Home Voters, House Prices 
and the Political Geography of Zoning

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Abstract: A number of countries have seen large house price increases over the years preceding the recent real estate bust. As has been argued by Glaeser et al. (2003) for the case of the US, these booms may also have been driven by the desire of homevoters – voters who are homeowners – to restrict supply, or “zone”. Only, homevoters’ observed role for the price of housing is ambiguous at best. This paper argues that this is because the underlying true role is. While homevoters have an interest in higher rent whereas tenants would prefer rent to be lower, these interests do not simply aggregate. As the paper’s model explains, neither do homevoter-ruled cities always wish to engage in, nor do tenant-ruled cities always wish to refrain from, zoning. – The paper motivates its model by presenting evidence on Germany’s large scale demolition (one particularly striking instance of zoning) joint with data on homevoters, zoning, housing attributes, and rents.

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1Access to Mikrozensus data at the Federal Statistical Office’s Center in Berlin (FDZ) is gratefully acknowledged.


1 Motivation

A number of countries have seen large house price increases over the years preceding the recent real estate bust. As has been argued by Glaeser et al. (2003) for the US and by Ortalo-Magné/Pratt (2008) more generally, these booms may also have been driven by the desire of homevoters – voters who are homeowners – to restrict supply, or “zone”. Only, homevoters’ observed role for the price of housing is ambiguous at best. This paper argues that this is because the underlying true role is. While homevoters have an interest in higher rent whereas tenants would prefer rent to be lower, these interests do not simply aggregate. As the paper explains, neither do homevoter-ruled cities always wish to engage in, nor do tenant-ruled cities always wish to refrain from, zoning. The paper motivates its analysis by presenting evidence on Germany’s demolition (one particularly striking instance of zoning) joint with data on homevoters, zoning, housing attributes, and rents.

In the empirical literature on homevoters, zoning and the price of housing (or rent for short), demonstrating a link either (i) between zoning and rent or (ii) between homevoters and zoning has not been a straightforward exercise, let alone identifying a causal relationship running all the way from homevoters to the price of housing. Earlier studies, as surveyed in Quigley/Rosenthal (2005), have frequently failed to identify a significantly positive effect of intensifying land use regulation on the price of housing. It is only in more recent work that tighter regulation consistently appears to drive house prices (e.g. Glaeser/Gyourko/Saks (2003), Ihlanfeldt (2005), Zabel/Dalton (2011), Magliocca et al. (2012)). Similarly, evidence on the relationship between zoning and homeownership (as a proxy to homevotership) is ambiguous also. Hilber/Robert-Nicoud (2009) find weak support of a positive relationship at best, whereas Glaeser/Ward (2009) actually identify a weakly negative relationship.

This paper argues that homevoters may exert substantial influence on rent nonetheless. The model brings together three explanations – one well-known, two apparently novel – of why a link between homevoters and house prices is not easily observed in empirical work. All three explanations depart from urban economics’ building block of rent increases in any given city driving away the more mobile among that city’s tenants. While homevoters’ strength in one city may tempt that city to zone, the rent increases thus triggered tend to spill into every other city, too. Rent changes dissipate, and this dissipation is one well known explanation of why cross-sectional differences in rent belittle the true underlying impact of zoning on rent. Or put differently, rent dissipation makes it difficult to trace rising rent back to its geographical origin.

Rent dissipation has long been established as one explanation of why a city’s rent may not be driven visibly by that city’s homeownership (e.g. White (1975), Hamilton (1978), Glaeser/Ward (2009)). The paper’s model offers two extra explanations that, if intimately related, do seem novel and are distinct. The first extra explanation is that homeowners’ strength in one city may imply supply regulations showing up in – another. If homeowner-dominated cities alternate with tenant-dominated cities, then homeowners strong in one city should find it advantageous to bribe tenants strong in its neighbor, to have extra restrictions imposed there that they themselves either have put in place
already or choose not to put in place at all. As emphasized by the literature on fiscal externalities and federalism (e.g. Oates (2011)), such decentralized trades are helped along by geographical proximity. For instance, homeowners in an agglomeration’s suburbs plus tenants in that agglomeration’s center might coalesce over extra regulation in the tenant-dominated center, complementing suburbs’ own zoning code.

The second extra explanation is offered by the non-linearity inherent to the relationship between homeowners and rent. Obviously, when the initial homeowner share in a city’s constituency is small then a small further rise in homeowners fails to establish political control, and hence does nothing to raise rent. If the homeowner share is close to the 50 percent threshold the same small increase in homeowner numbers will secure political control and may jumpstart zoning, giving rise to a sudden splurge in rent. And for an even higher homeowner share any further increase in homeowners will drive rents down. If homeowners are particularly numerous then tenants must be particularly few, and the homeowner majority’s incentive to exploit this minority of tenants must be weaker. While this non-linear relationship is intuitive at the city level, the paper’s model shows that this relationship also carries over to an urban system-wide, non-cooperative “zoning equilibrium”.

From a positive perspective, the paper’s model is an attempt to explain why homeowners, zoning and land rent empirically do not go together in obvious ways. From a normative perspective, and as more than just a side aspect of the analysis, the paper’s model also explores the welfare effects of zoning, effects that are non-too-obvious either. In the setup suggested just above the preceding paragraph, for instance, a tenant-ruled center is bribed by the landlord-ruled suburbs. The zoning induced in the center makes better off tenants in the center (who receive transfers that more than make up for the inconvenience of rising rent), makes better off landlords in both the center and the suburbs (whose benefits from rising real estate incomes exceed the transfer mentioned), yet makes worse off tenants in the agglomeration’s suburbs (who face a rising cost of living). In short, zoning affects the different groups of society in different ways, and these differences need not emerge along the fault lines of the absentee landlord single open city model.

If one way to introduce this paper’s two contributions is to point to its links to the literatures on zoning or on fiscal externalities and fiscal federalism, yet another way could be to emphasize its role in the debate on the social merits of homeownership. While homeownership does generate a number of positive externalities (e.g. Glaeser/diPasquale (1999)), at least one critical piece of the literature also suggests that pervasive homeownership leaves households more exposed to adverse shocks on the labor market (Oswald (1996)). – Now, a number of building blocks of the earlier theoretical literature enter this paper’s modeling setup as well. From Brueckner (1998) this paper’s model receives the idea that housing supply restrictions in one city interact with restrictions imposed in another. From Ortalo-Magné/Pratt (2007) and Hilber/Nicoud (2009) this paper inherits the explicit emphasis on the political struggle between the various stakeholders in the urban economy. And as Brueckner (1998) this paper also assumes congestion externalities away, and thus zoning’s potential role in correcting these.

At the same time, in contrast to Brueckner (1998) and Hilber/Nicoud (2009), land-
lords are not absentee but are a subset of the local electorate. In contrast to Hilber/Nicoud (2009), restrictions to the local housing supply are not assumed ad hoc but are explicitly modeled. In contrast to Ortalo-Magné/Pratt (2007), tenants are not assumed immobile but may migrate between cities. Finally, in contrast to Calabrese/Epple/Romano (2007) as another important contribution to the political economy of zoning, households are not heterogeneous in terms of their incomes, to the effect that zoning here is never meant to deter poorer households. – At the more technical level, a distinct feature of the model presented here is its extensive use of circulant matrices, matrices that are not commonly encountered in the economics literature. Not only are circulant matrices a natural reflection of this model’s circular setup. Circulants also enable us to rigorously derive the properties of the model’s non-cooperative “zoning equilibrium”.

Lest this paper’s theory of zoning appear artificial a case study precedes the model. This motivation focuses on demolition, with demolition understood to be an extreme variant of zoning, given that nothing is (permitted to be) built on the empty plots left behind. While a number of countries today are debating demolition of housing on a larger scale (e.g., the US (Glaeser/Gyourko (2008)) or Ireland (Irish Independent (2011))) our case study recounts Germany’s tremendous demolition program. In 2002 an estimated 1.4 million out of East Germany’s 7.4 million flats were vacant; and local governments have torn down over a quarter of a million units of public housing since. Officially this is to both “level obsolete and vacant housing” and “stabilize housing markets” (BMVBS (2006)). Yet the apparent contradiction between these two objectives – How can demolition restrict itself to vacant and obsolete housing if it is meant to stabilize housing markets? – raises the question of demolition’s true underlying rationale.

The paper’s empirical part presents three stills, on (i) zoning and rent, (ii) homeownership and zoning, and (iii) homeownership and rent, all three of which testify to the importance of a political economy perspective. (A full econometric analysis of these relationships is delegated to a follow-up paper, which can then test for those features of the present model that are not directly inspired by the motivational evidence presented here.) The first still is demolition’s positive correlation with rent; demolition does appear to “stabilize housing markets”. In previous analysis East Germany’s rents were found to develop more robustly than West Germany’s over those very years during which demolition was most intense (Dascher (2012)). By adding another wave of data on homeowners and tenants, housing attributes, and rent, this paper’s empirical part corroborates this finding.

The second still is demolition’s negative correlation with homeownership. Oddly, counties that demolish most tend to be tenant-ruled, rather than homeowner-ruled. In view of tenants’ supposed interest in low rent (via abundant supply) this would seem a paradox if demolition efforts were not simultaneously coupled with substantial state matching grants. Finally, the third still is homeownership’s failure to correlate positively with rent in a cross-sectional regression. – All three snapshots motivate the model’s interest in the circumstances under which homeowner-ruled cities do not, and tenant-ruled cities do, embrace zoning. Section 2 provides the paper’s case study. Ultimately the paper’s aim is not to uncover the rationale for East Germany’s demolition, but to contribute to
the larger political geography of zoning. This general analysis is in section 3. Section 4 concludes.

2 The Motivation

Our case study on zoning/demolition builds on a large micro data set of East and West German households (the Mikrozensus), available for the years 1998, 2002 (the beginning of East Germany’s demolition program) and 2006 (well into the program). For every responding tenant household we have information on the total rent paid and the floor space consumed, on the age of the building, on the number of the building’s stories, on the type of heating, on housing tenure, on county of residence and on degree of agglomeration of both city and county of residence. The Mikrozensus’ regional information permits us not just to relate apartment rents to county characteristics but also to connect average county level observations across different years, thereby creating a set of “true” panel data that ultimately permits us to control for those unobservable county effects that are invariant over time.3

Joining the resulting 305 county observations in both 2002 and 2006 gives a pooled data set of 610 counties. The benchmark model is the difference-in-differences approach employed in Dascher (2012). Here East Germany is the treatment (i.e. demolition) region and West Germany is the control:

\[
\ln q_{it} = \gamma_0 + \beta' x_{it} + \delta d2 + \gamma dT + \alpha (dT \cdot d2) + v_{it}
\]

(1)

where \(i\) is the county index, \(t\) indexes time, \(x_{it}\) is a vector of \(K\) covariates characterizing attributes of location and quality of that county’s average flat, \(\beta' = (\beta_1, \ldots, \beta_K)\) is the vector of these attributes’ coefficients, \(\gamma_0\) is the intercept, \(dT\) is a dummy variable that takes on the value of 1 if the county in question is located in East Germany and zero otherwise, and \(d2\) is a time dummy equal to one if the observation belongs to the second period.

The estimated coefficient on the interaction term, \(\hat{\alpha}_2\), reflects the OLS estimate of how strongly the rent of a flat with given attributes grew stronger in Eastern Germany during 2002-2006 than it did in Western Germany over that same period. For reference we first briefly reproduce the results in Dascher (2012). Column (1) in Table 1 shows the estimates for only pooling the data for 2002 and 2006. In this column the estimated coefficient \(\hat{\alpha}_2\) is not significantly different from zero. However, our estimates are biased if unobservable county fixed effects interfere. To correct for this we reestimate (1) in terms of its first differences (FD). Column (2) in Table 1 gives the resulting estimate \(\hat{\alpha}_2\), which

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2 Data are provided by the Forschungsdatenzentrum (FDZ) Berlin, a branch of Germany’s Statistical Offices.

3 Unfortunately the Statistical Offices do not disclose the regional identifier at the community level (which would be preferable) so that we need to aggregate data to the county level. Moreover, Bavaria’s Statistical Office does not even disclose the county identifier so that data on Bavaria are dropped. Finally, observations on Berlin and Hanover, where administrative boundaries changed, have been dropped, too. Then we map the 166,734 observations on tenant households for 1998 into 343 county averages, the 136,186 observations for 2002 into 328 county averages, and the 118,462 individual observations for 2006 into 341 county averages.
is positive and significant. Less susceptible to bias, this estimate informs us that for fixed flat quality Eastern Germany’s rents grew by 2.6 percentage points over and above West Germany’s trend (which was negative).

We want to rule out that the Eastern excess in rent change between 2002 and 2006 is due to some long-run underlying Eastern trend not connected to demolition. For instance, such a (negative) long-run Eastern excess could follow from the fact that the vacancy rate in East is much higher than that in West. Or, such a long-run (positive) Eastern excess might somehow reflect East’s convergence to West German levels. To control for this possibility we also make use of the available information on 1998. We first reestimate equation (1) on pooled data for 1998 and 2002, both in terms of levels (column (3)) and first differences (column (4)).

In neither of these regressions does the coefficient on the interaction term differ significantly from zero. This indicates that over the preceding period 1998-2002 – during which essentially no housing was demolished – Eastern rents did not develop differently from rents in West. The sudden increase in the Eastern excess in the rent change during the following period 2002-2006 may indeed reflect the effect of demolition.

To investigate this further we pool observations from all three waves 1998, 2002 and 2006, to then estimate the following full hedonic model:

$$\ln q_{it} = \gamma_0 + \beta' x_{it} + \delta_2 d2 + \delta_3 d3$$

$$+ \gamma dT + \alpha_2 (dT \cdot d2) + \alpha_3 (dT \cdot d3) + u_i + v_{it}$$

(2)

where $d2$ now is defined to equal one in both 2002 and 2006, and $d3$ is equal to one in 2006 only, to the effect that $\alpha_2$ then captures the initial deviation of the change in Eastern

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<thead>
<tr>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tr>
<td>$\gamma$</td>
<td>-0.184</td>
<td>-</td>
<td>-0.147</td>
<td>-</td>
</tr>
<tr>
<td>$(\hat{\gamma})$</td>
<td>(0.024)</td>
<td>(0.037)</td>
<td>(0.017)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>$\delta$</td>
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<td>-</td>
<td>-0.330</td>
<td>-</td>
</tr>
<tr>
<td>$(\hat{\delta})$</td>
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<td>(0.049)</td>
<td>(0.015)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>0.026</td>
<td>0.024</td>
<td>-0.048</td>
</tr>
<tr>
<td>$(\hat{\alpha})$</td>
<td>(0.015)</td>
<td>(0.008)</td>
<td>(0.015)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>Obs.</td>
<td>610</td>
<td>305</td>
<td>610</td>
<td>305</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.61</td>
<td>0.18</td>
<td>0.71</td>
<td>0.12</td>
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</table>

Table 1: Rent growth in East and West Germany (Standard errors in parentheses)

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4 Estimate $\hat{\gamma}$ shows by how many percent the Eastern rent lags the Western rent in the base period. Estimate $\hat{\delta}$ indicates by how many percent Western rent grew from the base period to the following period. These estimates conform to intuition. For example, the estimates imply an East-West rent gap of more than 18% in 2006 and a cumulative decrease in Western rent between 2002 and 2006 of almost 7%.

5 The large R-squared is due to the fact that the time dummy captures well the redefinition of rent which took place between 1998 and 2002, with rent in 1998 featuring payments that are no longer included in 2002 and 2006.
Table 2: Rent growth in East and West Germany (Standard errors in parentheses)

<table>
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<tbody>
<tr>
<td>$\hat{\gamma}$</td>
<td>$-0.106$</td>
<td>$-0.106$</td>
<td>$-0.106$</td>
</tr>
<tr>
<td></td>
<td>$(0.029)$</td>
<td>$(0.029)$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\alpha}_2$</td>
<td>$-0.031$</td>
<td>$-0.069$</td>
<td>$-0.031$</td>
</tr>
<tr>
<td></td>
<td>$(0.031)$</td>
<td>$(0.027)$</td>
<td>$(0.031)$</td>
</tr>
<tr>
<td>$\hat{\alpha}_3$</td>
<td>$0.040$</td>
<td>$0.032$</td>
<td>$0.040$</td>
</tr>
<tr>
<td></td>
<td>$(0.021)$</td>
<td>$(0.015)$</td>
<td>$(0.021)$</td>
</tr>
</tbody>
</table>

| $\bar{R}^2$ | 0.73 | 0.84 | 0.74 |

Obs. | 951 | 610 | 951 |

rent from the change in Western rent in 1998-2002; while $\alpha_3$ captures by how much East’s deviation from the West arising 2002-2006 exceeds that arising 1998-2002. Finally, the $u_i$ denote unobservable effects common to any given county’s observations. This latter model (2) we estimate in terms of levels (Table 2, column (1)), first differences (column (2)) and fixed effects (column (3)).

If East trended persistently differently from West, then the Eastern excess in 1998-2002, $\alpha_3$, should equal the Eastern excess in 2002-2006, $\alpha_2$, i.e. $\alpha_2 = \alpha_3$. In a two-sided t-test the results in Table 2 joint with the estimate of the parameter estimators’ covariance refute this hypothesis clearly though, for any of the first three columns. Besides, in the full model in equation (2) we once more see that ongoing demolition cannot have had no supply-constraining effect. We perform a one-sided test of the hypothesis that the Eastern excess did not increase from 1998-2002 to 2002-2006, i.e. $\alpha_3 \leq 0$. Making use of the estimates supplied within any of Table 2’s three columns, this hypothesis is easily rejected.

For a close-up on demolition we retrieve data on most Eastern counties’ cumulative demolition activities over the years 2002-2007 first.\(^6\) These data on demolition we combine with information on each Eastern county’s share of owner-occupied housing in total housing 1995 ($\text{homeownershare}$) and share of vacant housing in total housing 1995 ($\text{vacancyrate}$).\(^7\) We compute the share of sqm demolished during 2002-2007 in sqm of total housing in 1995 ($\text{dem}$), and then simply regress $\text{dem}$ on $\text{homeownershare}$ and $\text{vacancyrate}$. This yields the following descriptive equation (with an $\bar{R}^2$ of 0.33, a total of 69

\(^6\)Data on counties in Brandenburg, Saxony and Thuringia are provided by these states’ Statistical Offices. Demolition data are restricted to the years 2002-2007 because administrative boundaries were redrawn for Saxony’s counties in 2007. Data on counties in Saxony-Anhalt and Mecklenburg-Vorpommern unfortunately were not available for 2002-2007.

\(^7\)These latter data are taken from the Gebäude- und Wohnungsstättenzählung (GWZ) 1995, representing a complete count of all structures in East Germany in 1995. Again data were provided by State Statistical Offices.
observations and standard errors in parentheses):

\[
\hat{\text{dem}} = 0.121 - 0.381 \cdot \text{homeownershiprate} - 0.179 \cdot \text{vacancyrate} \tag{3}
\]

(0.020) (0.167) (0.031)

Counties that feature a larger share of tenants are found to level a greater fraction of their initial stock, even as one controls for the vacancy rate as a proxy for counties’ “need”. Equation (3) almost certainly is misspecified, but nonetheless is taken to suggest that tenant-ruled cities do demolish more than do homeowner-ruled cities – even after controlling for variables omitted. Let us next equate a county’s demolition with the funding that that county receives through state and federal government matching grants.\(^8\) Then the estimated equation (3) may suggest that state and federal government redistribute from landlords to tenants. This raises the question of which purpose such redistribution serves in the first place.

The third and last snapshot directly relates homevoters to rent. Said Mikrozensus also offers information on household members’ ages. From this we construct the number of homevoters as all those individuals who live in a household that owns its shelter. Dividing this number by the total number of individuals in households interviewed gives the homevoter share. We construct homevoter shares for all counties, and then regress county rent on all the county controls listed in \(x_t\) in (1) joint with the homevoter share for 1998 (as the earliest available wave). As mentioned above, the resulting first-guess estimate on the homevoter share is not significant. Other, similar specifications do not yield positive estimates either, even when accounting for homeownership’s dependence on rent by introducing the population’s age structure in 1993 as an instrument for homeownership in 1998. None of these non-results are reported here; yet they are an important part of the subsequent model’s motivation. (Detailed results are available on request.)

To summarize, demolition is less intense in counties where homeowners are stronger; rents do not rise stronger in counties where homeowners are stronger; yet being close to where demolition takes place does appear to push rent. This suggests a setting in which rents, and zoning efforts even, are contagious.

3 The Model

Subsection 2.1 sets out an urban system composed of many cities, each of which is open to individuals from neighboring cities. Cities are arranged on a circle, giving rise to an external spatial structure reminiscent of the Hotelling model. At the same time each city’s internal spatial structure conforms to the standard v. Thünen layout. Cities are monocentric, and land rents only settle at households’ maximum bids. Into this economic geography subsection 2.2 then introduces a political antagonism between landlords and tenants that goes beyond the standard economic conflict over rent that is typical of all voluntary transactions in the housing market. In contrast to parts of the urban economics literature landlords are not absentee but do reside within the city.

\(^8\)A total of 2.5 billion euro were spent on demolition between 2002 and 2010, a sum that exceeded the pure cost of demolition.
Landlords and tenants wrestle for control over the decision on whether to limit land supply, or “zone”. Cities with a tenant majority never ponder introducing such zoning but cities with a majority of landlords do. In Nash equilibrium, all cities with a landlord majority zone. Subsection 2.3 then explores the dissipation of the rent effect of one landlord-ruled city’s zoning throughout the urban system. While in the literature this dissipation is typically considered as restraining any given city’s incentive to zone, here it is treated as an externality that may make city zone nonetheless – if for a very different reason. Rather than zone to increase her own land revenues, a tenant-ruled city may zone to increase her neighbors’ revenues.

3.1 The Economic Geography

A total of $n$ cities are laid out on a circle, where $n$ is even throughout. Generally it is true that cities have at least two neighbors. Our circular setup retains this important property’s essence, by assigning two neighbors to each city. Cities are indexed $i = 0, \ldots , n - 1$, in clockwise fashion. If $n = 8$, for example, then city 0 has cities 7 and 1 as immediate neighbors. More generally, if $s$ is a positive integer we address city $i$’s two neighbors reached after moving $s$ cities either in clockwise fashion (“down the circle”, as a shorthand) or in counterclockwise fashion (“up the circle”). Then on the one hand $i + s$ identifies city $i$’s neighbor reached after moving $s$ cities down provided we properly redefine $i + s$ as the remainder when $i + s$ is divided by $n$, or $(i + s) \mod n$. On the other hand, $i - s$ properly identifies city $i$’s neighbor reached after moving $s$ cities up as long as we reinterpret $i - s$ as the – positive – remainder when $i - s + n$ is divided by $n$, or $(i - s) \mod n$ for short again. As an example of computing this positive remainder, moving 7 cities up from city 3 takes us to city $4 = ((3 - 7) + 8) \mod 8$.

Similarly, if we move from $i$ to $j$ in clockwise fashion then the distance between these two cities is $j - i$, where $j - i$ implicitly always is properly redefined as

$$j - i = (j - i) \mod n$$

(4)

If city $i$ is 6 and city $j$ is 3, then the clockwise distance between cities 6 and 3 is $5 = (3 - 6) + 8 \mod 8$. Alternatively, if we move from $i$ to $j$ in counterclockwise fashion then the counterclockwise distance is $i - j$, again if properly redefined. In the example this is $3 = ((6 - 3)) \mod 8$. Finally, the shortest distance between the two cities, denoted $d_{ij}$, is the smaller of the two available distances, $d_{ij} = \min \{j - i, i - j\}$. For example, the shortest way to go from city 1 to city 7 is to go up, covering a distance of only $2 = d_{17}$. – All through the paper city indices arising from sums or differences of indices or integers are thought to have undergone prior proper adjustment.

Into any given city a total of $L_i$ landlords and $T_i$ tenants are born, making $P_i = L_i + T_i$ its population total. Landlords are immobile across cities, while tenants are assumed mobile. However, and as explained in detail below, half of the tenants will only ever migrate to the upper neighbor, while the other half will only ever consider the lower one. And while all tenants are mobile in principle, they do differ with respect to their individual

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9This is sometimes also written as $(i + s) \mod n$. E.g., see Graybill (1983, section 8.10.)
migration costs. We assume that individual migration cost \( m \) is distributed uniformly on \([0, \bar{m}_i/2]\), with \( \bar{m}_i > 0 \). Tenants do not own any land. Instead landlords own, and between them equally share the proceeds of, all urban land. Moreover, the landlord class also own all undeveloped land in the city’s vicinity. Cities are spaced sufficiently far apart from each other to make this assumption meaningful.

In that landlords own the land that their tenants occupy, they truly are landlords. Yet in that landlords also own the land they themselves occupy, they should rightfully also be considered homevoters. This dual landlord-homevoter role is not an unpleasant artefact of the model’s ambition to simplify but must be seen as an implication of housing ownership’s not being divided up equally in society. If some households are tenants only, then some other household(s) must be both homeowner and landlord. Only in that we suggest that tenants’ plots are owned by homeowners resident in the same, rather than some other, city do we impose an assumption that might be subject to discussion later. For now, in view of landlords’ dual role the paper refers to landlords as homevoters whenever that latter concept is more appropriate.

The city’s CBD, or central business district, is where everybody must go for work. From the CBD the city extends into one direction, at unit width. There are no structures, to the effect that housing equals land. Thus if the marginal plot is \( r \) units away from the CBD then \( r \) units of housing are occupied altogether. All workers need to commute to the CBD from where they reside. Commuting cost per unit of distance travelled daily is \( 2t \), and the daily urban wage earned at the CBD is \( w \) and constant across cities. Rent paid (or earned) at distance \( r \) from the CBD is \( q_i(r) \) for the single unit of housing that households occupy. Hypothetical rent at the CBD is \( q_i(0) \), which is shortened to \( q_i \). In the city’s internal spatial equilibrium, all tenants must be equally well off, so that the urban-wide structure of rents is given by \( q_i(r) = q_i - 2tr \).

The model assigns landlords an important role in local elections and for this it must treat these landlords as residents. But dispensing with the absentee landlord framework raises the question of where exactly landlords reside. In this paper landlords always populate the city’s inner ring, while tenants are relegated to the outer ring. (We might assume that landlords were born into the city first, populating the plots closest to the CBD.) Naturally, a landlord may consider moving out himself for to then rent out an inner ring plot to a tenant in order to receive greater rent. But note that in our simple framework the attendant gain (represented by the tenant’s greater rent bid) is just offset by the loss wrought by added travel (imposed on the landlord-household now having to commuting longer). We conclude that landlords have no incentive to leave the inner ring.

As agricultural bid rent is set to zero both, the urban boundary and the city’s total supply of housing, equal \( q_i/2t \). The \( L_i \) units closest to the CBD are inhabited by the city’s landlords-homevoters. Local politics might decree to not let the remainder be rented out in its entirety. Such a decree is referred to as zoning. (Note though that this may also correspond to a policy of outright demolition of existing structures.) Zoned land is denoted \( z_i \), and by assumption is always located at the city’s periphery. More precisely, zoned land comprises all land between the urban fringe, \( q_i/2t \), on the one hand and \( q_i/2t - z_i \) on the
other. With this, net housing supply to the tenant class becomes \( q_i/(2t) - L_i - z_i \) units of land. Tenant utility is \( w - q_i \), while landlord utility is \( w \) plus the representative landlord’s share in overall urban rent.

We turn to tenants’ location decisions. We assume that one half of tenants are oriented towards city \( i - 1 \): Should these tenants ever emigrate, they would only do so to city \( i - 1 \). The other half of tenants, correspondingly, are oriented towards to city \( i + 1 \). Now first suppose that \( q_{i-1} < q_i \). A tenant native to \( i \), oriented towards \( i - 1 \), endowed with migration cost \( m \) and swapping \( i - 1 \) for \( i \) now receives \( w - q_{i-1} - m \) instead of \( w - q_i \). Thus a tenant native to \( i \), oriented towards to \( i - 1 \) and endowed with migration cost equal to just \( q_i - q_{i-1} \) is indifferent between leaving and staying while every tenant native to \( i \), oriented towards \( i - 1 \) and endowed with migration cost short of this rent differential leaves for \( i - 1 \). Total emigration out of \( i \) and into \( i - 1 \), denoted \( M_{i,i-1} \), becomes

\[
M_{i,i-1} = (q_i - q_{i-1}) T_i / \bar{m}_i. \tag{5}
\]

Alternatively, suppose that \( q_i < q_{i-1} \). Then total immigration from \( i - 1 \) into \( i \) is \( (q_{i-1} - q_i) T_{i-1} / \bar{m}_{i-1} \). A reasonable assumption here is that migration cost’s variance is increasing in the tenant class’ size. While some cities may have larger tenant classes than others, these cities also exhibit greater \( \bar{m}_i \). Specifically, we assume that the ratio \( T_i/\bar{m}_i \) is constant for all \( i = 1, \ldots, n \). But then \( M_{i,i-1} \) not just represents the migration flow from city \( i \) to \( i - 1 \), if positive. Also, it represents the migration flow from city \( i - 1 \) to \( i \), if negative. Letting \( i \) run from 0 through \( n - 1 \) in (5) gives the bilateral migration flows among all city-pairs conceivable.

Total demand for \( i \)'s housing comes from native landlords, native tenants still present, and immigrant tenants, and can be written as natives \( P_i \) plus overall net immigration \( (M_{i+1,i} - M_{i,i-1}) \). Equating this sum with net housing supply \( q_i/2t - z_i \) and making use of the migration flow definition in (5) twice yields the market clearing condition for housing in \( i \),

\[
q_i/(2t) - (T_i/\bar{m}_i) (q_{i-1} + q_{i+1} - 2q_i) = P_i + z_i. \tag{6}
\]

Letting \( i \) run from 0 through \( n - 1 \) traces out all \( n \) conditions for local housing market clearing. These conditions can only be solved simultaneously. Since the \( n \) left hand sides in (6) are linear in all of the urban system’s rents \( q' = (q_0, \ldots, q_{n-1}) \) we define the coefficient matrix \( A \) with entries \( a_{ij} \) such that

\[
a_{ij} = \begin{cases} 
  a = 2T_i/\bar{m}_i + 1/(2t) & \text{if } i = j \\
  -b = -T_i/\bar{m}_i & \text{if } i = j + 1 \text{ or if } i = j - 1 \\
  0 & \text{else.}
\end{cases} \tag{7}
\]

with \( i, j = 0, \ldots, n-1 \). (For an illustration of this matrix see (22) in the Theory Appendix. Variables \( a \) and \( b \) are only referenced in this Appendix.) Here the first row relates to city 0, and the \( i \)-th row relates to city \( i - 1 \).

\[\text{In view of the plethora of ways to zone (emphasized by Quigley/Rosenthal (2005)), while this paper’s might appear a very crude way to model zoning it should capture the essence of zoning nonetheless.}\]
Now, $A$ is an $n \times n$ matrix with a strong pattern. Non-zero entries are only found on the main diagonal, on the two diagonals below and above the main diagonal, and as elements $a_{n-1,1}$ and $a_{1,n-1}$. All other elements are zero. As the Theory Appendix shows in detail, $A$ is symmetric, diagonally dominant, and circulant (Lemma 1), where $A$ being circulant implies that elements along any given diagonal are constant. These three properties confer a great deal of structure on the inverse of $A$, and the various other matrices built around that inverse later. $A$’s dominant diagonal implies that all of its inverse’s entries are strictly positive; and $A$’s being circulant implies, among other things, that the sum of the elements of any of its inverse’s rows (or columns) is the same (Lemma 2).

If we let $d' = (P_0 + z_0, \ldots, P_{n-1} + z_{n-1})$ be the vector of right-hand sides in (6) we can rewrite this system more compactly as $Aq = d$. Due to our assuming the ratio $T_i/m_i$ to be constant across all $i$ (even as $T_i$ and $m_i$ are free to vary), none of the elements of $A$ will be determined within the model. Moreover, $A$’s inverse $A^{-1}$, henceforth denoted $C$, exists so that the solution to (6) simply is $Cd$. Rent in any city $i$ is a linear function not just of the parameters governing city $i$ but of the parameters governing all cities. Specifically, rent in city $i$ equals

$$q_i = \sum_{j=0}^{n-1} c_{ij} (P_j + z_j)$$  

(8)

where the $c_{i0}, \ldots, c_{i,n-1}$ are the $n$ successive entries in $C$’s row $i$. In the case where the urban population $P_j$ is the same in every city and equal to $P$ and where there is no zoning anywhere, the rent vector emerging for housing market clearing equals $P \sum_{j=0}^{n-1} c_{ij}$. In the Theory Appendix it is shown that for every row of $C$ the sum of entries, as the second term in this latter product, equals $2t$ (Lemma 2, Property (v)). But then every city’s rent for such symmetric housing market clearing is $2tP$. – More generally and for reference, Proposition 1 collects two intuitive yet important results regarding the responses of rents to changes in zoning efforts.

**Proposition 1: (Rent Response and Rent Dissipation)**
We identify the following responses of rents to zoning:

$$(i) \quad \frac{\Delta q_i}{\Delta z_i} > 0 \quad ; \quad (ii) \quad \frac{\Delta q_j}{\Delta z_i} > 0$$  

(9)

where $i, j = 0, \ldots, n-1$ and $i \neq j$. Hence,

(i) (Rent Response) Rent in city $i$, $q_i$, is strictly increasing in zoning in city $i$, $z_i$.

(ii) (Rent Dissipation) Rent in city $j$, $q_j$, is strictly increasing in zoning in city $i$, $z_i$.

Proposition 1 exploits the fact that the ratio of $\Delta q_i$ to $\Delta z_j$, being the response of rent in city $i$ to extra zoning in city $j$, is just the entry found in row $i$ and column $j$ of $C$ – all elements of which are strictly positive. On the one hand, extra zoning in city $i$ raises that city’s rent. On the other, extra zoning in city $i$ also lifts rent everywhere else, despite migration linking a city with its immediate neighbors only rather than linking it with every other city. Much as in the filtering literature (e.g. Sweeney (1974)), rent changes originating in one housing segment (quality or spatial) diffuse into other segments, too. Of
course, this has long been recognized in the literature on the extent to which communities are substitutes to each other (e.g. White (1975), Hamilton (1978)).

Additional properties follow from the setup introduced so far. For instance, the spillover from a zoning city \( k \) felt \( s \) cities down from \( k \) is the same as the spillover felt \( s \) cities up. Only, we want to focus on those properties that are important later. The mere fact that rent spills over into neighboring cities raises two fundamental questions familiar from the theory of positive externalities. On the one hand, if inter city cooperation is little then spillovers should take away from a given city’s incentive to zone. On the other hand, if inter city cooperation is intense then the disincentive tied to spillovers may actually be mitigated by inter city negotiations followed by concomitant Pareto-improving transfers. Both these themes are pursued below, with Proposition 4 exploring the first theme and Proposition 5 addressing the second. – Proposition 2 next analyzes the extent to which rent changes dissipate in the urban system, and on how this dissipation depends on inter city mobility. (The proofs of all following propositions are in the Theory Appendix.)

**Proposition 2: (Distance Decay and Inter City Mobility)**

We identify the following additional responses of rents to extra zoning:

(i) \[ \frac{\Delta q_i}{\Delta z_k} > \frac{\Delta q_j}{\Delta z_k} \quad \text{if} \quad d_{ik} < d_{jk} \]

(ii) \[ \frac{\Delta q_i}{\Delta z_k} \to \frac{2t}{n} \quad \text{as} \quad b \to \infty. \]

(i) (Distance Decay) In response to extra zoning in city \( k \), rent in any city \( i \) closer to \( k \) changes by more than rent in any other city \( j \neq i \) even further away from \( k \) does.

(ii) (Inter City Mobility) As intercity mobility grows \((b = T_i/m_i \text{ rises as all the } m_i \text{ fall suitably})\), any city’s rent changes induced by city \( k \)’s zoning converge to \( 2t/n \).

If city \( k \) zones and if city \( i \) is closer to \( k \) than city \( j \) is then the rent change induced in \( i \) will exceed that triggered off in \( j \). More generally, rent increases gradually die out as one moves away from the source of extra zoning. The degree to which these rent increases die out is not fixed, however. We may roughly distinguish between two cases. If mobility is limited such that the \( m_i \) are large and \( b \) consequently is small then rent increases fade away quickly and near their source, with the impact of extra zoning in \( k \) largely being absorbed by its closer neighbors. In contrast, if mobility is unlimited so that \( b \) is large, rent increases travel throughout the urban system and are even evenly distributed across the entire urban system. We return to this latter property when discussing whether inter city trades in spillovers can really arise, showing that such trades do arise if household mobility is sufficiently strong.

**An Example:** Let \( n = 8, a = 10, b = 4.5 \) and hence \( 2t = 10 \). Then matrix \( C \)'s fourth column, \( C_3 \), looks as follows:

\[
C'_3 = (0.08, 0.11, 0.16, 0.24, 0.16, 0.11, 0.08, 0.07)
\] (10)

Following this column, zoning in city 3 will drive up all cities’ rents, given that all of the column’s entries are strictly positive. Rent goes up strongest at the source, by 0.24 Euro. City 2’s and city 4’s rent increases are next largest, at 0.16 Euro each. Rent increases in other cities yet further away are smaller still, equal to 11 cents only. Finally, the rent increase is smallest at the city opposite city 3, generating a mere 7 cents extra.
3.2 The Political Geography

As noted above, if the native population $P_i$ is the same in every city and if initially there is no zoning then competitive rents $q_i$ are the same in every city. Thus in such an initial symmetric competitive equilibrium there is no inter city migration at all. Depending on the initial respective sizes of landlord class and tenant class, $L_i$ and $T_i$, either landlords or tenants govern $i$. If $T_i > L_i$ then tenants rule, and to city $i$ we will refer to as a tenant city. A tenant city will not want to restrict urban land, will set $z_i$ to zero, and will have competitive rent obtain. Conversely, if $T_i < L_i$ then homevoter–landlords rule, and city $i$ becomes what we will call a homevoter city. There landlords toy with the idea of zoning. Because restraining access to developable land raises rent on those remaining units ultimately rented out, such “monopoly zoning” may raise landlords’ aggregate rental income as long as remaining tenants are sufficiently numerous.

There is no restriction with regards to whether or not any particular city is landlord ruled. Nor do we restrict the overall number of homevoter cities. But we do assume that homevoter cities are spaced equally far apart. That is, the distance when going from one homevoter city to the next is always the same. For example, if $n = 8$ we either may have eight homevoter cities, we may have four homevoter cities spaced two cities apart, or we may have two homevoter cities spaced four cities apart. We address these equidistant homevoter cities by defining an ordered set $I$ that collects homevoter cities’ indices, in the ordering implied by starting with the smallest homevoter city index and then sliding along the urban circle. For example, if cities 5, 3, 7 and 1 are homevoter cities then $I = \{1, 3, 5, 7\}$.

When focusing on some homevoter city $i$ in this set $I$, let us assume that $\delta_i$ is the cost of zoning one unit of housing there. (This becomes particularly meaningful if zoning is interpreted as demolition. In the case of zoning $\delta_i$ reflects the cost of monitoring vacant plots, to ensure that they stay vacant.) The landlord class’s revenue sums to

$$ R_i = \left( q_i/(2t) - L_i - z_i \right) \cdot \left( 2tz_i + (q_i - 2tL_i) \right) / 2 - \delta_i z_i $$

The first term in brackets gives housing actually rented out. The second term in brackets represents average rent earned on this latter stock when making use of the following facts: (i) the lowest rent earned on any plot is $2tz_i$ (at the border between tenant occupied housing and zoned, peripheral land), (ii) the highest rent earned on any plot is $q_i(0) - 2tL_i$ (at the border between landlord occupied housing and tenant occupied housing), and (iii) the urban rent gradient $q_i(r) = q_i - 2tr$ is linear so that average rent is just the average of the two rent extremes.

The landlord class maximize total rent in (11) with respect to $z_i$, taking zoning efforts in all other homevoter cities as given. Prior to finding revenue’s first derivative, simplifying it and setting it equal to zero we quickly recall that rent’s derivative $dq_i/dz_i$ is a constant and also written as $c_i$ or $c_0$. Itself implied by the linearity of the solution to competitive housing rents in (8), this property considerably simplifies our subsequent analysis. We write homevoter city $i$’s first order condition as

$$ q_i c_0/(2t) - 2tz_i = \delta_i + c_0 L_i, $$
from which we see that zoning efforts are strategic complements. Extra zoning in some homevoter city other than \( i \) results in greater rents anywhere, and hence also in \( i \). But then the marginal benefit to extra zoning in \( i \) rises, too. Further, note that the second derivative of aggregate rental income with respect to zoning \( z_i \) equals \( c_0^2/(2t) - 2t \), which is easily shown to be strictly negative.\(^{11}\) With the s.o.c. strictly negative, the f.o.c. does characterize optimum zoning on the part of \( i \)'s government.

We next replace \( q_i \) featuring in (12) by the expression obtained for housing market clearing, spelt out in (8). Making use of the assumption that local population \( L_i + T_i = P \) is the same in any city, noting that \( z_j = 0 \) whenever \( j \notin I \) and rearranging the resulting condition translates into

\[
\frac{c_0}{(2t)} \left[ \sum_{j \in I} c_{ij} z_j \right] - 2t z_i = \delta_i - c_0 T_i
\]

for all homevoter cities \( i \in I \).

The resulting system of equations has \( \pi = |I| \) equations, where \( \pi \leq n \). To rewrite this as a matrix equation we first define the coefficient matrix \( D \) with entry \( d_{ij} \)

\[
d_{ij} = \begin{cases} 
    c_{ij} \cdot \frac{c_0}{(2t)} - 2t & \text{if } i = j \\
    c_{ij} \cdot \frac{c_0}{(2t)} & \text{else},
\end{cases}
\]

where \( i \) and \( j \) are restricted to those indices contained in \( I \), making us retain the initial city labels in what follows. Moreover, let \( g \) be the \( \pi \times 1 \) vector of right hand sides in (13). And denote by \( z \) the \( \pi \times 1 \) vector of zoning efforts across homevoter cities. Then (13) can more compactly be rewritten as \( Dz = g \). Let \( F \) denote \( D \)’s inverse, should it exist. Then this system’s solution \( Fg \), or \( \tilde{z} \) more briefly, is our zoning equilibrium. Corresponding rents are subsequently found by substituting the solution \( \tilde{z} \) back into housing market clearing conditions (8), and are written as \( \tilde{q}_i \). Further details are given in Proposition 3.

Proposition 3: (Zoning: Existence, Uniqueness, Symmetry, Rents and Welfare)

Let \( n < \pi \) and suppose that \( c_0 T_i - \delta_i \) is the same positive constant for all \( i \in I \). Then:

(i) (Zoning Existence and Uniqueness) A unique zoning equilibrium exists in which homevoter cities’ zoning efforts are strictly positive.

(ii) (Zoning Symmetry) Homevoter cities’ uniform zoning efforts are

\[
\tilde{z}_i = \frac{c_0 T_i - \delta_i}{2t - \frac{c_0}{(2t)} \cdot \sum_{j \in I} c_{ij}}.
\]

(iii) (Zoning and Homevoter City Rents) Rents in homevoter cities are identical.

(iv) (Zoning and Tenant City Rents) A tenant city’s rent falls short of homevoter cities’ rent. Further, a tenant city’s rent is decreasing in distance to the nearest homevoter city.

(v) (Zoning and Welfare) The symmetric zoning equilibrium is Pareto-inefficient.

\(^{11}\)The sum of all elements of any row of \( C \) is \( 2t \) (Lemma 2 in the Theory Appendix). But then \( c_0 \), as just one entry in that row, is smaller than \( 2t \).
At the technical level, the proof makes repeated use of the fact that $D$ inherits being circulant from $C$, and even is diagonally dominant, giving rise to its inverse $F$ being a circulant with strictly negative entries only. Proposition 3 first emphasizes that a Nash equilibrium in zoning efforts among homevoter cities exists.\(^\text{12}\) Existence of a non-cooperative equilibrium suggests that homevoter cities may indeed artificially restrict the supply of housing to sustain income from local housing even as their “monopoly zoning power” is constrained by households’ potential flight to neighboring cities nearby. Also, this existence is at odds with the idea that a homevoter city discard the option of sitting back in order to free ride on other homevoter cities’ zoning efforts. Now, if the excess of demolition costs $\delta_i$ over the gross rent gain $c_0T_i$ is the same in any homevoter city then homevoter cities’ zoning efforts in (15) are the same, too. Obviously homevoter cities’ zoning efforts are decreasing in the number of resident tenants that can possibly be exploited, $T_i$.

Since $D$ can be shown to have positive elements only, any increase in $\delta_i$ makes the solution decrease. While this is a reassuring feature of the model, if $\delta_i$ is too large then $Fg$ turns negative. So we add that $Fg$ represents a zoning equilibrium only if it is positive. Otherwise $\bar{z}$ is zero. Moreover, can we be sure that cities that start out as homevoter cities continue to be homevoter cities in zoning equilibrium? As we will see next, the answer is yes. In zoning equilibrium homevoter cities are more expensive than tenant cities so that very mobile tenants will have left whereas immobile landlords have stayed on. This asymmetry in migration reinforces the prexisting homevoter majorities in homevoter cities.

Formally, equilibrium rents are found by solving for $\bar{z}$ in (15) first and for $\bar{q}$ in (8) then. We first evaluate the impact equilibrium zoning efforts have on equilibrium rents. We find that any homevoter city charges a higher rent than any tenant city does, and for any comparable distance between residential location and CBD. Moreover, tenant cities’ rents can be ranked in a natural way. For every tenant city we may identify its next-nearest homevoter city. Then it is true that a tenant city closer to its next-nearest homevoter city exhibits a higher rent than a tenant city further away from its next-nearest homevoter city, again at comparable distance from the CBD.

If we go on to assume that all cities initially have the same landlord population $L_i$ then in zoning equilibrium homevoter cities actually house a smaller overall population – even as their developable area is larger. This of course reflects the loss of habitat, or the extra in sprawl, imposed by the local zoning. Put differently, on average tenant cities house tenants further out than homevoter cities do. It will not come as a surprise here that tenant cities’ marginal residents – those living right next to agriculture – could then be easily made better off, without making anyone else worse off. This can be achieved by reallocating these residents to that part of any homevoter city’s zoned area that is closest – and also closer – to a CBD. (The zoning equilibrium’s Pareto-inefficiency admittedly is due to the fact that no externalities in need of correction distort the no-zoning equilibrium.)

– The following numerical example illustrates a number of Proposition 3’s properties.

**The Example Continued:** Suppose that there are $\pi = 4$ homevoter cities, cities

\(^{12}\)To be sure, to prove existence and uniqueness we do not need to impose the strong assumption that $c_0T_i - \delta_i$ is constant across cities. These differences merely need to be positive.
1, 3, 5 and 7. Suppose further that $\delta_i = 0$, $P = 7$ and that $L_i = 4$ for all $i \in I$. Then equilibrium zoning amounts to 0.83 units of housing in each of the four homevoter cities. Corresponding equilibrium rents are

$$q' = (7.39, 7.43, 7.39, 7.43, 7.39, 7.43, 7.39, 7.43)$$

Due to the dissipation of the impact of the four homevoter cities’ zoning on rents, homeowner cities are not much more expensive than tenant cities. At the same time, if zoning had not taken place then rents would be 7 Euro throughout, hence much smaller.

The example demonstrates how the cross-sectional rent change observed in the data once the zoning equilibrium has emerged very much (in the example: roughly ten times) understates the rent change that the city-wise before-after comparison attaches to zoning. From this perspective, the cross-sectional approach to zoning embraced in a number of empirical studies does not seem to be well suited to uncover zoning’s full impact on real estate markets. Now, because homevoter cities have a higher cost of living than those tenant cities surrounding them and since the wage is always the same, remaining tenants are worse off than tenants living in cities ruled by tenant-governments. In contrast to the open city literature, utility is not the same throughout the entire urban system. This is why homevoter cities house less tenants than tenant cities do.

There are various ways in which both the spatial structure and pervasiveness of homevoter dominance play into the comparative statics of zoning and rents. We first analyze an exogenous increase in the number of homevoter cities. As a caveat to our assumption of homevoter cities being equally distant to each other we may not always be able to increase the number of homevoter cities by just one. For example, if $n = 12$ then going from 3 to 4 equidistant homevoter cities is possible, but from 4 to 5 is not. Moreover, a homevoter city’s identity may change as the number of homevoter cities grows, an identity change that is problematic in a time-series context. At the same time, this identity change poses no particular challenge in the context of cross-sectional comparisons. Proposition 4’s first part below documents how an exogenous change in the number of homevoter cities resonates in terms of extra zoning and higher rents (Property (i)).

Building on this result we analyze the exogenous and gradual increase in a homevoter city’s homevoters, rising up from a very small initial level to encompassing the entire local population. To this end let some city in the set $I$ be a tenant city initially, with a single homevoter-landlord only. As we increase the number of homevoters in this city, by gradually having tenants turn homevoter, we find that zoning efforts and rents initially do not respond at all (because homevoters yet fail to conquer the town hall), then rise very abruptly (as the homevoter share in the local electorate reaches one half), to then fall continuously as the homevoter share in the local population tends to 1. Proposition 4 summarizes the resulting non-linearity in zoning’s and rents’ responses to homevoter numbers (Property (ii)). This latter property, too, advises against a simple linear estimation strategy when analyzing cross-sectional data on homevoters and zoning or rents.

Proposition 4: (Zoning Equilibrium and Homevoter Strength)

(i) (Homevoter Cities’ Number, Zoning and Rents) Let the number of equidistant homevoter
cities increase from \( n_1 \) to \( n_2 \). Then for \( \Delta \pi = n_2 - n_1 \) and \( i \in I \),

\[
\frac{\Delta \tilde{z}_i}{\Delta \pi}, \quad \frac{\Delta \tilde{q}_i}{\Delta \pi} > 0.
\] (17)

(ii) (Homevoter Numbers, Zoning and Rents) If the number of homevoters \( L_i \) rises in a given homevoter city while that city’s overall native population \( P \) remains the same, then

\[
\frac{\Delta \tilde{z}_i}{\Delta L_i}, \quad \frac{\Delta \tilde{q}_i}{\Delta L_i} \begin{cases} 
= 0 & \text{if } L_i \leq T_i \\
< 0 & \text{if } L_i > T_i
\end{cases}
\] (18)

No formal statement is included here to indicate what happens with tenant city rents as homevoter cities become increasingly numerous. Yet intuitively it must be clear that an increase in homevoter cities’ number \( \pi \) should not just raise rents in tenant cities also, but should even depress the difference between homevoter city rents on the one hand and tenant city rents on the other. After all, as homevoter cities’ prevalence goes up, any tenant city’s distance to its next nearest homevoter city goes down also, so that the distance decay in rents becomes less pronounced. Put more succinctly, the greater homevoters’ dominance in society is the less this dominance shows up in rent’s cross-sectional variation. This, joint with a number of additional properties mentioned, is illustrated in the sequel to our example:

**The Example Continued:** If there are merely \( \pi = 2 \) homevoter cities (cities 2 and 6, say), then zoned areas in these two cities equal 0.78 < 0.83 units of housing, and the resulting equilibrium rents are

\[
q' = (7.17, 7.18, 7.24, 7.18, 7.17, 7.18, 7.24, 7.18)
\] (19)

This illustrates how with less homevoter cities rents are lower than in the previous example (see (16)) throughout – though still in excess of the 7 Euro that would be observed in the absence of any zoning. Moreover, we see how rent changes in tenant cities decay as we move away from one homevoter city – from 7.24 via 7.18 to 7.17 – yet rise again – from 7.17 via 7.18 back to 7.24 – as we approach the next. Finally, observe how the variation in rent rises as the number of homevoter cities falls.

### 3.3 Voluntary Tenant City Zoning

Let the number of homevoter cities reach \( n/2 \). Then homevoter cities alternate with tenant cities along the urban circle. The pair of homevoter cities now encircling each tenant city could be viewed as an agglomeration’s suburbs or periphery, with the tenant city that periphery’s central city or center. (It is true that such agglomerations overlap yet this does not alter the analysis that follows.) In terms of our earlier example of four homevoter cities altogether, if the center zones one unit then homevoters earn 16 cents on every plot rented out while tenants resident with the tenant city zoning lose 24 cents (see the list (10)). If the center’s tenant majority is tiny, if the peripheries’ homevoter majorities are tiny also and if native populations all equal \( P \), then the tenant city’s aggregate loss in tenant welfare is \( 0.5\hat{P} \cdot 0.24 \) while the periphery’s aggregate homevoter welfare gain is \( 2 \cdot 0.5\hat{P} \cdot 0.16 \) – which is larger.
The positive fiscal externality joint with the geographical fact that all three cities are close to each other (so that transfers become but yet another item in the existing fiscal center-periphery relations, not modeled here) joint with the political fact that it is a jurisdiction’s well-defined right to zone suggests that homevoter suburbs offer a transfer to the tenant center, equal to some sum within $[0.12P, 0.16P]$ in exchange for one unit of zoning in the center.\textsuperscript{13} This already points to the environment that is fertile in encouraging this type of trade, an environment featuring not just similar homevoter numbers but also suburb governments’ complete disregard for their respective tenant minorities.

More generally, consider any given tenant city $i$ (the center) and the two homevoter cities adjacent to it $i-1$ and $i+1$ (the periphery). Suppose that the center zones $z_i$ units of housing there. Departing from zoning equilibrium $\tilde{z}$ makes suburb $i-1$ gain

$$t\left(q_{i-1}/(2t) - L_{i-1}\right)^2 - t\left(\tilde{q}_{i-1}/(2t) - L_{i-1}\right)^2$$

where $q_{i-1}$ ($\tilde{q}_{i-1}$) is the rent obtained after (before) the center has zoned. These terms indicate homevoter incomes after and before center zoning has set in.

A similar gain applies to the other suburb. And given that in zoning equilibrium no center tenant remaining will want to flee to the periphery (which is more expensive according to Property (iv) of Proposition 3), all of those $\tilde{q}_i/(2t) - L_i$ center tenants lose $q_i - \tilde{q}_i$ each. Summing the two welfare terms corresponding to the two suburbs (one of them spelt out in (20) already), subtracting center tenants’ loss just mentioned, adding zoning cost, taking the first derivative of the resulting sum with respect to $z_i$ and evaluating that derivative at the zoning equilibrium gives the net benefit from marginal “Coasian zoning” in the center,

$$\left(\tilde{q}_{i+1}/(2t) - L_{i+1}\right) c_1 + \left(\tilde{q}_{i-1}/(2t) - L_{i-1}\right) c_1 - \left(\tilde{q}_{i}/(2t) - L_i\right) c_0 - \delta_i$$

where $c_1$ is shorthand for $dq_{i-1}/dz_i = dq_{i+1}/dz_i$. The first two terms indicate the two suburbs’ rent gains, approximately equal to these cities’ outer rings multiplied by the rise in rent. The third term captures the center’s welfare loss, equal to that city’s initial number of tenants times the rent rise these tenants have gone through.

If rent spillovers from center zoning $c_1$ are sufficiently large then suburbs indeed have an incentive to pay the center for starting to zone. Technically, the rent spillover $c_1$ can always be made arbitrarily close to the rent change in the zoning city, $c_0$ by requiring mobility to be sufficiently high (Proposition 2, Property (ii)).

**Proposition 5: (Zoning in Cities with Tenant Majority)**

Suppose $\pi = n/2$, $L_i = L_{i-1} = L_{i+1}$. A tenant city $i$ will zone in exchange for a Coasian transfer from the two homevoter cities $i-1$ and $i+1$ surrounding it if mobility is sufficiently large and zoning cost $\delta_i$ sufficiently small. This trilateral trade makes majority voters in each of the three cities involved better off, at the expense of all those tenants in the urban system that are not resident in $i$.

\textsuperscript{13}This principle may be generalized. If homevoter neighbors 3, 5 or even $s$ ($s$ odd) cities away from $i$ are part of the coalition also then the compensation paid out to the center may be even bigger. Our objective here is to show that trading in zoning occurs even if the number of bribers is small, provided that mobility is sufficiently high and transaction costs are sufficiently low.
Put differently, if zoning cost is zero then it is always possible to identify a level of mobility beyond which tenant city zoning is beneficial to all of the agglomeration’s three cities. Proposition 5 also suggests that the distinction between homevoter cities’ zoning and tenant cities’ zoning becomes blurred as soon as zoning in tenant cities no longer just is simply zero. Moreover, as the number of homevoter cities reaches the threshold $n/2$ zoning will intensify in homevoter cities (Proposition 4, Property (i)).\footnote{While Proposition 5 emphasizes the incentive to zone in a tenant city, this proposition does not specify an equilibrium in which tenant cities zone. At little extra cost in terms of additional assumptions, such an equilibrium can be shown to exist though (even if it is no longer unique). For reasons of space exposition this equilibrium is not included here. – One possible application of Proposition 5 comes in the case of a four region (city) setup. Let us divide Germany very roughly into the following four equal-sized regions: West (Northrhine-Westphalia, Rhineland-Palatinate) and East (Saxony, Saxony-Anhalt, Thuringia, Mecklenburg-Vorpommern, Berlin) both as tenant-ruled regions, and South (Bavaria, Baden-Wuerttemberg, Hessia, Saarland) and North (Lower Saxony, Schleswig-Holstein, Hamburg, Bremen) as homeowner-ruled regions both. Proposition 5 suggests that South and North would bribe East to zone as soon as reunification became fact.} Note, though, that Proposition 5 does not suggest that tenant cities zone more than do homevoter cities.

4 Conclusions

A case study (section 2) provides the starting point for this paper’s theory of zoning. During the demolition period 2002-2006, East Germany’s rent grows by more than West Germany’s rent does, while it fails do so during the prior, no-demolition period 1998-2002. This difference-in-difference-in-differences perspective rules out two types of concerns commonly raised. Neither is the sudden excess in Eastern rent 2002-2006 due to some trend underlying both East and West (a trend which surely should not be associated with demolition); nor is it due to some trend affecting East even before demolition set in (a trend which should not be associated with demolition either). Second, demolition and its funding focus on tenant-ruled cities, rather than on homevoter-ruled cities. And third, homevotership fails to explain rent in the baseline cross-sectional setup.

The paper’s theory part (section 3) casts the political geography of zoning as the outcome of a – political – conflict of interests between homevoters and tenants. Any increase in zoning ultimately is due to homevoters reaching yet another threshold. The model identifies three different thresholds at least. Obviously, if a single city’s local home-owner share exceeds half of the local population then that city begins to zone (Proposition 3); less obviously, if the share of homeowner-ruled cities exceeds half of the overall number of cities then all tenant-ruled cities enclosed begin to zone (Proposition 5); and if the national homeowner share exceeds half of the national population then the federal government provides incentives to all of its constituent cities – be they homeowner- or tenant-ruled – to zone.

Hence not only does the overall number of homeowners matter; homeowners’ spatial distribution matters, too. It must be misleading to speak of – the – homeownership rate here. Besides, increasing homeownership need not just drive zoning up; it also drives zoning down once tenant numbers have sufficiently dwindled (Proposition 4). – An important question left open in the paper is whether if homevoters should have played a role
in driving the recent real estate boom homevoters may not also have played a role in the even more recent bust. Addressing this question suggests making a household’s tenure choice explicit (left to a follow-up paper). Intuitively, augmenting the model by a tenure decision may generate a self-defeating boom in real estate prices, as higher rent feeds into rising homevoters numbers while rising homevoter numbers feed back into lower rent.
5 References


Irish Independent (2011) Nama boss says foreign banks may bulldoze ghost sites, June 24th 2011.


6 Theory Appendix

Lemma 1: (Properties of the Coefficient Matrix, $A$)

The coefficient matrix $A$ can be written

$$
\begin{pmatrix}
  a & -b & 0 & 0 & \ldots & 0 & 0 & -b \\
  -b & a & -b & 0 & \ldots & 0 & 0 & 0 \\
  0 & -b & a & -b & \ldots & 0 & 0 & 0 \\
  \vdots \\
  0 & 0 & 0 & -b & \ldots & a & -b & 0 \\
  0 & 0 & 0 & 0 & \ldots & -b & a & -b \\
  -b & 0 & 0 & 0 & \ldots & 0 & -b & a
\end{pmatrix}
$$

(22)

once we set $b = T_i/m_i$ and $a = 2T_i/m_i + 1/(2t) = 2b + 1/(2t)$. Note that $a - 2b$, or $1/(2t)$, is the sum of all elements of any given row or column. Now, $A$ has the following properties:

(i) $A$ is symmetric.

(ii) $A$ is diagonally dominant.

(iii) $A$ is circulant. That is, element $a_{ij}$ equals $a_{j-i}$ or, as stated in the text and emphasized here one more time, more properly $a_{(j-i)\mod n}$ (Graybill (1983), section 8.10).

Proof of Lemma 1

(i) We first focus on the elements on the diagonal that comes just below the main diagonal. Consider an element of that diagonal, $a_{j+1,j}$. By (7), this equals $-b$. We compare this element with $a_{j,j+1}$, being an element on the diagonal that comes just above the main diagonal. Again by (7), this equals $-b$. But then we have shown $a_{j+1,j} = a_{j,j+1}$. Proceeding in this fashion shows that $A$ is symmetric. □

(ii) Consider the first column of matrix $A$ in (22). Given that $a - 2b = 1/(2t) > 0$, the entry on the main diagonal dominates all other entries of that column. All other columns can be treated in similar fashion. □

(iii) A matrix is circulant if the elements on any given diagonal are identical to each other. As (22) illustrates, matrix elements are zero except for the five diagonals. For the two diagonals located in the bottom left and top right corner there is nothing to show. Only for the elements on these diagonals do we need to show that elements are identical. Yet this is obvious from the definition of $A$ in (7). □

Lemma 2: (Properties of the Inverse of the Coefficient Matrix, $C$)

Consider the inverse of $A$, with $A^{-1}$ denoted $C$ as in the text.

(i) $C$ exists.

(ii) $C$ is symmetric.

(iii) $C$ is circulant. That is, $c_{ij} = c_{j-i}$.

(iv) The sum of all elements of any column or row of $C$ is $2t$.

(v) For any given row of $C$, elements satisfy $c_k = c_{n-k}$ for $k = 1, \ldots, n - 1$.

(vi) For any given row of $C$, the following ranking applies:

$$
c_0 > c_1 > \ldots > c_{n/2}.
$$

(23)

(vii) $C$ has strictly positive elements only.
Proof of Lemma 2

(i) Since $A$ is diagonally dominant it is non-singular and hence $C$ exists (Graybill (1983, Theorem 8.11.2)). □

(ii) Since $A$ is symmetric, so is its inverse. □

(iii) Since $A$ is circulant, $C$ is also (Graybill (1983, Theorem 8.10.4)). □

(iv) Let $v_1$ be a column vector of ones, of dimension $n$. We form the matrix product $v_1 A C_0$, where $C_0$ is the first column of $C$. This matrix product is a number (i.e. $1 \times 1$-matrix), and this number must equal 1, by $C$ being $A$'s inverse. Moreover, this number also equals

\[
\sum_{i=0}^{n-1} \sum_{k=0}^{n-1} a_{ik} c_{k1} = \sum_{k=0}^{n-1} c_{k1} \sum_{i=0}^{n-1} a_{ik} = \sum_{k=0}^{n-1} c_{k1} (a - 2b)
\]

where the second equality follows from the fact that each column of $A$ sums to $(a - 2b)$ (see Proof of Lemma 1, Property (ii)). But then $\sum_{k=0}^{n-1} c_{k1} = 1/(a - 2b) = 2t$. The proof for any other column of $C$ is similar. □

(v) Because $C$ is circulant (Property (iii)), row 0 is $(c_0, c_1, \ldots, c_{n-1})$ while column 0 is $(c_0, c_{n-1}, \ldots, c_1)'$. And because $C$ is symmetric (Property (ii)), $c_1 = c_{n-1}, c_2 = c_{n-2}$, etc. To summarize,

\[c_k = c_{n-k}\quad\text{for}\quad k = 1, \ldots, n-1.\quad(24)\]

(vi) Consider matrix $A$ as in (22). Consider $n/2$ as the index of the city “facing” city 0, in the circular context. Information on this city’s housing market is contained in row $n/2$ of $A$. Multiply this row by the first column of $C$, i.e. by $(c_0, c_{n-1}, \ldots, c_1)'$. This gives

\[-b(c_{n/2-1} + c_{n/2+1}) + ac_{n/2} = 0\quad(25)\]

Property (v) reveals that $c_{n/2+1} = c_{n/2-1}$. Employing this and rearranging yields

\[c_{n/2} = 2 \alpha c_{n/2-1}\quad(26)\]

where $\alpha = b/a$. Note that because $a - 2b > 0$, $\alpha$ is strictly smaller than 1/2 so that $c_{n/2} < c_{n/2-1}$. Next multiply row $n/2 - 1$ by the first column of $C$. This gives $-b(c_{n/2-2} + c_{n/2}) + ac_{n/2-1} = 0$ or, after making use of (26) and rearranging,

\[c_{n/2-1} = \frac{\alpha}{1 - 2\alpha^2} c_{n/2-2} \quad(27)\]

Given $\alpha < 1/2$ we also have $1 - 2\alpha^2 > 1/2$. But then $\alpha < 1 - 2\alpha^2$ and hence $c_{n/2-1} < c_{n/2-2}$. Continuing on in this fashion we see that in the list \{c_1, \ldots, c_{n/2}\} each element is strictly smaller than its predecessor – except for the first.

To sort out this first element multiply row 0 of $A$ by column 0 of $C$. This gives $-b(c_1 + c_{n-1}) + ac_0 = 1$. From $c_1 = c_{n-1}$ (Property (v) again)

\[c_0 = 1/a + 2ac_1. \quad(28)\]

The right hand side of the equation is strictly greater than $c_1$. This is easily verified when making use of $c_1 < 2t$ (Property (iv)). But then the left hand side of this equation, $c_0$, is strictly greater than $c_1$ also. This completes the ranking of rent responses in (23). □

(vii) Since $A$ is diagonally dominant, all of $C$'s elements are non-negative (Graybill (1983), Theorem 11.4.2 (Property (7)) joint with Theorem 11.43 (Property (3))). It remains to confirm that all entries in $C$ are strictly positive.

By this Lemma’s Property (iii) it is sufficient to prove that all elements of the first column of $C$ are strictly positive. Let us focus on column 0 thus. Multiplying the first row of $A$ by the first column of its inverse $C$, and making use of this Lemma’s Property (v), gives $ac_0 - 2bc_1 = 1$.\[\]
Given that $c_1 \geq 0$ it is not possible that $c_0 = 0$. Hence $c_0 > 0$. Multiplying the second row of $A$ by the first column of its inverse $C$ and again making use of Property (v) gives $ac_1 - b(c_0 + c_2) = 0$. Given that $c_0 > 0$ it is not possible that $c_1 = 0$. Hence $c_1 > 0$. Etc. Continuing in this fashion reveals that all entries of $C$'s first column are strictly positive. □


Proof of Proposition 2: (Distance Decay and Inter City Mobility)

(i) (Distance Decay) The ratios mentioned in the proposition, $\Delta q_i/\Delta z_k$ and $\Delta q_j/\Delta z_k$, equal $c_{ik} = c_{k-1}$ and $c_{jk} = c_{k-j}$, respectively. If either $k - i$, $k - j$, or even both, exceed $n/2$ we map those indices in excess of $n/2$ into indices smaller than $n/2$ by invoking Lemma 2’s Property (v). So without loss of generality let us assume that $k - i$ and $k - j$ are $n/2$ or smaller. But then the assumption $k - i < k - j$ joint with the ranking (23) implies $c_{ik} = c_{k-i} < c_{k-j} = c_{jk}$. □

(ii) (Inter City Mobility) From (26) we see that the ratio $c_{n/2}/c_{n/2-1}$, or $2a = 2b/a = 2b/(2b + 1/(2t))$, not only grows but also converges to 1 as $b \to \infty$. Similarly, in (27) the ratio $c_{n/2-1}/c_{n/2-2}$ converges to 1 as $b \to \infty$, too. Likewise, all other ratios of elements not involving $c_0$ can be shown to increase monotonically, and to converge to 1. Put differently, with the exception of $c_0$ all of $C$’s elements become arbitrarily close to one another as $b \to \infty$.

But then it is also possible to show that all elements of $C$ converge as $b$ tends to infinity. Suppose $c_0$ is not monotonically decreasing. Then increasing $b$ must at least once fail to depress $c_0$. Combine this with the fact that distance decay – each element $c_i$ in the list $\{c_0, \ldots, c_n/2\}$ is smaller than its predecessor $c_{i-1}$ (Lemma 2, Property (vi)) – becomes less and less pronounced as $b$ rises. Thus the sum of elements in the set $\{c_0, \ldots, c_n/2\}$ and hence also the sum of all entries of any of $C$’s columns grow. This is in contradiction to that sum being equal to $2t$ throughout (Lemma 2, Property (iv)).

We conclude that $c_0$ must be monotonically decreasing in $b$. To this we add the fact that $c_0$ is bounded from below, by zero. Hence $c_0$ must converge to some limit as $b$ tends to infinity. Yet if $c_0$ is convergent then $c_1$ must be convergent, too. After all, using (28) $c_1 = (ac_0 - 1)/2b$ so that $c_1$ is the sum of $c_0$, $a/2b$ and $1/2b$. The latter term obviously converges to zero while the former is the product of a convergent sequence, $c_0$, with another convergent sequence, $a/2b$. By the rules on convergent sequences and given the fact that $a/2b$ converges to 1 the sequence $c_1$ converges, too, and to the same limit as $c_0$.

Continuing on in this fashion shows that all elements in $\{c_0, \ldots, c_n/2\}$ not only converge but even share the same limit. Let $\bar{c}$ denote this common limit. But then $a\bar{c}$ must equal $2t$, the sum of any given column’s elements. This yields the limit given in the proposition. □

Lemma 3: (Circulant Symmetric Submatrix)

We construct a submatrix of $C$ called $\bar{C}$ that is circulant and symmetric, too. Let homevoter cities be evenly spaced such that the common distance between any two neighboring homevoter cities, $s = n/\pi$, is an integer. Let $k$ be the city index of the first homevoter city encountered when starting at city 0 and moving down clockwise. Then cities $k, k + s, k + 2s, \ldots, k + n - s$ are homevoter cities. Their indices are collected in $I$, the ordered set of $\pi$ homevoter cities’ indices.

Now let us reduce $C$, by eliminating all rows and columns that do not have an index in $I$. This lemma claims that the resulting $\pi \times \pi$-submatrix $\bar{C}$ is circulant and symmetric.

Proof of Lemma 3:

For the proof we need to label rows and columns in $\bar{C}$ by indices $r = 0, \ldots, \pi - 1$ and $p = 0, \ldots, \pi - 1$, respectively. The element in row $r$ and column $p$ of $\bar{C}$ is $c_{k+r\sigma,k+p\sigma}$ or, given that $C$ is circulant, $c_{(p-r)\sigma}$. We show that all the other elements on the same diagonal are the same, too. Now, the element $\sigma$ rows further down (or up, if $\sigma < 0$) and $\sigma$ columns further to the right (or to the left, if $\sigma < 0$) is $c_{k+(r+\sigma)\sigma,k+(p+\sigma)\sigma}$ or, given that $C$ is a circulant, simply $c_{(p-r)\sigma}$, too. Thus $\bar{C}$ is circulant. □
Proof of Proposition 3: (Zoning Efforts in Zoning Equilibrium)

(i) (Zoning Existence) Matrix $D$ is diagonally dominant. This essentially follows from the definition of $D$ in (14). Given that $c_{i1} < 2t$ clearly main diagonal elements must be strictly negative; while given that $c_{ij} > 0$ (Lemma 2, Property (vii)) off-diagonal elements are strictly positive.

Now, the sum $\sum_{i=0}^{n-1} c_{ij}$ just equals $2t$ (Lemma 2, Property (iv)). Omitting one (or more) of these elements (corresponding to tenant cities) while making remaining elements smaller (by multiplying them with $c_0/2t$, a number smaller than one) implies that it is sufficient that $\delta_i - c_0 T_i$ to be positive it is sufficient that $\delta_i < c_0 T_i$. □

(ii) (Zoning Symmetry) Matrix $D$ equals $(c_0/2t) \cdot \mathcal{C} - 2t \cdot E$, with $E$ an $\pi \times \pi$ identity matrix. $D$ is circulant, because $\mathcal{C}$ and $E$ are (Lemma 3) and because a weighted sum of circulant matrices is also circulant (Graybill (1983), Theorem 8.10.1). Moreover, all elements in column $j$ of matrix $D$ sum to $c_0/(2t) + \sum_{i \in I} c_{ij} - 2t$.

Since $D$ is circulant, this latter sum is the same in every column. Moreover, since the inverse of a circulant also is a circulant (Graybill (1983), Theorem 8.10.4) $F$ is a circulant, too. This in turn implies that for any given column its elements sum to

$$\frac{1}{c_0/(2t) + \sum_{i \in I} c_{ij} - 2t}.$$

Multiplying $F$ with the vector of right hand sides $(\delta_i - c_0 T_i, \ldots, \delta_i - c_0 T_i)$ gives (15). □

(iii) (Zoning and Homevoter City Rents) In the symmetric zoning equilibrium the right hand sides in (6) become $P$ for $i \notin I$ and $P + \bar{z}_i$ if $i \in I$ (where, to be sure, $\bar{z}_i$ is constant for all $i \in I$). Let $x$ be an $n \times 1$ index vector such that the entry at row $i$ is 1 if $i \in I$ and 0 otherwise, and let $\iota$ be an $n \times 1$ vector of ones. Then $\iota P + x \bar{z}_i$ is shorthand for the right hand sides of (6), and

$$\tilde{q} = C(\iota P + x \bar{z}_i) = C \iota P + C x \bar{z}_i. \quad (30)$$

On the right hand side the first term equals the $n \times 1$-vector $\iota \cdot 2t P$, which has identical entries across cities; whereas the second term equals the $n \times 1$-vector $(C x) \bar{z}_i$, now also denoted $y$, which clearly does not exhibit identical entries across cities.

The entry $y_i$ we obtain by summing all elements of $C$ in row $i$’s successive columns $j \in I$. That is, for $y$ we have

$$y_i' = \left( \sum_{j \in I} c_{0j}, \ldots, \sum_{j \in I} c_{(n-1)j} \right) \cdot \bar{z}_i. \quad (31)$$

Recall that $\mathcal{C}$ is circulant (Lemma 3). That is, for rows $i \in I$ the elements in $\mathcal{C}$ sum to the same number. But then the entries $(\sum_{j \in I} c_{ij}) \bar{z}_i$ in rows $i \in I$ of $y$ are equal to each other. Hence the $\tilde{q}_i$ are the same for all $i \in I$, too. □

(iv) (Zoning and Tenant City Rents) The proof is in two steps. First we establish that rents in tenant cities of the same distance to the next nearest homevoter city are the same (Step I). Then we show that the proposition’s claim tenant cities closer to the next homevoter city are dearer (Step II).

(Step I): We claim $q_k = q_{k+s} > q_{k+1} = q_{k+s-1}$ with $k,k+s \in I$. I.e. consider the tenant cities bounded by two successive landlord cities, $k$ and $k+s$. Suppose one moves one city away from $k$
(away from $k + s$), in the direction of $k + s$ (in the direction of $k$). To find the overall effects of the various zoning efforts onto these two tenant cities note that we may ignore the effects of zoning on the part of all homevoter cities other than $k$ and $k + s$ onto the two tenant cities $k + 1$ and $k + s - 1$ because these effects will be the same anyway, given the overall symmetry.

Moreover, note that not just does the effect of zoning in $k$ on $k + 1$ equal the effect of zoning in $k + s$ on $k + s - 1$. (After all, in both cases distance is the same, equal to one city.) Also, the effect of zoning in $k$ on $k + s - 1$ equals the effect of zoning in $k + s$ on $k + 1$. (Again, in both cases the distance is the same, now equal to $s - 1$ cities.) More formally, this is because

\[
\begin{align*}
c_{k+1,k} &= c_{n-1} = c_1 = c_{k+s-1,k+s} \\
c_{k+s-1,k} &= c_{n-s+1} = c_{s-1} = c_{k+s-1,k+s}
\end{align*}
\]

where the equality in the middle of either (32) or (33) follows from Lemma 2’s Property (v) and where all other equalities merely apply the definition of a matrix being circulant.

We conclude that $\tilde{q}_{k+1} = \tilde{q}_{k+s-1}$. Repeating this argument reveals that for any $l \in \{1, \ldots, s/2 - 1\}$ we must have $q_{k+l} = q_{k+s-l}$

(Step II): Consider the tenant city located at the center of the segment $\{k, \ldots, k+s\}$, city $k+s/2$. We assume that adjacent cities $k+s/2-1$ and $k+s/2+1$ are tenant cities, too, lest the proof is complete. These adjacent tenant cities must exhibit the same rent (Step I). But then

\[q_{k+s/2} < q_{k+s/2+1} = q_{k+s/2-1}\]

must be true. Suppose not. Then while the introduction of zoning in homevoter cities would make rents, and also housing supply, go up, housing demand would not go up because neighboring cities’ rents would be cheaper, and hence would pull mobile residents away. The housing market in $k+s/2$ would no longer balance. Hence (6) must be true. Continuing this argument proves the ranking indicated in the proposition. □

(v) Zoning and Welfare: Let us focus on (any) one homevoter city and (any) one tenant city. In the model’s zoning equilibrium, a tenant city is more expensive than any homevoter city (Proposition 3, Property (iv)). Hence the tenant city’s population exceeds the population of any homevoter city. This in turn implies that $\tilde{q}_j/(2t) > \tilde{q}_i/(2t) - \tilde{z}_i$ for $j \notin I$ and $i \in I$.

Thus there are locations $r'$ in the homevoter city’s zoned area that are closer to the CBD than is the tenant city’s marginal plot at the tenant city’s urban boundary $\tilde{q}_j/(2t)$. Reassigning a perfectly mobile tenant (who surely exists given that until now there has only been immigration into the tenant city) at the tenant city’s boundary to the vacant plot $r'$ units away from the homevoter city’s CBD will release part of that tenant’s commuting cost while making none worse off. □

**Proof of Proposition 4: (Zoning Equilibrium and Homevoter Strength)**

(i) (Homevoter Cities’ Number, Zoning and Rents) As $\pi$ rises the index set $I$ includes even more indices and, hence, the sum in the denominator on the right hand side of (15), or $c_0/(2t) \sum_{i \in I} c_{ij}$, now includes even more elements. Moreover, as homevoter cities necessarily now are positioned closer to each other each element that was included in that sum beforehand now gets replaced by an element that is larger.

The proof of rents rising in every city follows the same pattern as the proof of the preceding property, and thus is only sketched here. As extra homevoter cities appear on stage and begin to zone, incumbent homevoter cities not only receive an extra housing price stimulus from these extra zoners. Also, given greater closeness to each other they also receive even greater stimuli from incumbent zoners. Thus equilibrium rents rise. □

(ii) (Homevoter Numbers, Zoning and Rents) Without loss of generality, let us consider changing $T_{ik}$ in city $k$, where $k$ is the first index contained in $I$. On the one hand, as long as homevoters have not captured the majority of votes then there is no zoning in $k$. 27
On the other hand, if homevoters constitute the majority of $k$’s electorate then $k$ becomes a homevoter city so that zoning assumes a strictly positive value. To further assess the effect of a variable $T_k$ we describe a non-symmetric zoning-equilibrium first. Let $\delta_i - c_0 T_i$ be identical for all $i \in I \setminus \{k\}$, and denote this common value by $v$. Then decompose $\delta_k - c_0 T_k$ into $v + w$, where $w = (\delta_k - c_0 T_k) - v$.

Multiplying $F$ by the vector of right hand sides $(v + w, v, \ldots, v)$ gives an $\pi \times 1$ column vector with first element

$$\frac{v}{c_0/(2t) \cdot \sum_{i \in I} c_{ij} - 2t} + f_0(\delta_k - c_0 T_k - v),$$

representing $\tilde{z}_k$, where $f_0$ is a typical, strictly negative element on the main diagonal of circulant $F$. Straightforward comparative statics then reveal that $\tilde{z}_k$ is strictly increasing in $T_k$.

To assess the effect of a change in $T_k$ on rent we briefly note that that change brings about equidirectional changes in all other homevoter cities’ zoning efforts, too. But then rents in all homevoter cities must surely rise. □

**Proof of Proposition 5: (Tenant-City-Zoning in Homevoter-Tenant-Coalitions)**

Let us ignore $\delta_i$ for the moment. Exploiting the fact that $\tilde{q}_{i-1} = \tilde{q}_{i+1}$ (Proposition 3, Property (iv)), making use of $L_{i-1} = L_i$, dividing (21) through by $c_0$ and rearranging for $c_1/c_0$ yields a strictly positive residual iff

$$\frac{c_1}{c_0} > \frac{\tilde{q}_i/(2t) - L_i}{2(\tilde{q}_{i-1}/(2t) - L_i)} \quad (34)$$

For $b$ sufficiently large the left hand side is strictly larger than the right hand side:

As $b$ tends to infinity the ratio on the left hand side tends to 1 (Proposition 2, Property (ii)). And as $b$ tends to infinity both $\tilde{q}_i$ and $\tilde{q}_{i-1}$ tend to $\eta = 2t P + \pi \pi$. Thus the numerator on the right hand side tends to $\tilde{q}_i/(2t) - L_i$, while the denominator tends to $2(\eta/(2t) - L_i)$. So the right hand side approaches one half. □