower bounds can be inferred from consulting corresponding shaded areas. In panels (a) and (b), the lower bound on opponents amounts to half the shaded area below the density’s graph (blue on screen); whereas in panel (c) the lower bound on proponents amounts to half the shaded area above the density’s graph (in red).

Panel (d) adds an “inverted-U city” to our little city morphology. Moscow, for example, appears to exhibit just this shape (Bertaud/Renault (1997), cf. Fig. (1b)). The “inverted-U” city is different from the three preceding stylized city shapes. Neither is one of the two lower bounds zero, nor are lower bounds just as easily read off the shaded areas. In panel (d), and following the principles outlined above, the lower bound on opponents is equivalent to half the area obtained after subtracting the smaller, and doubly, shaded (orange) area from the larger, singly shaded, (blue) one. While early ring differences are negative, later ring differences are overwhelmingly positive. Including those later, and positive, differences in our lower bound (i.e. a cumulative sum) is preferable even if that comes at the cost of also including those earlier, and negative,
Panel (d)’s “inverted-U city” illustrates the principles underlying our lower bound on landlord proponents, too. Here this latter lower bound occurs where $D$ vanishes, or where $f$ and its reflection $f(\tilde{r} - r)$ intersect in the Figure.\footnote{Minimizing $\int_0^\infty D(r)dr/2$ requires an interior solution, denoted $b^*$, to satisfy $f(b^*) = f(\tilde{r} - b^*)$.} The resulting lower bound is half the cumulative sum of all ring differences from the city center up to the intersection, or half the panel’s doubly shaded (or orange) area above the density’s graph.\footnote{In their policy simulation, Bertaud/Brueckner (2005) show how the introduction of a floor-to-area restriction (FAR) reduces that city’s skew. The city shape observed prior to the policy’s implementation (Fig. 7) resembles our “inverted-U city".} Generally we should expect city shapes to be the more informative the more asymmetric they are. A city of symmetric shape, i.e. in which ring differences are zero always, reveals next to nothing of its politics to the observer. The following section will revisit this idea, replacing asymmetry with skew. Also, note how in panels (a) through (d) there always is at least one of the two lower bounds that is active (i.e. strictly positive), as predicted by Proposition 1’s Part (iv).

Figure 1’s panel (a) also illustrates one scenario where landlord opponents are decisive. Its shaded area is well in excess of half the area below the graph of commuting density. This city cannot help but turn down the ring road proposal. Not a single constellation of landlord portfolios exists that could collapse the anti-ring-road majority that is the inescapable consequence of that city’s shape. In that sense we do expect to find the interests of a majority of Parisians (or Parisian landlords at least) to always go “back to the center”, as suggested by the introductory quote. The same cannot be said for the “hat city” in Figure 1’s panel (b). Alternatively, if $l^p$ exceeds $1/4$ then it is the landlord proponents of the ring road who will prevail. Panel (a) provides one example.

4 The Skewness of the City’s Shape

Different city shapes may give rise to similarly sized bounds. It may not so much be the entire commuting distribution that matters to the urban majority but one particular aspect of it. Perhaps a simple suitable indicator of asymmetry, rather than the previous section’s more intricate bounds $l^p$ and $l^p$, could provide an assessment of the city’s political economy, also. This section pursues these ideas. We suggest the commuting distribution’s skewness as an indicator that (i) itself bounds lower bounds from below and (ii) is visually appealing at that. Ultimately it is skewness (form) that drives decisions on the ring road proposal (function): Function follows form (Proposition 2 later).

To start us on this idea we offer an alternative definition of skewness $\sigma$, i.e.,

$$\sigma = \int_0^{\tilde{r}/2} D(r) \left( \tilde{r}/2 - r \right) dr. \quad (6)$$

Here $\sigma$ is a weighted sum of those ring differences $D(r)$ introduced in the previous section, with a given ring difference’s (positive) weight equal to the two underlying rings’ common distance to midtown.\footnote{In its reliance on $\tilde{r}$, the length of the commuting distribution’s support, $\sigma$ differs from the various definitions of skewness found in the literature. Let $\rho$ denote the commuting distribution’s mean, that is $\rho = \int_0^\infty f(r)r dr$. Next} We justify $\sigma$ by pointing to its visual appeal. In formula (6), early ring differences
(associated with distances close to 0) receive large weights while late ring differences (associated with distances close to \(\tilde{r}/2\)) only benefit from small weights. That an indicator of skewness should reward early ring differences makes sense. Surely how the very distant first and last ring compare to each other frames our perception of the commuting density’s (i.e. city shape’s) skewness by more than how the two adjacent rings right on either side of midtown \(\tilde{r}/2\) compare to each other. Generally, for a city shape to exhibit strong positive skew, two properties contribute. First, ring differences should more often than not be positive (true for our stylized “classical city”, “hat city” and “inverted-U-city” (Figure 1 again), though not true in our “edge city”). And second, these positive ring differences better occur early (close to the CBD) rather than late (close to midtown). The “inverted-U city” displays visibly smaller skew than our “hat city” precisely because it lacks those early positive ring differences. We argue that these properties support our choice of \(\sigma\) as one plausible indicator of skewness.

At the same time, and as the paper’s next proposition, \(\sigma\) also allows us to bound city politics. City skewness connects the graphic with the political, by exploiting both concepts’ common mutual connection with the physical. A compact statement of this idea runs through the following short sequence of inequalities:

\[
\sigma = \int_0^{\tilde{r}/2} D(r) \left( \tilde{r}/2 - r \right) dr \leq \max_{b \in [0, \tilde{r}/2]} \left[ \int_0^b D(r) \left( \tilde{r}/2 - r \right) dr \right] \tag{7}
\]

\[
\leq \max_{b \in [0, \tilde{r}/2]} \left[ \int_0^b D(r) \left( \tilde{r}/2 \right) dr \right] \tag{8}
\]

\[
= \tilde{r} \max_{b \in [0, \tilde{r}/2]} \left[ \int_0^b D(r) dr / 2 \right] = \tilde{r} l_\rho, \tag{9}
\]

Inequality (7) exploits the fact that the integral over weighted ring differences is greatest if the integral’s upper limit is chosen freely, rather than being invariably fixed at \(\tilde{r}/2\). And equation (9) makes use of the simple fact that a monotonic transformation of the maximand does not affect the maximization procedure’s solution. This leaves us with inequality (8). Recall that generally \(f\) need not be monotonic in \(r\). But then ring differences \(D(r)\) cannot be signed. A ring difference may be anything: positive, zero, or even negative. So replacing \((\tilde{r}/2 - r)\) by \(\tilde{r}/2\), as we do when going from the r.h.s. of (7) to the r.h.s. of (8), not necessarily increases the integral.

And yet increase is precisely what that integral does. As the formal proof in the Appendix shows, inequality (8) is true indeed. Its proof really relies on one single important insight. By definition, the upper limit of the integral \(b\) on the r.h.s. of (7) is chosen to render the expression in square brackets as large as possible. Let \(r^*\) denote the underlying maximizer. In the resulting integral, i.e. in

\[
\int_0^{r^*} D(r) \left( \tilde{r}/2 - r \right) dr, \tag{10}
\]

note that \(\sigma\) may be rewritten as \(\int_0^\tilde{r} f(r)(\tilde{r}/2 - r)dr\), which in turn simplifies to \(\tilde{r}/2 - \rho\). Hence \(\sigma\) simply is midtown distance to the CBD minus mean distance to the CBD. This is not the same as any of the standard definitions of skewness.

Additional shapes could be drawn to illustrate how \(\sigma\) conforms with our intuition on skewness. Symmetric distributions, for instance, are characterized by skewness being equal to zero (as they should be). For symmetric distributions, ring differences \(D\) are all zero and hence so is skewness.
ring differences may well alternate in sign, but late ring differences at distances just short of \( r^* \) must be positive. Why else would they have been included in the sum (10)? Yet these late, and positive, ring differences close to \( r^* \) are also those where replacing \((\tilde{r}/2 - r)\) with \(\tilde{r}\) has greatest impact. After all, the change in weight applied when going from the r.h.s. of (7) to the r.h.s. of (8) is \( r \), and hence is largest if \( r \) is close to \( r^* \). So intuitively positive ring differences come to enjoy a greater extra in weight than negative ring differences do. On balance replacing weights serves to increase the overall sum.\(^{20}\) To summarize, replacing \((\tilde{r}/2 - r)\) by \(\tilde{r}/2\) does contribute to raising the r.h.s. of (7), and so inequality (8) is true.

We summarize the overall inequality implied by the succession of inequalities (7) through (9), and combine it with \( l^o \leq l^o \) (Proposition 1), in Proposition 2’s first part. There we state that landlord opposition to the ring road proposal is bounded from below by skew \( \sigma \) adjusted for “city size” \( \tilde{r} \), or \( \sigma/\tilde{r} \). The more skewed the city is the more confident we can be of the ring road proposal’s meeting landlord resistance. Proposition 2’s first part also states that the negative of adjusted city skew bounds landlord proponents from below.\(^{21}\) We conclude that in sufficiently positively skewed cities (Figure 1’s panel (a), (b) and (d)), part (i)’s first inequality may be useful, while in sufficiently negatively skewed cities (Figure 1’s panel (c)) we exploit the second inequality instead. Either way one of the proposition’s two inequalities must be helpful except when the city’s shape is symmetric.

**Proposition 2: (Function Follows Form)**

(i) (Physical and Visual): Adjusted city skew \( \sigma/\tilde{r} \) bounds landlord opponents via \( \sigma/\tilde{r} \leq l^o \), and bounds landlord proponents as in \( -\sigma/\tilde{r} \leq l^p \).

(ii) (Function Follows Form): If \( \sigma/\tilde{r} > 1/4 \) (or \( -\sigma/\tilde{r} > 1/4 \) alternatively) the center retains its retail and employment function (the periphery takes over jobs and shops).

Proposition 2’s Part (ii) sets out the paper title’s causality from city shape (form) to buildings’ residential vs. commercial uses (function). On the one hand, if \( \sigma/\tilde{r} \) exceeds 1/4 then this is not just true for \( l^o \) but a fortiori also for \( l^p \). On the other hand, if \( -\sigma/\tilde{r} \) exceeds 1/4 then so does \( l^p \). Buildings in the city traditionally house retail and office uses. If city skew is strong enough then traditional uses are certain to be preserved, while if city skew is sufficiently negative then uses are sure to be reversed, i.e. city center buildings become residential and it becomes peripheral structures’ turn to take over the retail and office function.\(^{22}\) Of course, for “amorph cities”, with \( \sigma/\tilde{r} \) in the closed interval \([-1/4,1/4]\), equilibrium policy cannot be assessed further.

\(^{20}\) The formal proof in the Appendix generalizes this intuitive idea to city shapes for which ring differences’ signs alternate more often than just once (i.e. finitely many times).

\(^{21}\) The formal proof is similar to that just presented. A sketch appears in the Appendix.

\(^{22}\) To pick up on an earlier footnote, shape and skyline coincide in peninsula (linear) cities. Intuitively, New York and San Francisco appear to have both: positively skewed skylines-shapes and strong, confident CBDs. According to Proposition 2, this is not a coincidence but an implication of the city’s shape. We further pursue this idea in the following section. We also briefly comment on the evidence in Burchfield et al. (2006) according to which there is “almost no correlation between the extent to which residential development is scattered and that to which employment is decentralized” by noting that Proposition 2 relies on the skew of the commuting distribution, rather than on the extent to which development is scattered.
5 Shape Origins and Skyline Convexity

This section traces the city’s shape back to urban (i) topography and (ii) technology. To best discuss these ultimate determinants of urban form (and hence, by virtue of Proposition 2, urban functions), we make two stylized additions to the model. First, land supply is capped in lagging, though not in leading, rings. Only fraction $\alpha$ represents developable land available in lagging rings. Below, an exogenous drop in $\alpha$ roughly represents the transition to a more “linear” city.

And second, residential housing only begins at $\tilde{r}$, rather than at 0, where $\tilde{r} \geq 0$ generally and $\tilde{r} = 0$ initially. Differences in $\tilde{r}$ may reasonably be thought to reflect differences in CBD size, cross-sectional variation in federal zoning (e.g., prohibiting residential uses near the CBD) or the presence of a green belt. We might add that much of World War II aerial bombing afflicted central city, rather than peripheral, housing (starting with Germany’s bombing of Warsaw and Rotterdam, e.g. Lampugnani (1985), Brakman et al. (2004)), and hence may fit the increase in $\tilde{r}$ analyzed below. Besides, changes in $\alpha$ and $\tilde{r}$ are not purely cross-sectional only. A climate change induced rise in the sea level, for instance, will also induce variation in coastal contours, and hence variation in $\alpha$.

Now, ring differences, housing market equilibrium, and our lower bound become

$$D(r, \tilde{r}, \alpha) = 2\pi \left( rh(r) - \alpha (\tilde{r} - r) h(\tilde{r} - r) \right),$$

$$1 = 2\pi \left( \int_{\tilde{r}}^{\hat{r}/2} rh(r) \, dr + \int_{\hat{r}/2}^{\tilde{r}} \alpha r h(r) \, dr \right),$$

$$l^o(\alpha, \tilde{r}) = \max_b \int_b^b D(r, \tilde{r}, \alpha) \, dr / 2,$$

respectively. For completeness we add that changes in $l^o(\alpha, \tilde{r})$ simply go into the opposite direction of any changes in $l^o(\alpha, \tilde{r})$, and so there is no explicit discussion of these former changes below.

Changes in $\alpha$ or $\tilde{r}$ have direct effects on our lower bound on landlord opponents that are easily gauged from consulting (13). But there are also indirect effects, operating through the adjustment of $\tilde{r}$ implied by having to maintain equilibrium in the housing market (12). Fortunately, there is no need to account for the adjustment in the optimum upper limit entering $l^o$, denoted $b^*$. By its definition (in (6)), $l^o$ is a (maximum) value function, and so its derivatives with respect to $\alpha$ and $\tilde{r}$ may be approached with the envelope theorem in hand. We thus have

$$\frac{d l^o}{d \alpha} (\alpha, \tilde{r}) = \int_0^{b^*} \left( \frac{\partial D(r, \tilde{r}, \alpha)}{\partial \alpha} + \frac{\partial D(r, \tilde{r}, \alpha)}{\partial \tilde{r}} \frac{\partial \tilde{r}}{\partial \alpha} \right) \, dr / 2,$$

$$\frac{d l^o}{d \tilde{r}} (\alpha, \tilde{r}) = \int_0^{b^*} \frac{\partial D(r, \tilde{r}, \alpha)}{\partial \tilde{r}} \frac{\partial \tilde{r}}{\partial \alpha} \, dr / 2.$$

These derivatives inform us about changes in the lower bound on landlord opponents, and so we do not know how actual landlord opponent numbers change. Nonetheless we will think of an increase in $l^o$ joint with a simultaneous decrease in $l^p$ as indicating that decentralization is becoming less “likely”.

Both derivatives feature the common partial derivative $\partial D / \partial \tilde{r}$. This derivative is positive if the skyline $h$ is convex (Lemma 2, in the Appendix). An increase in $\tilde{r}$ (as implied by a shock to $\alpha$ or $\tilde{r}$) raises rent throughout the city, yet has two opposing effects on ring differences $D$. On the one
hand, population in each leading ring, at $r$, grows because buildings rise in height when rents go up. On the other hand, population in each lagging ring, at $\tilde{r} - r$, grows because that ring now has shifted out, and hence commands a greater area. If the city skyline is convex, then the extra floor space obtained from building higher up near the center exceeds the extra floor space provided by even more bungalows at the city’s periphery.\textsuperscript{23} Note that historically, skylines have often been found to be convex. Bertaud (2003, p. 12) shows a number of population density plots that best are approximated by strictly convex densities (Paris, Bangkok, Jakarta). The negative exponential that often is successfully fitted to empirical population densities in the literature (e.g. Mcdonald (1989), Bertaud/Malpezzi (2014), Duranton/Puga (2015)) is strictly convex.\textsuperscript{24}

Consider a city with a convex skyline (Proposition 3). First, a circular city will be more inclined to decentralize than a slightly less circular one. Taking land away from lagging rings has the urban boundary shift out, so that rents rise throughout the city. Because leading rings add more population than lagging rings do (given convexity), ring differences, and our lower bound on landlord opponents, rise. As an application, and somewhat speculatively, Europe may have decentralized jobs and shops less than the US because its more rugged topography presses its cities into more of a linear mold. And second, a city embraces decentralization less when its CBD expands. As housing shifts out to the city periphery, rents rise throughout the city. Due to the skyline’s convexity, this will again raise population in leading rings by more than it will raise that in lagging rings. The lower bound on landlord opponents increases. Skyline convexity emphasizes the importance of being able to build up in the city’s leading rings in response to central housing loss if a city is to retain its traditional center. Building height requirements often prevent such adjustment (Bertaud/Brueckner (2005), Glaeser (2011)).

**Proposition 3: (The Origins of Form, and Function)**

Consider a city with a convex skyline. Let $\alpha$ fall, marking a transition from a more circular to a more linear city, or let $\tilde{r}$ rise, driving residential housing away from the center. In either case, our lower bound on landlord opponents $l_o$ rises, and our lower bound on landlord proponents $l_p$ falls.

### 6 Extensions

The paper’s model rests on a closed-city-resident-landlords framework. Resident landlords’ political interests we unveil by inspecting the city’s shape. City shape’s skew turns out to be an urban property endowed with the merits of (i) data availability and (ii) predictive power. Admittedly, exposing this property relies on a number of debatable assumptions. Not all cities are closed; landlords may own more, or less, than two apartments; landlords may vary by the extent to which they own multiple properties; decentralization need not be beneficial if commuting costs also rise; building a ring road may seem a very special case of an urban policy; and so forth. In any event, note that our assumptions give conditions sufficient for the paper’s propositions. Mild deviations

\textsuperscript{23}For now we take skyline convexity as given. But one might wish to relate the skyline’s convexity to its proximate causes. Building height restrictions surely do not contribute to convexity (and in fact may cause the skyline to be concave rather than convex.)

\textsuperscript{24}We add that, Kim (2007), however, also supplies evidence according to which density gradients in US urban areas may have “flattened”, i.e. become less convex.
from these assumptions not necessarily overturn them. In fact, mild variations may even strengthen these propositions.

Consider the role of the closed city assumption. Treating the city as closed if really it is open biases our results. If the city is open then constructing a ring road opens up a host of new property developments on land that was out of reach previously. This land will be attractive to the mobile among other cities’ residents, and a competitive housing industry will develop residential housing on it. If rents earned from all of this extra housing accrue to the city’s indigenous landlords then these indigenous landlords obviously have an – added – incentive to decentralize, and this continues to be true if indigenous landlords only appropriate a fraction of these gains. Moreover, we still may treat mobile, indifferent tenants as abstaining from the ring road poll. An open city must be particularly skewed if it is to withstand this extra temptation and stick to its traditional center.

We have argued that the one landlord–one tenant assignment exposes the city shape’s role for urban political economy at minimum cost. Relaxing this assumption can enrich the model in interesting ways, too. E.g., suppose that a fraction \( 1 - \alpha \) of all housing belongs to owner-occupiers, while the remaining share \( \alpha \) is owned by landlords owning two properties each. Suppose further that owner-occupiers only, and exclusively, live in apartments located beyond \( r_1 > \tilde{r}/2 \). This probably is a fair assumption for many cities’ suburban housing. Owner-occupiers’ interest in decentralization is obvious, but those \( \alpha/2 \) landlords’ political interests are not. Suppose the last ring were inhabited by owner-occupiers only. Consulting (3), now all housing in the first ring can safely be assumed to point to landlord opponents. When constructing the first lower bound, no correction other than dividing by 2 is called for. Ultimately it merely is the skewness of the city’s shape over the “intermediate rings” that matters.

Our analysis has focused on replacing the CBD with a string of shopping districts and office parks along the ring road, in one swipe. One might object that this is too radical a policy. But our analysis may apply to other urban transportation policies, too. We might, for example, analyze an increase in \( t \) brought about by a tax on commuting or a neglect of radial roads. Taking the first derivative of landlord utility \( \omega + t(\tilde{r} - r_i - r_j) \) with respect to \( t \) gives \( (\tilde{r} - r_i - r_j) \). A landlord votes for the tax on commuting precisely if \( i + j \leq n \), which just repeats the familiar condition (3) We conclude that our \( l^n \) also is a lower bound on landlords supporting an increase in \( t \). When tenants have less political clout than landlords this observation could be useful. Not all landlords vote for the commuting tax. But those who do clearly do so for the rise in rent implied (as in Borck/Wrede (2005)). Likewise, \( l^p \) may be a lower bound on landlords endorsing a commuting subsidy.

Average commuting cost \( \rho \) could be considered one useful measure of welfare. The city decentralizes for sure if \(-\sigma/\tilde{r} > 1/4 \) (Proposition 2, Part (ii)), or if \( \rho > (3/4)\rho \) equivalently given that \( \sigma = \tilde{r}/2 - \rho \). Decentralization transforms each distance \( r \) into \( \tilde{r} - r \), and hence also transforms average commuting cost \( \rho \) into \( \tilde{r} - \rho \). Yet if average commuting cost \( \rho \) exceeds \( (3/4)\tilde{r} \) initially, then average post-decentralization commuting cost must fall short of \( \tilde{r}/4 \). Building costless ring roads, on which commuters later travel at no cost, makes negatively skewed cities better off. At the same time, ring roads are certainly not costless, and neither is travel on a ring road. Suppose, for instance, that \( t \) rises to \( t' > t \) as the city decentralizes. If aggregate commuting costs equal \( t\rho \) before, they amount to \( t'(\tilde{r} - \rho) \) after, decentralization. Whether society is better off depends on whether \( t'/t < (\tilde{r} - \rho)/\rho \). This may, or may not, hold. Welfare reducing decentralization is a possible
outcome of an extended model.

Throughout the paper we have held population fixed. If we allow for immigration, extra layers of peripheral rings will form, and rents throughout the city will rise. Again it is the city skyline’s convexity that helps predict how ring differences, and hence landlord interests, respond. Building height restrictions, in particular, will prevent the central city housing height adjustment that may counter the extra political weight that the proposition of decentralization inevitably attracts.

7 Conclusions

While city policy obviously shapes urban form, this paper argues that urban form also shapes city policy. The more skewed a city’s shape, the less conceivable a majority of residents that prefer replacing the traditional center at the CBD by a succession of office parks and shopping malls along a ring road. This theory also relates to two long-standing themes in both architectural theory and economics. It connects to architectural theory because it completes the relationship between function and form. As Frank Lloyd Wright notes (quoted in Saarinen (1954)), “Form follows function – that has been misunderstood. … Form and function should be one, joined in a spiritual union.” And it connects to mainstream economics. Economics as a field is weary of generalizing an aggregate’s properties on towards the aggregate’s component members lest it commit a “fallacy of division”. By linking the built environment (a society aggregate) to the preferences of at least a majority of its landlords (a decisive subset of society’s members) we provide an example of where inferring dominant residents’ properties does seem justified after all. Whenever the city’s shape “leans towards” the city center (a majority of) resident landlords are inclined to maintain, and hence “lean towards”, the city center. Conversely, if the city “leans towards” the periphery (a majority of) resident landlords are inclined to develop, and hence “lean towards”, the periphery. No fallacy is involved when assessing a city’s politics by its shape.
8 Literature


9 Appendix

Proof of Proposition 1:

Part (i) (Lower Bounds, and Existence): Consider a landlord who resides in ring \( i \) yet rents out the extra property in ring \( j \). This is a “match” \( \{i,j\} \). Let matrix \( B \) collect the frequencies with which matches \( \{i,j\} \) occur. For example, \( b_{1,3} \) is the number of times a landlord owning, and living in, an apartment in the first ring also owns an apartment in ring 3. Note that, with this definition, the sum of all entries in row \( i \) plus the sum of all entries in column \( i \) just yield the apartment total in ring \( i \).

\[
B = \begin{pmatrix}
  b_{1,1} & b_{1,2} & b_{1,3} & \ldots & b_{1,n-2} & b_{1,n-1} & b_{1,n} \\
  b_{2,1} & b_{2,2} & b_{2,3} & \ldots & b_{2,n-2} & b_{2,n-1} & b_{2,n} \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  b_{n-1,1} & b_{n-1,2} & b_{n-1,3} & \ldots & b_{n-1,n-2} & b_{n-1,n-1} & b_{n-1,n} \\
  b_{n,1} & b_{n,2} & b_{n,3} & \ldots & b_{n,n-2} & b_{n,n-1} & b_{n,n}
\end{pmatrix}
\]  

(16)

In view of condition (3), \( B \)'s counterdiagonal (comprising all the elements on the diagonal stretching from the bottom left corner to the top right hand corner) collects all those matches that leave landlords indifferent to the ring road. In contrast, entries above (below) \( B \)'s counterdiagonal collect all those matches that involve landlord opponents (landlord proponents).

Since \( (s_1 - s_n)/2 \) is an obvious first lower bound, let us make precise the second lower bound discussed in the main text instead, or \( (s_1 + s_2 - (s_{n-1} + s_n))/2 \). In (16), \( s_1 \) and \( s_2 \) are the sums of all entries given in the first row and column and second row and column, respectively. It is clear that this sum overstates the number of landlord opponents; some of its elements are found on, or below, our counterdiagonal. The implied error amounts to

\[
\left( b_{n,1} + b_{n-1,2} + b_{n,2} \right) + \left( b_{1,n} + b_{2,n-1} + b_{2,n} \right).
\]  

(17)

This error collects a subset of all the matches linking apartments in the last two rings to apartments in the first two rings, indicating landlords that do not oppose the ring road at all. We take care of these by subtracting all apartments in the last two rings from \( (s_1 + s_2) \), i.e. by subtracting \( (s_{n-1} + s_n) \). Similar reasoning applies to subsequent lower bounds on landlord opponents, or to any lower bound on landlord proponents.

As shown in the main text, the desirable lower bound on landlord opponents and the desirable lower bound on landlord proponents are the maximum of \( \ell^p(b) \) and that of \( \ell^p(b) \), respectively. Being integrals of continuous functions, \( \ell^p(b) \) and \( \ell^p(b) \) are differentiable on the compact interval \([0, \bar{r}/2]\). Hence both these maxima always exist. \( \square \)

Part (iv) (Useful Bounds): The proof is by contradiction. Thus suppose both lower bounds are useless. I.e., suppose

\[
\max_{b \in [0,\bar{r}/2]} \int_0^b D(r) \, dr = 0 \quad \text{and} \quad \min_{b \in [0,\bar{r}/2]} \int_0^b D(r) \, dr = 0.
\]

From the first equation we gather that \( \int_0^b D(r) \, dr \leq 0 \) for all \( b \in [0,\bar{r}/2] \) (else \( \ell^p \) would need to positive, contradicting our assumption); whereas from the second equation above we infer that \( \int_0^b D(r) \, dr \geq 0 \) for all \( b \in [0,\bar{r}/2] \) (else \( \ell^p \) would have to be positive, contradicting our assumption). Joining these latter two inequalities gives

\[
\int_0^\bar{r} D(r) \, dr = 0
\]

19
for all $b \in [0, \tilde{r}/2]$. This implies that the integral in the last equation is a constant function of $b$. Hence its derivative with respect to $b$, or $D(b)$, must equal zero for all $b$. So $f$ is symmetric. Finally, if $f$ is symmetric both bounds obviously are useless (zero). □

**Part (vi) (Special Bounds):** If $f$ is decreasing (increasing) on $[0, \tilde{r}/2]$, then $\tilde{r}/2$ maximizes $P(b)$ ($P(b)$).

### Proof of Proposition 2:

**Part (i) (Physical and Visual):** Following the discussion in the main text, to complete the proof it remains to show that inequality (8) is true. We introduce some auxiliary notation first. Let the signs of ring differences $D(r)$ alternate on $[0, r^*)$. Consider all those intervals on which $D$ retains its sign. We pair off these intervals into groups of two. That is, we divide $[0, r^*)$ into $n$ consecutive intervals (or rings) $[0, r_1^*], [r_1^*, r_2^*], \ldots, [r_{n-1}^*, r^*]$ such that the $i$-th such interval (ring) decomposes into one subset on which $D < 0$, denoted $[r_i^*, r_{i+1}^*]$, and another on which $D > 0$, written $[\tilde{r}_i, r_i^*]$. We also adopt $r_0^* = 0$ and $r_n^* = r^*$.

Now, by the mean value theorem of integration, there must be numbers $c'_i$ and $c''_i$, satisfying $\tilde{r}/2 \geq c'_i \geq c''_i > 0$ as well as $c'_i \geq c'_{i+1}$ for all $i$, such that

$$\max_{b \in [0, \tilde{r}/2]} \left[ \int_0^b D(r) \left( \tilde{r}/2 - r \right) dr \right] = \sum_{i=1}^n \left[ \int_{r_{i-1}^*}^{\tilde{r}_i} D(r) \left( \tilde{r}/2 - r \right) dr + \int_{\tilde{r}_i}^{r_i^*} D(r) \left( \tilde{r}/2 - r \right) dr \right]$$

$$= \sum_{i=1}^n \left[ c'_i \int_{r_{i-1}^*}^{r_i^*} D(r) dr + c''_i \int_{\tilde{r}_i}^{r_i} D(r) dr \right]$$

$$\leq \sum_{i=1}^n \left[ c'_i \int_{r_{i-1}^*}^{r_i^*} D(r) dr + c'_i \int_{r_{i-1}^*}^{r_i^*} D(r) dr \right]$$

$$= \sum_{i=1}^n c'_i \int_{r_{i-1}^*}^{r_i^*} D(r) dr. \quad (18)$$

Lemma 1 (following this proof) shows that $\int_{r_i^*}^{r^*} D(r) dr \geq 0$ for any $j = 1, \ldots, n$. That is, summing over any $j$ last ring differences gives a non-negative number. We continue with the r.h.s. of (18) making repeated use of this. I.e.,

$$\sum_{i=1}^n c'_i \int_{r_{i-1}^*}^{r_i^*} D(r) dr = \sum_{i=1}^{n-1} c'_i \int_{r_{i-1}^*}^{r_i^*} D(r) dr + c'_n \int_{r_{n-1}^*}^{r^*} D(r) dr \leq \sum_{i=1}^{n-1} c'_i \int_{r_{i-1}^*}^{r_i^*} D(r) dr + c'_{n-1} \int_{r_{n-2}^*}^{r_{n-1}^*} D(r) dr \leq \ldots \leq c_1 \int_{r_0^*}^{r^*} D(r) dr$$

Note how successive inequalities repeatedly exploit Lemma 1, for increasingly larger values of $j$. It remains to add that

$$c_1 \int_0^{r^*} D(r) dr \leq \int_0^b D(r) \left( \tilde{r}/2 \right) dr \leq \max_{b \in [0, \tilde{r}/2]} \left[ \int_0^b D(r) \left( \tilde{r}/2 \right) dr \right]$$

Putting all consecutive inequalities in this paragraph together completes the proof of inequality (8). □
Next we turn to the proof of the second inequality in Part (i). This latter inequality gives a related, if independent, result. Proving it relies on a sequence of inequalities akin to that in (7) through (9).

Compactly stated,
\[
\sigma = \int_0^{\tilde{r}/2} D(r) \left( \tilde{r}/2 - r \right) dr \geq \min_{b \in [0, r/2]} \left[ \int_0^b D(r) \left( \tilde{r}/2 - r \right) dr \right]
\]
(19)
\[
\geq \min_{b \in [0, r/2]} \left[ \int_0^b D(r) \left( \tilde{r}/2 \right) dr \right]
\]
(20)
\[
= -\tilde{r} \max_{b \in [0, r/2]} \left[ \int_0^b \left( -D(r) \right) dr / 2 \right] = -\tilde{r} \bar{l}^p.
\]
(21)

In this sequence of inequalities steps (19) and (21) are obvious again. It is inequality (20) that plays the crucial, and not-so-intuitive, part. Much as above, moving from the r.h.s. of (19) to the r.h.s. of (20) is not trivial because \( D(r) \) still cannot be signed. It is possible to prove that inequality (20) holds nonetheless. By implication, \( \sigma \geq -\tilde{r} \bar{l}^p \). Rearranging this inequality and combining it with \( \bar{l}^p \leq \bar{l} p \) (Proposition 1, Part (i)) yields the \(-\sigma / \tilde{r} \leq \bar{l} p\) also stated in the Proposition’s Part (i). □

**Lemma 1:**

For all \( j = 1, \ldots, n \), it is true that
\[
\int_{r_{n-j}}^{r^*} D(r) dr \geq 0.
\]
(22)

Ring differences in any last \( j \) rings sum to a non-negative number.\(^{25}\)

**Proof of Lemma 1:** The proof makes use of the notation introduced in the proof of inequality (8) in the main text. We first show that \( \int_{r_{n-1}}^{r^*} D(r) dr \geq 0 \). As our point of departure, recall that surely
\[
0 \leq \int_{r_{n-1}}^{r^*} D(r)(\tilde{r}/2 - r) dr
\]
because otherwise \( r^* \) could not be the optimizer. (The \( n \)-th ring should not have been included in \( l^0 \) \( ) \). But then
\[
0 \leq c'_n \int_{r_{n-1}}^{\tilde{r}} D(r) dr + c''_n \int_{r_n}^{r^*} D(r) dr
\]
\[
\leq c'_n \int_{r_{n-1}}^{\tilde{r}} D(r) dr + c'_n \int_{r_n}^{r^*} D(r) dr = c'_n \int_{r_{n-1}}^{r^*} D(r) dr
\]
Since \( 0 < c'_n \) this proves \( \int_{r_{n-1}}^{r^*} D(r) dr \geq 0 \). Next we show that \( \int_{r_{n-2}}^{r^*} D(r) dr \geq 0 \). Following similar reasoning as above, clearly
\[
0 \leq \int_{r_{n-2}}^{r^*} D(r)(\tilde{r}/2 - r) dr
\]
But then
\[
0 \leq c'_{n-1} \int_{r_{n-2}}^{\tilde{r}_{n-1}} D(r) dr + c''_{n-1} \int_{r_{n-1}}^{\tilde{r}_{n-1}} D(r) dr + c'_{n-1} \int_{r_{n-1}}^{r^*} D(r) dr
\]
\[
\leq c'_{n-1} \int_{r_{n-2}}^{\tilde{r}_{n-1}} D(r) dr + c'_{n-1} \int_{r_{n-1}}^{r^*} D(r) dr + c'_{n-1} \int_{r_{n-1}}^{r^*} D(r) dr = c'_{n-1} \int_{r_{n-2}}^{r^*} D(r) dr
\]
\(^{25}\)This property also is illustrated by the simple example of Figure 1’s panel (d) where positive ring differences dominate negative ring differences.
Since $0 < c'_{n-1}$ this proves $\int_{n-2}^{r} D(r)dr \geq 0$. Proceeding along these lines proves Lemma 1. □

**Lemma 2**: If building height $h(r)$ is convex then the following inequality holds:

$$\frac{\partial D(r, \tilde{r}, \alpha)}{\partial \tilde{r}} > 0.$$

**Proof of Lemma 2**: We note three properties of the building height (or skyline) function $h$. First, $h(r)$ also depends on $\tilde{r}$. A greater urban boundary $\tilde{r}$ increases pressure on rent, and this in turn raises building height. Second, $h(\tilde{r} - r)$ does not depend on $\tilde{r}$. A greater urban boundary $\tilde{r}$ does not increase pressure on rent in a ring $\tilde{r} - r$ that sees its relative position (its position relative to the urban boundary) unchanged. And third, the increase in building height implied by the urban boundary shifting out by one unit of distance equals the change in building height induced by approaching the CBD by one unit of distance, or:

$$\frac{\partial h(r)}{\partial \tilde{r}} r = - \frac{\partial h(r)}{\partial r}. \quad \text{(23)}$$

Differentiating $D(r, \tilde{r}, \alpha)$ as in (11) with respect to $\tilde{r}$, accounting for the first two Ricardian rent properties just listed and also dividing by $2\pi$ gives

$$\frac{1}{2\pi} \frac{\partial D(r, \tilde{r}, \alpha)}{\partial \tilde{r}} = \frac{\partial h(r)}{\partial \tilde{r}} r - \alpha h(\tilde{r} - r). \quad \text{(24)}$$

Now, by the convexity of $h$, we have

$$\frac{\partial h(r)}{\partial \tilde{r}} r < \frac{\partial h(\tilde{r} - r)}{\partial \tilde{r}} r < h(\tilde{r}) - h(\tilde{r} - r)$$

for $r \in [0, \tilde{r}/2]$. Combining this with the fact that $h(\tilde{r}) = 0$, making use of the third of Ricardian rent’s properties listed in (23) and exploiting $1 \geq \alpha$ gives

$$\frac{\partial h(r)}{\partial \tilde{r}} r > \alpha h(\tilde{r} - r).$$

This proves that either expression in (24) is strictly positive. □

**Proof of Proposition 3**: Implicitly differentiating (12) reveals the partial derivatives of $\tilde{r}$ with respect to parameters $\alpha$ and $\tilde{r}$. These derivatives’ signs are

$$\frac{\partial \tilde{r}}{\partial \alpha} < 0 \quad \text{and} \quad \frac{\partial \tilde{r}}{\partial \tilde{r}} > 0.$$

Assuming convexity of $h$, we infer the corresponding sign of $\partial D/\partial \tilde{r}$ by making use of Lemma 2. Finally, we write down the obvious property that

$$\frac{\partial D(r, \tilde{r}, \alpha)}{\partial \alpha} < 0. \quad \text{(25)}$$

Combining the various derivatives’ signs then reveals that

$$\frac{d l^{\alpha}(\alpha, \tilde{r})}{d \alpha} < 0 \quad \text{and} \quad \frac{d l^{\alpha}(\alpha, \tilde{r})}{d \tilde{r}} > 0.$$

The corresponding signs for the implied changes in $l^{\alpha}$ are simply the negative of the signs given above. □