Urban Centrists according to the Greatest Cumulative Ring Difference

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Abstract: On a wide range of issues, urban voters’ interests can be shown to align according to whether their average property is close to the city center (voters that we label centrists) or near the periphery (decentrists). We bound the unobservable number of centrists by its observable minimum. This is the title’s greatest cumulative ring difference. Along similar lines we bound decentrists by (minus) the least cumulative ring difference. Both bounds we then take to cities’ involvement in (i) opposing decentralization, (ii) fighting climate change, (iii) introducing zoning regulation on minimum lot size and building height, (iv) imposing growth controls and (v) encouraging homeownership.

Keywords: Greatest Cumulative Ring Difference, Decentralization, Climate Change, Homeownership, Zoning Regulation

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1 Introduction

Urban housing ownership traditionally comes along in one of two forms. Under “common ownership”, all citizens receive an equal share of overall rental income (e.g., Pines/Sadka (1986)). Or landlords are “absentee” (e.g., Wildasin (1986), Kanemoto (1980)). Somewhat paradoxically, the first regime amounts to assuming that every resident is a landlord; while the second goes as far as to assume that no resident is one. Of course, these regimes are meant to keep urban political economy tractable. Yet it might also be fair to say that they do not model housing ownership at its most consistent. Either regime removes the housing ownership heterogeneity at the heart of “real cities”. Either regime defines away the large class of urban conflicts in which it is landlords (rather than landlords and their tenants) who oppose, and defy, one another.

This paper parts with the common-ownership-or-absentee-landlord tradition. Our focus is a regime in which landlords are both (i) resident (not absentee) and (ii) heterogeneous (rather than identical) in their endowments of housing. Resident landlords are allotted varying allocations of tracts of urban land. Then their interests naturally tie in with where their properties locate on average. Their interests align according to whether their average properties are close to the city’s center (voters we label as centrists) or far away from it (decentrists). Centrists have a stake in the city center, while decentrists are more inclined towards the city periphery. For a flavor of this antagonism, consider three brief examples from fields as diverse as retail location, climate policy, and zoning.

Consider a policy of shifting urban retail out to the city’s peripheral ring road. Such a policy will surely be supported by decentrists. Yet just as surely centrists will oppose it. Second, less obviously, consider the proposal to tax carbon emissions. Centrists will anticipate an increase in the value of their real estate that more than offsets the increase in their commuting cost. Decentrists will expect their real estate value to also rise, albeit by less than what their commuting costs do. Ultimately centrists vote for, while decentrists vote against, the carbon tax. Finally, consider limiting building height. Centrists must be opposed to such a regulation, expecting it to depress their incomes. But decentrists should welcome the induced intra city migration flowing towards them.

While urban retail, climate policy and urban zoning are important policy issues, ignoring their centrist-decentrist dimension could be justified on the grounds that centrists and decentrists do not easily reveal themselves. They certainly do not reveal themselves by where they live.\textsuperscript{2} Centrists and decentrists only reveal themselves by where they own. And because individual land holdings are rarely observable, we might conclude that centrists and decentrists are an empirically vacuous, wholly intractable concept.\textsuperscript{3} As this paper argues, however, this would be an opportunity foregone. This is because we could, and should, exploit the constraints embodied by the observable spatial structure of the

\textsuperscript{2}And this is one of the reasons why a simple map of urban voting outcomes by city district so often fails to impress. Landlords’ voting behavior not just accounts for the policy effects on the property they reside in but also accounts for the policy effects on the (unknown) properties they do not reside in (i.e., rent out).

\textsuperscript{3}This conclusion would turn us back to one of the two housing ownership regimes set out above.
city’s overall housing. Centrists and decentrists must be consistent with aggregate urban housing. Intuitively, in a city with a lot of housing near the periphery, say, there cannot be many centrists.

This simple idea informs the entire paper. We may not be able to compute centrists’ or decentrists’ numbers. But we may be able to bound them. We will derive best lower bounds on centrists and decentrists from the spatial distribution of the city’s housing. Then three sets of results emerge from this. First, we will identify and compute the smallest number of centrists that can conceivably be packed into the city, by following a simple, easily implementable formula: (i) we divide the city into \( n \) rings around the central business district (CBD); (ii) we define as “ring difference \( i \)” those housing units in ring \( i \) minus housing units in ring \( n+1-i \) (where \( i \leq n/2 \)); (iii) we let the \( j \)-th cumulative sum of ring differences add up the first \( j \) ring differences; and (iv) then we choose the greatest from among all \( n/2 \) cumulative sums.

That is, the minimum conceivable number of centrists in the city coincides with the greatest cumulative ring difference. Much of the paper’s modeling energy goes into providing a formal proof of this. This proof builds on the linear programming nature of minimizing centrisim when allocating housing to landlords. Of course, we will also identify the minimum number of decentrists, not least because it can be turned into an upper bound on centrists. Resulting “bands of confidence” provide estimates (predictions) of the urban political equilibrium for a number of diverse policy fields. Hence, and second, we will offer empirically testable predictions on the effects of exogenous bounds changes.

For example, we will predict that the larger the greatest cumulative ring difference is (the larger is the least possible number of centrists and hence) the likelier we observe (i) strong opposition to urban decentralization and sprawl, (ii) endorsement of an emission-abating tax on urban commuting and (iii) relaxation of controls of housing development in the more central part of the city. Finally, and third, we will look into the greatest cumulative ring difference’s proximate causes. We will predict our indicator to be greater, (i) the less building height is regulated, (ii) the less minimum lot size regulation is binding, (iii) the more homeownership is discouraged, (iv) the stricter suburban growth-controls are, and (v) the less important central city urban blight is.

Here we will also address the interaction between the factors that cause, and those that are caused by, the greatest cumulative ring difference. Along the lines of Brueckner/Glazer (2008) and Kahn (2011), we will suggest that a majority of centrists will attempt to cement its underpinnings. A majority of centrists will avoid policies that likely encourage the future share of decentrists to rise. An initial majority of centrists will surely refrain from regulating building heights, from subsidizing homeownership, from relaxing suburban growth controls, etc. (A similar point applies to decentrists, of course.) Let us illustrate this idea by briefly digressing. In 2014, a majority of voters struck down the proposal to redevelop Berlin’s former Tempelhof airfield for residential housing.\(^4\)

Various interest groups publicly supported this decision prior to the vote. Elderly citizens pointed to the airfield’s importance for the Berlin Airlift 1948, when the Soviet Union

\(^4\)For further details see Nitsch (2009).
blocked all roads between Berlin’s West and the West of Germany; younger activists
highlighted the Tempelhof airfield’s role as public park; environmentalists emphasized its
role as a potential “urban cold island”. At the same time, the referendum’s outcome also
had its critics. The former airfield is very large, and is very close to the city’s center. And
it is endowed with an abundance of infrastructure. Two subway lines, a commuter railway
line, and a highway run along its edges. Nowhere else would housing make more sense
than here, in the immediate vicinity of a transportation hub connecting residents fast to
many of Berlin’s jobs. However: If, as may well have been the case, a narrow majority of
voters was decentrist initially, this majority must have been loth to develop the airfield,
for fear of pushing centrists’ voter share (close to one half still) beyond the 50% threshold
and thereby putting an end to the decentralization already underway.5

A second anecdote relates the centrist/decentrist-divide to climate change. Consider the
proposal to introduce a tax on carbon-consuming commuting, and allow for a given share
of tenants to support this tax even as the cost of living is expected to rise (because
these tenants also would benefit from the perceived emissions abatement). Again the
centrist/decentrist-distinction is helpful. Centrists support the tax while decentrists op-
pose it. (E.g., for centrists commuting costs rise, but rental incomes are likely to rise, too.)
Cities with more centrists are likelier to support the tax, and since more “compact” cities
inevitably house more centrists, such cities are also more likely to fight climate change.
This may explain why Europe fights climate change harder than the US.6

To the best of the author’s knowledge, none of the results here have been discussed before.
And while they build on earlier work by the author (Dascher (2017)), they do merit the
extra attention given here. In terms of methodology, first, the lower bound on centrists
introduced in Dascher (2017) is now uncovered to be: best. The greatest cumulative ring
difference is the best of all lower bounds to the number of centrists; it is the minimum
conceivable of the number of centrists. This is neither a trivial point nor a mere technicality
because it provides confidence in the bound’s efficiency, which is important when applying
the concept to actual cities.7 And in terms of thematic reach, second, our distinction
between centrists and decentrists now also extends to issues beyond urban decentralization.
Arguably, interesting aspects of urban development and taxation, zoning and housing
tenure, follow a centrist/decentrist-divide, too.

Accordingly, the paper has five sections. Section 2 sets out the basic framework and lays
out two examples. Section 3 provides the general treatment of any given city. This section
departs from the aim of minimizing the number of centrists given the city’s commuting
distribution. Section 4 explores the issues to which the centrist/decentrist-distinction – and

5 This interpretation is reminiscent of the view entertained by Brueckner/Glazer (2008) (also applicable
here), that voters suspect immigrants’ of preferences that are different from their own and hence prefer to
block immigration altogether (even if this is costly now).
6 This is quite different from arguing that more compact cities may emit less carbon dioxide because
their inhabitants commute a smaller average distance (e.g. Glaeser/Kahn (2004), Brueckner (2005),
Bento/Franco/Kaffine (2006)).
7 Also note that the greatest cumulative ring difference is the solution to a simple yet nonetheless
parametric linear optimization problem. I.e., it is the optimal solution to not just some specific numeric,
but in fact any, commuting distribution.
hence the greatest cumulative ring difference as best lower bound – may contribute. This includes policies both affected by this distinction and driven by it. Section 5 concludes.

2 Model

A representative, closed and monocentric city (as pioneered by Wheaton (1973), Pines/Sadka (1986) and Brueckner (1987)) juts \( \tilde{r} \) units of distance out from the CBD. Daily commuting costs for a resident living at distance \( r \) from the CBD are \( tr \). Ricardian rent follows the familiar \( q(r) = t(\tilde{r} - r) \). The city’s overall population is \( s \), and the daily urban wage is \( w \). Residents are assumed to consume exactly one unit of housing. Apartments are built by profit maximizing investors. One unit of capital \( k \) combined with one unit of land yields \( h(k) \) units of housing, where \( h' > 0 \) and \( h'' < 0 \) (again, Brueckner (1987)).

If \( p \) is the price of capital, investors choose \( k \) so as to satisfy the \( q(r)h_k(k) = p \) necessary for maximum profit. The optimal capital depends on rent \( q \) and price \( p \), and so can be written as \( h_k(t(\tilde{r} - r), p) \). Let \( h(r) \) be shorthand for the building height obtained for this optimal capital choice. Then the city boundary \( \tilde{r} \) is implicitly defined by the condition that the housing market clear,

\[
\int_0^{\tilde{r}} a(r)h(r) \, dr = s, \tag{1}
\]

where \( a(r) \) is land available in a ring of unit width \( r \) units of distance away from the CBD, and where \( a(r)h(r)/s \), also written \( f(r) \), indicates the share of households commuting from within that ring to the CBD. Let \( F(r) \) denote the share of households commuting \( r \) or less.

Subsequent analysis deviates from the standard urban model in two ways. First, commuting density \( f(r) \) is exogenous for most of the analysis. We will observe commuting density determine urban politics, rather than urban politics determine commuting density. As in Dascher (2017), the underlying contention is that adjustment speed for buildings is small (“clay”), whereas politics’ speed of adjustment is great (“putty”). Second, landlords are resident, not absentee. To best introduce their housing endowment, we now shift to a discrete model variant. Divide the city into \( i = 1, \ldots, n \) concentric rings \( (\tilde{r} \text{ even}) \) around the CBD, of equal width \( \tilde{r}/n \). And presume rings narrow enough to justify treating rent, building height, commuting times etc. as identical across ring \( i \)'s plots. Then apartments in ring \( i \) can be approximated by \( f(r_i)s \), also denoted \( s_i \).

Empirically, tenant shares fluctuate widely. This variation is often translated into a model tenant share of 0 (a homevoter society) or 1 (an absentee landlord society where landlords have nowhere in the city to live). But these choices risk abstracting from the very real opposition either to rising costs of commutes (bothering home voters) or to increases in the cost of living (bothering tenants). To address such fault lines of urban political economy, we impose an aggregate tenant share of \( 1/2 \) instead. Of course, many different ownership

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We assume \( a \) is continuous in \( r \). As \( h \) is (differentiable and hence) continuous in \( r \), so is \( f \).
patterns give rise to a 1/2 tenant share. But the most natural starting point is to endow half the population (called landlords) with an identical apartment endowment, of 2.9

So each landlord owns the property she resides in herself as well as the one she rents out to her only tenant. These properties, to be sure, in no way need to (nor typically will) locate in the same ring.10 Fundamentally, information on landlords’ property portfolios is not public, and so landlord attitudes towards the ring road are not easily gauged by policy makers (or us). At the very least, however, we quickly see that landlords do not constitute a homogeneous group. Consider the – unknown – match matrix $X = (x_{ij})$, which collects the frequencies with which the various matches between landlords and tenants occur. Let $i (j)$ indicate the landlord’s (tenant’s) location. Then $x_{13} = 7$, for example, indicates that seven landlords living in ring 1 also rent out to a tenant in ring 3.

We identify as “centrists” those landlords whose average property is closer to the center than to the periphery. For centrists, average portfolio distance to the center $(r_i + r_j)/2$ falls short of “midtown’s” distance to the center $\tilde{r}/2$. Equivalently, $i + j \leq n$. Centrists’ occurrence is associated with entries located strictly above the counter diagonal of $X$, i.e. the diagonal that stretches from $X$’s bottom left corner to its top right one. While we are focussed on assessing centrists’ overall number, we will also be interested in landlords whose average property is closer to the periphery than to the center, i.e. “decentrists”.11 Decentrists are landlords for whom $i + j \geq n + 1$. Decentrists are tied to entries below the counterdiagonal. As emphasized earlier, it turns out a remarkable fact (section 4) that our centrist/decentrist-distinction can address a broad range of seemingly unrelated urban issues.

Briefly pause to note a peculiar property of match matrix $X$. Housing in any given ring $i$ can shelter both, tenants or landlords. Entries in row $i$ represent landlords living in ring $i$, while elements of column $i$ represent tenants housed in $i$. To capture the overall number of households inhabiting ring $i$ we need to sum over all of $X$’s entries in both, row $i$ and column $i$. The resulting sum equals ring $i$’s given stock of apartments, $s_i$. With this ring $i$’s housing constraint reads $\sum_{j=1}^{n} (x_{ij} + x_{ji}) = s_i$.

Summing over all centrist-related entries in $X$ gives $\sum_{i=1}^{n-1} \sum_{j=1}^{n-1} x_{ij}$, the true number of centrists, $l^c$. Contrast this with the smallest number of centrists conceivable, $\bar{l}^c$. Such a number bounds the true yet unknown number of centrists $l^c$ from below. To identify $\bar{l}^c$, we minimize the number of centrists given ring housing constraints and the non-negativity

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9We leave alternative arrangements of resident landlordship (e.g. a monopoly landlord surrounded by owner-occupiers and tenants, etc.) to subsequent work.

10We could settle anywhere between two extremes here. Either one could require each landlord to live in the same ring as (or even next to) the tenant she rents out to (to save on transaction costs, say). Or landlords could be thought to hedge the risk from a change in the structure of urban rents, diversifying their portfolio across space whenever possible.

11Even as decentrists by definition are invested into properties that on average are close to the city extremes, “extremists” (despite its spatial connotation) probably is not a better term.
requirements $x_{ij} \geq 0$. This translates into the more compact statement

$$\min_{x_{ij}} \sum_{i=1}^{n-1} \sum_{j=1}^{n-i} x_{ij} \quad \text{s.t.} \quad \sum_{j=1}^{n} (x_{ij} + x_{ji}) = s_i \quad (i = 1, \ldots, n)$$

$$x_{ij} \geq 0 \quad (i, j = 1, \ldots, n)$$

(2)

as the linear program to be solved.

We run two eight-ring city examples next. These are examples to offer some intuition on how a feasible, and even optimal, solution to program (2) plays out. But they really are more than just examples. They motivate a trial solution that quickly generalizes to any given city as well. To keep these examples as simple as possible we assume that landlords will always occupy the ring that is closer to the CBD, leaving the ring further out to her tenant. This implies that $x_{ij} = 0$ as long $i > j$, or that match matrix $X$ is upper triangular (as in (3 or (4)) below). This is an innocuous assumption, and one that we may easily relax later.

**Example 1.** Our first city has commuting distribution $s = (38, 36, 30, 10, 12, 8, 4, 2)$. Matrix $X_1$ in (3), in highlighting eight non-zero entries, already suggests a basic feasible solution. We briefly illustrate feasibility. Adding up all entries in row 1 and column 1, for instance, gives $20 + 18 = 38$ or $s_1$, while adding up all entries in row 7 (consisting of zeros only) and column 7 gives just $0 + 4$ or $s_7$. Our feasible solution here displays one feature that we might expect of an optimal solution, notably that (3) assigns the maximum weight possible to entries on the counterdiagonal. This forces centrists numbers down as best as we can (relevant entries on screen shown in red). We get $x_{18} = \min\{s_1, s_8\} = 2$. Similarly, $x_{27} = 4$, $x_{36} = 8$ and $x_{45} = 10$.

Put differently, whenever possible we allocate a peripheral apartment in some given outer ring $j$, $5 \leq j \leq 8$, to a proprietor who owns her second apartment in corresponding inner ring $9 - j$. This must be a necessary property of a centrist-minimizing allocation. Suppose $X_1$ violated it, i.e. suppose $x_{18} = 1 < 2 = \min\{38, 2\}$. Since no apartment in any earlier ring is capable of successfully turning a landlord in ring 1 – someone who would otherwise be a centrist – into a decentrist, an opportunity to reduce centrists would have irrevocably passed up. At the same time, of course, not all apartments in a given peripheral ring $j$ may be assignable to a landlord in corresponding ring $n - j + 1$. In ring 5, for example, only 10 out of 12 apartments are.

There are $(s_1 - s_8) = 36$ apartments in ring 1 still waiting to be allocated, as are $(s_2 - s_7) = 32$ apartments in ring 2 and $(s_3 - s_6) = 22$ in ring 3. We apportion these remainders to landlords owning both their properties within the same ring. Since any match on the main diagonal accounts for two apartments, we set $x_{11} = (s_1 - s_8)/2 = 18$, $x_{22} = (s_2 - s_7)/2 = 16$ and $x_{33} = (s_3 - s_6)/2 = 11$. Note that $x_{44} = 0$, given that $x_{45} = 10$ already and that row 4 and column 4 must add up to $s_4 = 10$. 

6
Applying the simplex algorithm would show that the solution set out in (3) above not just is feasible but also optimal. Instead of going through the details here, we offer a systematic treatment that applies to any city below (section 3). At this stage we simply claim that the trial number of centrists suggested by (3) also is the – minimum – number of centrists given the specific distribution of housing $s$ in hand.

Adding up centrists is simple enough. We merely need to collect the few non-zero entries above the counterdiagonal. These are conveniently located on the upper half of the main diagonal (blue on screen). This gives $\sum_{i=1}^{3} (s_{1} - s_{9-i})/2$ or 45 minimum centrists. Minimum centrists’ share in city population becomes 45/140. Further, should tenants not vote (as explained below) minimum centrists’ share in the electorate becomes 45/70, passing for the majority. Computing minimum centrists provides valuable information here. It is not possible for the true number of centrists to fall short of 45 but it is quite possible (if not utterly likely) for the true number of centrists to surpass 45. (The latter occurs should true matches deviate from one of the optimal allocations). In the first example city, centrists prevail – whatever – the assignment of houses in the city.

Example 2. Our second city exhibits stocks $s = (38, 14, 30, 10, 12, 8, 26, 2)$. We take a step towards generalization by introducing the concept of ring difference $\delta_i$, 

$$\delta_i = s_i - s_{n+1-i},$$

as the number of apartments in “leading” ring $i$ minus that in “lagging” or “antagonist” ring $n + 1 - i$. It is defined for $1 \leq i \leq 4$. In our second example city, $\delta_i$ is positive for $i$ equal to 1 or 3 (since there we have a “surplus”) and it is negative if $i$ equals 2 or 4 (because then there is a “deficit”). (The first three ring differences of the first example city are positive.)

True to our strategy of emphasizing the counterdiagonal, match matrix $X_2$ in (4) assigns as many apartments as possible in lagging rings to owners in corresponding leading rings. And because we have a surplus in rings 1 and 3, for these rings this works just fine. All apartments in rings 8 and 6 can be assigned to landlords living in rings 1 and 3, respectively. And while this works less well for apartments in lagging rings 5 and 7, remaining apartments are not lost on us. Ring 2’s deficit of $(-s_2 - s_7) = 12$ we can “save up for”, or “post to”, the next best successive ring boasting a surplus. In our

$^{12}$It is not, however, a unique optimal solution. Letting any landlord trade apartments with her or his tenant, for example, generates many other feasible solutions. More on this below.

$^{13}$This could be dubbed the “Northeast-Corner-Rule”.

$$X_1 = \begin{pmatrix} 18 & 0 & 0 & 0 & 0 & 0 & 2 \\ 16 & 0 & 0 & 0 & 0 & 4 & 0 \\ 11 & 0 & 0 & 8 & 0 & 0 & 1 \\ 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$
example, this is ring 3 (where \( s_3 - s_6 = 22 \)). Those 12 apartments reflecting ring 2’s deficit can valuably be employed to offset the better part of ring 3’s surplus.

\[
X_2 = \begin{pmatrix}
18 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 14 & 0 & 0 \\
5 & 0 & 0 & 8 & 12 & 0 \\
0 & 10 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(4)

And so we set entry \( x_{37} \) in \( X_2 \) to \( x_{37} = s_7 - s_2 \), or 12. Intuitively, the 12 ring 7-apartments not assignable to ring 2-landlords now are assigned to landlords in ring 3, in the attempt to at least turn those off centrism. Everything else parallels our discussion of the first example. We balance the first three rings’ housing constraints by setting \( x_{11} = (s_1 - s_8)/2 = 18 \), \( x_{22} = 0 \) and \( x_{33} = (s_3 - (s_6 + (s_7 - s_2)))/2 = 5 \). Again, moreover, the basic feasible solution, set out in (4), also is the optimal one. Minimum centrists are found to sum to 23, if only to see their share in the landlord total attain a mere 23/70. This is less than what is needed to permit the strong prediction on the voting outcome in our first example city.\(^{14}\)

Let us briefly revisit our first example city. Surely its 45 minimum centrists could also be written as the cumulative sum of the first three ring differences, \( \sum_{i=1}^{3} (s_1 - s_{9-i})/2 \), much as the 23 minimum centrists identified now could be written as the cumulative sum of the first three ring differences, \( \sum_{i=1}^{3} (s_1 - s_{9-i})/2 \). Note that it makes sense to include \( \delta_2 \) in either example even as its sign is positive first and negative then. Including \( \delta_2 \) when it is positive acknowledges the fact that \( (s_2 - s_7)/2 \) ring 2-landlords can never be turned away from centrism. And including \( \delta_2 \) when it is negative emphasizes the fact that \( (s_7 - s_2)/2 \) ring 3-landlords can. At the same time, of course, \( \sum_{i=1}^{3} (s_1 - s_{9-i})/2 \) excludes \( \delta_4 \) in either city. This is because \( \delta_4 \)’s negative sign indicates that the planner can afford each landlord in ring 4 a ring 5-apartment that counters that landlord’s impulse to “go centrist”. With no centrists to collect in the fourth ring, minimum centrists’ sum should stop short of it.

By way of our examples, we come across two tentative ideas: Minimum centrists can be represented by a cumulative sum of ring differences. Moreover, this cumulative sum should include successive ring differences up to the point where it ceases to grow. That is, we suspect the centrists’ minimum to coincide with the greatest cumulative ring difference. In the general city discussion that follows next, these ideas form the basis of (i) a trial solution that (ii) can then be shown to be optimal.

\(^{14}\) However, recall that while minimum centrists fall short of threshold 36, their true number not necessarily does.

8
3 The Greatest Cumulative Ring Difference

Allow for any \( n \times 1 \) vector \( s \) now, and put the corresponding linear program (2) into standard form. We first stack all \( n \) columns of \( X \) into one long \((n^2 \times 1)\) vector \( x \). This gives \( x' = (x_{11}, \ldots, x_{1n}, \ldots, x_{n1}, \ldots, x_{nn}) \). To address the objective function in (2) in matrix notation, let \( c_i \) equal an \( n \times 1 \) vector consisting of ones only except for the last \( i \) entries, which are zero instead. For example, \( c_3 \) is a list of \( n-3 \) ones followed by three zeros, i.e. \( c_3' = (1, \ldots, 1, 0, 0, 0) \). Define an \( n^2 \times 1 \) list of weights \( c \) by setting \( c' = (c_1', \ldots, c_n') \). Then our objective function \( \sum_{j=1}^{n-1} \sum_{i=1}^{n-1} x_{ij} \) can be cast as the product \( c'x \).

Next, let \( \tau_i \) denote an \( n \times 1 \) vector featuring 2 in its \( i \)-th row and 1 in all other rows. For example, \( \tau_2' = (1, 2, 1, \ldots, 1) \). Moreover, let \( J_i \) denote what becomes of the \( n \times n \) identity matrix once row \( i \) has been replaced with \( \tau_i' \). Then the coefficient matrix \( A \) is \( A = (J_1, \ldots, J_n) \); it is \( n \times n^2 \). The tableau in Table (1) illustrates \( A \) in its bottom left part. This table also indicates our specific vector of objective function weights \( c \) (in its first row) as well as the vector of ring housing stocks (last column).

With the extra notation in hand, linear program (2) may equivalently be stated as \( \min_x c'x \) subject to \( Ax = s \) and \( x \geq 0 \), where the equality constraints may also be read off Table (1)’s rows. Now, this program’s dual is \( \max_y s'y \) such that \( y'A \leq c' \), where \( y \) is the dual’s \((n \times 1)\) vector of unknowns: \( y' = (y_1, \ldots, y_n) \). Table (1) also indicates the dual’s constraints; these can be read off its columns. For instance, the constraint complementary to \( x_{11} \) being positive simply is \( 2y_1 \leq c_{11} = 1 \) (see first column in Table (1)), while that associated with, say, \( x_{1n} \) is \( y_1 + y_n \leq c_{1n} = 0 \). Or, the very last of all these constraints is \( 2y_n \leq c_{nn} = 0 \) (see last column in Table (1)).

| 1 1 1 1 ... 1 0 | ... | 1 0 0 0 ... 0 0 | \hline
| 2 1 1 1 ... 1 1 | ... | 1 0 0 0 ... 0 0 | \hline
| 0 1 0 0 ... 0 0 | ... | 1 0 0 0 ... 0 0 | \hline
| 0 0 1 0 ... 0 0 | ... | 0 1 0 0 ... 0 0 | \hline
| ... | ... | ... | ... | \hline
| 0 0 0 0 ... 1 0 | ... | 0 0 0 0 ... 1 0 | \hline
| 0 0 0 0 ... 0 1 | ... | 1 1 1 1 ... 1 2 | \hline

Table 1: Matrix \( A \), objective function weights \( c \) and housing stocks \( s \)

Ring indices belong to set \( \{1, \ldots, n\} \), and hence ring difference indices are owned by \( \{1, \ldots, n/2\} \).

Recall that both of the previous section’s examples feature the cumulative sum of the first three ring differences, \( \sum_{j=1}^{3} \delta_j/2 \). There we suggested including ring differences up to when their cumulative sum ceases to grow. To address this idea even when ring housing stocks are parametric rather than numeric, we introduce the more flexible notation \( \sum_{j=1}^{i} \delta_j/2 \). As short-hand for this sum we write \( \Delta(i) \) (rather than \( \Delta_i \)), to make explicit its dependence on \( i \).

Any cumulative sum \( \Delta(h) \) may be preceded by some other cumulative sum \( \Delta(g) \), wherer
$g < h$, that is greater than it. In this case we say that the cumulative sum up to ring difference $h$ is "in the shadow of" the cumulative sum up to ring difference $g$, borrowing terminology established in the context of the “Rising Sun Lemma” (Spivak (1994)). Or, synonymously but also more briefly, ring $h$ is in the shadow of, or overshadowed by, ring $g$.

Of those rings never overshadowed, let ring

$$i^* = \arg \max_i \sum_{j=1}^{i} (s_j - s_{n+1-j}),$$

be the one exhibiting the greatest cumulative sum, $\Delta(i^*)$. Note that the last ring difference featuring in that sum must be positive, i.e. $\delta_{i^*} \geq 0$, else $i^*$ could not be the maximizer. For a similar reason, $\delta_{i^*+1} < 0$.

Aided by (5) we identify centrists’ minimum in three steps. We first inspect a city in which all rings up to $i^* - 1$ display negative ring differences and are overshadowed (subsection 3.1). Then we extend our analysis to the case where, while all rings prior to $i^* - 1$ continue to be overshadowed, their ring differences may exhibit any sign (subsection 3.2). Third, we assemble the general city case from our understanding of the previous two cases (in subsection 3.3). Our focus is the solution set out in subsection 3.1, subsections 3.2 and 3.3 then fill in the details. Let us now briefly preview this section’s insight: Irrespective of city’s commuting distribution $s$ and in full ignorance of match matrix $X$, centrists’ minimum equals the observable and hence computable $\Delta(i^*)/2$ (Proposition 1).

### 3.1 Ring Differences Almost All Negative

Our point of departure is the “fairly, but not fully” general city spelt out in Table (2). That table’s header has the ring difference index $i$, the second row provides ring difference $\delta_i$’s sign, and the third row indicates whether or not the corresponding cumulative ring difference $\Delta(i)$ is overshadowed (• is a suggestive shorthand) or not (◦). In this particular city, all ring differences both prior to $i^*$ and beyond $i^* + 1$ are negative and overshadowed.

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>...</th>
<th>$i^* - 1$</th>
<th>$i^*$</th>
<th>$i^* + 1$</th>
<th>$i^* + 2$</th>
<th>...</th>
<th>$n/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_i$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Delta(i)$</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>◦</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>

Table 2: A Parametric City

**Basic Feasible Solution.** We set out a basic feasible solution next. (Entries of $X$ not addressed below are zero.) We begin by considering the elements on the counterdiagonal

\[\Delta(0) = 0.\] Even the first ring difference may be overshadowed, by $\Delta(0) \equiv 0$. This occurs whenever $\delta_1 < 0$.\footnote{We define $\Delta(0) = 0$. Even the first ring difference may be overshadowed, by $\Delta(0) \equiv 0$. This occurs whenever $\delta_1 < 0$.}

\[\text{Suppose we were to collect all ring differences in some other (yet not necessarily identical) ring difference's shadow, giving rise to set of ring differences } S = \{i : \Delta(i) < \Delta(h) \text{ for some } h < i\}. \text{ To illustrate, } S = \{4\} \text{ in example 1, while } S = \{2, 4\} \text{ in example 2.}\]
of match matrix $X$. Here we set

$$x_{i,n+1-i} = \min \{s_i, s_{n+1-i}\} \quad (i = 1, \ldots, n/2).$$

(6)

Given our assumed signs of $\delta_i$ in Table (2), this entails setting all entries $x_{1,n}$ "up" to $x_{i-1,n+2-i}$, and again from $x_{i+1,n-i}$ to $x_{n/2,n/2+1}$, equal to the leading ring’s stock, $s_i$. Only $x_{i,n+1-i}$ becomes the lagging ring’s stock, $s_{n+1-i}$. Note how this assignment makes as many owners of property in leading rings (voters who otherwise likely are centrists) as possible disavow centrism.

Moreover,

$$x_{i,n+1-i} = (s_{n+1-i} - s_i) \quad (i = 1, \ldots, i^*-1).$$

(7)

Note that the expressions on the r.h.s. represent ring deficits. Deficits originating in rings prior to $i^*$ are posted to leading ring $i^*$, as the earliest next ring offering up an excess. Excess apartments in rings up to $i^*$ then are matched up with apartments in ring $i^*$. This generalizes how we proceeded earlier when setting $x_{37}$ equal to 12 in example city 2.

Next, let

$$x_{i^*,i^*} = \left( s_{i^*} - (s_{n+1-i^*} + \sum_{k=1}^{i^*-1} (s_{n+1-k} - s_k)) \right)/2.$$  

(8)

or $\Delta(i^*)/2$. At first sight nothing seems to preclude $x_{i^*,i^*}$ from being strictly negative, in contradiction to primal variables’ non-negativity constraints. However, recall that $i^*$ maximizes the cumulative sum of ring differences. And so $\sum_{j=1}^{i^*-1} \delta_j \geq 0$, i.e. a non-negative number. But note that this latter number just coincides with the r.h.s. of (8). Put yet differently, ring excess $\delta_{i^*}$ is sufficient to offset the ring deficits $\delta_j$ associated with, and inherited from, all the rings prior to $i^*$. And so $x_{i^*,i^*}$ really is non-negative.

At last we set

$$x_{n+1-i,n+1-i} = (s_{n+1-i} - s_i)/2 \quad (i = i^* + 1, \ldots, n/2).$$

(9)

Ring deficits originating in rings following $i^*$ are relegated to main diagonal elements below the counterdiagonal, to the desired effect of contributing nothing to the number of centrists. Now, equations (6), (7), (8) and (9) set out an optimal solution of the primal. (Recall that elements of $X$ never addressed are zero.) To confirm feasibility first, we take a brief (nonetheless exhaustive) tour through the city’s rings (i.e. through all row/column pairs of $X$).

Consider any row/column pair (i.e. ring) with $i \in \{1, \ldots, i^*-1, i^*+1, \ldots, n/2-1\}$ first. The single positive entry here is the counterdiagonal entry, set equal to $s_i$ (see (6)). Next, focus on row and column $i^*$. Non-zero entries can be found on the main diagonal (see (8)), on the counterdiagonal (see (6)) and “towards the end of" row $i^*$ (see (7)). Summing over them, by design of (8), just yields $s_{i^*}$. This completes our discussion of housing constraints in leading rings.

We cover lagging rings next. Consider some lagging ring with $n+1-i \in \{n/2, \ldots, n-i^*\}$. We identify but two positive entries, on the main diagonal and the counterdiagonal (see
inequalities for which the l.h.s. may not exceed $y_i$ either. As is expected, they sum to $s_{n+1-i}$. All housing constraints are met, and this confirms feasibility. Let $\mathbf{y}$ denote this feasible vector.

**Basic Optimal Solution.** We invoke complementary slackness between the primal and the dual in order to check whether $\mathbf{y}$ could even be optimal. For $i \in \{1, \ldots, n/2\}$ entries on the counterdiagonal $x_{i,n+1-i}$ are strictly positive (see (6)), as is the main diagonal element $x_{i,i}$ (see (8)). By complementary slackness, the corresponding constraints of the dual – read off the corresponding columns of Table (1) – must be met with equality, and so

$$y_i = -y_{n+1-i} \quad (i = 1, \ldots, n/2) \quad \text{and} \quad y_i^* = 1/2.$$  \hfill (10)

These equations specify the weights on ring housing stocks $s_i$ in the dual’s objective. For $i \in \{1, \ldots, i^* - 1\}$, entries $x_{i^*,n+1-i}$ are positive also (see (7)). Again, by complementary slackness, corresponding constraint inequalities in the dual become binding, and according to Table (1), $y_{i^*} = y_{n+1-i}$. Combining this with $y_{n+1-i} = -y_i$ and the fact that $y_i^* = 1/2$ (see (10)) gives the first set of equations in (11). At last we make use of equations (9). For $i$ in $\{i^* + 1, \ldots, n/2\}$, constraint inequalities translate into $y_i = 0$. Joint with the first set of equations in (10), this in turn implies the second set of equations in (11):

$$y_i = 1/2 \quad (i = 1, \ldots, i^*) \quad \text{and} \quad y_i = 0 \quad (i = i^* + 1, \ldots, n).$$  \hfill (11)

Table (3), next, also spells out the full solution to equations (10) and (11), denoted $\mathbf{y}$ (ignore the table’s caption for the moment).

Fundamentally, should $\mathbf{y}$ be feasible then $\mathbf{x}$ is optimal (Chvatal (1980), Theorem 5.3). To check feasibility we apply $\mathbf{y}$ to all of the dual’s constraint inequalities. First consider inequalities for which the l.h.s. may not exceed 1 (i.e. where $i + j \leq n$), $y_i + y_j \leq 1$. Since either $y_i$ and $y_j$ at best equal $1/2$, their sum cannot exceed 1. Then consider constraint inequalities for which the l.h.s. may not even exceed 0 (i.e. inequalities where $i + j \geq n+1$), $y_i + y_j \leq 0$. Whenever $y_i = 1/2$ occurs this is because $i \leq i^*$. Hence $j \geq n+1 - i^*$. But then we have $y_j = -1/2$. And so then $y_i + y_j = 0$. Alternatively, whenever $y_i = 0$ then $i^* + 1 \leq i$. But then we must have $j \geq n - i^*$. And so $y_i + y_j$ equals zero or $-1/2$. We conclude that $\mathbf{y}$ is feasible after all, and that $\mathbf{x}$ is optimal.

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>$\ldots$</th>
<th>$i^*$</th>
<th>$i^* + 1$</th>
<th>$\ldots$</th>
<th>$n - i^*$</th>
<th>$n - i^* + 1$</th>
<th>$\ldots$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_i$</td>
<td>1/2</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-1/2$</td>
<td>$-1/2$</td>
<td>$-1/2$</td>
</tr>
</tbody>
</table>

| Table 3: The dual’s optimal solution |

We compute the objective function values for primal and dual, providing a check on optimality as well as, of course, the desired minimum number of centrists itself. On the one hand, summing over all entries above the counter diagonal the objective function value
in the primal gives $x_{i^*}$, as set out in equation (8), or

$$\Delta(i^*) = \max_i \sum_{j=1}^i (s_j - s_{n+1-j})/2.$$  \hspace{1cm} (12)

Precisely this is the paper’s greatest cumulative ring difference. On the other hand, computing the sum of ring stocks using the optimal weights in (10) and (11) yields the very same expression $\sum_{j=1}^{i^*}(s_j - s_{n+1-j})/2$. Hence, applying a standard fundamental argument at the core of duality theory, this common value represents: the minimum conceivable number of centrists.

None of all of the variations in the succession of ring differences allowed for below have an effect on the validity of formula (12), and so we state the general result here and now. Formula (12) gives a universal closed form solution for the best lower bound on urban centrism. It provides an observer of an arbitrary given city with a prediction of centrists’ minimum conceivable voter share.

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Proposition 1: (Greatest Cumulative Ring Difference and Centrism)

Centrists’ minimum conceivable voter share, $\lambda^c$, is given by the greatest cumulative ring difference $\lambda^c = \max_i \Delta(i)/s = \max_i \sum_{j=1}^i (s_j/s - s_{n+1-j}/s).^{18}$

3.2 Not All Ring Differences Negative

Nothing of substance changes if one (or more) of those shadow differences is (are) positive, rather than negative. To see this we turn to the city set out in Table (4) below, with the second ring the one ring to have flipped its sign. We assume that everything else remains the same, and so $\Delta(2) < \Delta(0)$ while $i^*$ keeps maximizing $\Delta(i)^{19}$.

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
<th>$i^* - 1$</th>
<th>$i^*$</th>
<th>$i^* + 1$</th>
<th>$i^* + 2$</th>
<th>...</th>
<th>$n/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_i$</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$\Delta(i)$</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
<td>⋯</td>
</tr>
</tbody>
</table>

Table 4: Negative and Positive Ring Differences

We introduce the following three (i.e. not numerous) changes to the primal’s solution:

(i) Entry $x_{2,n-1}$ ceases to be $s_2$ and turns into $s_{n-1}$. (ii) Entry $x_{2n}$ becomes $(s_2 - s_{n-1})$, replacing the zero it was before. (iii) Entry $x_{i^*,n}$ drops from ring difference $(s_n - s_1)$ to the

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18This result extends Dascher (2016) which, building on a more direct, non-LP approach, introduces $\Delta(i^*)$ as a lower bound to centrists’ true number. This section added that $\Delta(i^*)$ also is the best lower bound, making $\Delta(i^*)$ an interesting lower bound.

19These assumptions are not restrictive. First, if $\Delta(0) < \Delta(2)$, we would have to consider alternating spells of ring differences in the shadow and not in the shadow. This case is considered shortly. And second, if $i^*$ shifted due to $\delta_2$ flipping its sign, nothing would change in the argument below as long as $2 < i^*$.  

13
difference of ring differences \((s_n - s_1) - (s_2 - s_{n-1})\). These changes maintain feasibility, as is easily checked by consulting the housing constraints of the four rings affected by these changes.

Note that \(x_{i^*,i}\) is not among the entries changed. This particular entry continues to equal \(\Delta(i^*)/2\). Since this entry is the only one to enter the primal objective’s value, this formula does not change either. Note the role of ring 2 still being overshadowed here. While \(\delta_2\) is positive, it is not sufficiently so to offset the negative \(\delta_1\) that precedes it. And hence \((s_1 - s_n) + (s_2 - s_{n-1})\), or \(x_{i^*,n}\) indeed is positive. What about the implied changes for the dual? Since \(x_{2,n-1}\) and \(x_{i^*,n}\) continue to exceed zero, \(\Delta(i^*)/2\) continues to capture the dual’s value.

Put differently, while it is true that the objective’s value changes, it is also true that formula \(\Delta(i^*)/2\) continues to indicate the minimum number of centrists even in the modified city. Exploring a sign change for any other ring difference, or for extra ring differences, proceeds along similar lines. That is, \(\Delta(i^*)/2\) captures the minimum number of centrists whatever the signs of the ring differences in rings up to \(i^*\) as long as these ring differences are overshadowed. Example 1, for instance, exhibits \(\delta_i > 0\) on the first three rings, and \(i^* = 3\). The minimum centrists we identified then obviously coincide with what \(\Delta(i^*)/2\) reduces to in the special case. – The following subsection allows for alternating spells of rings overshadowed and rings not overshadowed.

### 3.3 Not All Ring Differences Overshadowed

What (if anything) changes if one (or more) of the ring differences were not overshadowed? We allow for the possibility that not all ring differences prior to \(i^*\) are overshadowed (see Table (5)). Let all ring differences from 1 up to \(i' - 1\) be in the shadow of ring 0, and all ring differences between \(i' + 1\) and \(i^* - 1\) be overshadowed by \(i'\), so that \(i^*\) is not in the shadow. One optimal feasible solution assigns \(\sum_{j=1}^{i'} \delta_j/2\) to \(x_{i',i'}\), and \(\sum_{j=i'+1}^{i^*} \delta_j/2\) to \(x_{i^*,i^*}\), and zero to any other element above the counterdiagonal. The corresponding minimum number of centrists becomes the sum of these two (only non-zero) terms. But this is just our familiar \(\Delta(i^*)/2\). Adding extra spells of ring differences in the shadow adds nothing of substance here.

<table>
<thead>
<tr>
<th>(i)</th>
<th>1</th>
<th>(i' - 1)</th>
<th>(i')</th>
<th>(i' + 1)</th>
<th>(\ldots)</th>
<th>(i^* - 1)</th>
<th>(i^*)</th>
<th>(i^* + 1)</th>
<th>(\ldots)</th>
<th>(n/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta(i))</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>

Table 5: Alternating Spells of Shadow and Light

At last we turn to the question of what happens if any ring differences following \(i^* + 1\) (rather than preceding \(i^*\)) exhibit a positive sign. Recall that, by definition of \(i^*\), ring differences beyond \(i^*\) must be overshadowed. Let one of these ring differences be positive, rather than negative, i.e. \(i^* + 2\) say. Being in the shadow of \(i^*\), the excess arising in ring difference \(i^* + 2\) must be swamped by the deficit in the previous ring difference at \(i^* + 1\). Once more, there is no change in the number of minimum centrists.
4 Urban Political Economy: Centrists vs. Decentrists

We bring in decentrists next. Our previous discussion has a straightforward decentrist analogue, of which we only give the result here. Where before we have used $s_{n+1-i}$ to swamp potential centrists in $i$ (as best as we could), conversely we now use $s_i$ to swamp decentrists in $n+1-i$ (as best as we can). If $\delta_i > 0$, we must reckon with the possibility that there are no decentrists in that latter ring at all. If $\delta_i < 0$, then $\delta_i/2$ decentrists (or less if ring differences prior to $i$ are positive) need to be reckoned with in ring $n+1-i$.

Let us define the corresponding maximizer

$$i^{**} = \arg \max_i \sum_{j=1}^i \left( -s_j - s_{n+1-j} \right), \quad (13)$$

so that $-\Delta(i^{**})/2$ captures the minimum number of decentrists; it is the best lower bound on decentrists (Proposition 2).

**Proposition 2: (Least Cumulative Ring Difference and Decentristism)**

Decentrists’ minimum conceivable voter share, $\lambda^d$, is given by minus the least cumulative ring difference $\lambda^d = \max_i -\sum_{j=1}^i (s_j/s - s_{n+1-j}/s) = -\min_i \Delta(i)/s$.

Decentrists’ minimum complements our discussion of centrists. This is because $1 - \lambda^d$ provides a natural upper bound on the true centrist voter share (Proposition 3, Part (i)), just as $1 - \lambda^c$ is one on the true decentrist voter share (Proposition 3, Part (ii)). Are these bounds consistent, in the sense of $\lambda^c + \lambda^d < 1$ or

$$\left( \sum_{j=1}^{i^*} \delta_j + (-\sum_{j=1}^{i^{**}} \delta_j) \right)/s < 1, \quad (14)$$

equivalently? Either $i^* > i^{**}$. Then the sum above reduces to $\sum_{j=i^{**}+1}^{i^*} \delta_j/s$, which surely is less than one. (It starts with the sum of only some (i.e. not all) leading rings’ housing shares and then even deducts corresponding lagging rings’ shares.) Or $i^{**} > i^*$. Then this sum becomes $-\sum_{j=i^*+1}^{i^{**}} \delta_j/s$, which must fall short of one also.

As a byproduct, the sum in (14) emerges strictly greater than zero if we exclude the (improbable) case of $i^* = i^{**} = 0$ (which would imply that all ring differences are zero and so that the commuting distribution is symmetric). Then clearly at least one of the two bounds must be strictly positive (Proposition 3, Part (iii)). Moreover, we can say something about the relationship between our two bounds. Again allow for two cases.

First suppose that $i^* > i^{**}$. Then the positive sum $\sum_{j=i^{**}+1}^{i^*} \delta_j/s$ must exceed the absolute value of the negative sum $\sum_{j=1}^{i^{**}} \delta_j/s$. Centrists’ minimum must be more than twice as large as decentrists’ minimum. Alternatively, $i^{**} > i^*$. Then the absolute value of the negative sum $\sum_{j=i^{**}+1}^{i^*} \delta_j/s$ must exceed the positive sum $\sum_{j=1}^{i^{**}} \delta_j/s$. Here decentrists’ minimum is more than twice as large as centrists’ minimum. Put slightly more generally, whichever best lower bound is greater, it is at least twice as large as the smaller of the two (Proposition 3, Part (iv)).
Proposition 3: (Bounding Centrists and Decentrists)

(i) $\lambda^c$ is bounded as in $\lambda^c \leq 1 - \lambda^d$.
(ii) $\lambda^d$ is bounded as in $\lambda^d \leq 1 - \lambda^c$.
(iii) At least one of the two, $\lambda^c$ or $\lambda^d$, is strictly positive.
(iv) Either $\lambda^c > 2\lambda^d$ or $\lambda^d > 2\lambda^c$.

Minimum centrists and decentrists can also be tied to the city’s physical form. Consider two cities (or two impressions of the same city at two different moments in time), and suppose that the c.d.f. of commuting distance in city 1 (first-order) stochastically dominates the commuting distance c.d.f. in city 2. I.e., $F_1(r) < F_2(r)$ for all $r$. Arguably in this case city 1 is less compact than city 2. Perhaps not too surprisingly, we predict that if a city is more compact then it also exhibits greater $\lambda^c$ and smaller $\lambda^d$ (Proposition 4).

Proposition 4: (City Compactness and Centrism)

Let city 1’s commuting distribution (stochastically) dominate city 2’s. (City 2 is more compact.) Then $\lambda^c_1 < \lambda^c_2$ and $\lambda^d_1 > \lambda^d_2$. A more compact city displays greater centrism.

5 Centrists vs. Decentrists

We explore the greatest cumulative ring difference’s implications for urban political economy. It is these that justify our interest in the greatest cumulative ring difference (and its cousin, minus the least cumulative ring difference) in the first place. We first address policies that directly follow the initial centrist/decentrist-balance (a ring road to facilitate suburban business location, a tax on commuting to abate carbon emissions). We then address policies that feature as determinants of centrism, but should also be seen as the outcomes of it (building height restrictions, minimum lot size, suburban growth control, homeownership).

Decentralization. Suppose a costless ring road could be built for free. Such a ring road that would encircle the city all along its boundary, and would permit firms and shops to connect with each other almost as well as when remaining in the traditional CBD. Firm employees in large office parks easily establish face-to-face contacts by fast car travel along the ring road. Shops’ customers could visit a multitude of shops, for example by frequenting large shopping centers with free parking. In that sense a ring road created a strong rival to the traditional CBD with all its physical proximity.

Imagine the ring road (rather starkly) to represent an immediate shift away from the CBD. Then instead of travelling $r_i$ to the CBD for work and shop, every resident in ring $i$ now

\[\text{Dascher (2016), where an indicator of urban skewness bounds } \lambda^d \text{ from below and } \lambda^d \text{ from above.}\]
commutes $\tilde{r} - r_i$ to the city’s periphery. And instead of paying rent $t(\tilde{r} - r_i)$ rent, every tenant in ring $i$ now instead pays $tr_i$. And yet, tenants’ cost of living $t\tilde{r}$, or the sum of commuting cost and rent, are unaffected, given that $\tilde{r}$ remains unfazed. We should expect tenants not to vote. The political decision on the ring road is fought out among resident landlords only.

**Proposition 5: (Decentralization)**

If $\lambda^c > 0.5$ (if $\lambda^d \geq 0.5$), the city votes for (against) shifting jobs and shops to the periphery. Compact cities are less likely to decentralize in the first place.

Let $\omega$ subsume aspects of city life that are identical across residents, such as the wage or public good benefits. For a landlord, without the ring road utility is $\omega + t(\tilde{r} - r_j - r_i)$ while it is $\omega - t(\tilde{r} - r_j - r_i)$ with it. The implied benefit is $2t(r_i + r_j - \tilde{r})$, which is strictly negative if and only if $i + j < n$. Equivalently, resident landlords vote for the ring road if they are decentrists, but they vote against it if they are centrists. If either $\lambda^c$ or $\lambda^d$ exceeds one half, we can predict which of the two interest groups prevails (Proposition 5).

Proposition 5 signals how sprawl starts (for details see Dascher (2016)). Combining it with Proposition 4 yields a “corollary”: compact cities are less likely to decentralize.

**Climate Change.** Let an increase in $t$ be proposed, equal to $\Delta t$ and brought about by a commuting tax. Taking the first derivative of the landlord utility $\omega + t(\tilde{r} - r_j - r_i)$ defined above with respect to $t$ gives $\tilde{r} - r_i - r_j$. A resident landlord votes for the commuting tax if and only if $i + j \leq n$, which just repeats the condition encountered in a previous paragraph. So centrists vote for the commuting tax, while decentrists vote against it. Of course, this ignores revenue from the tax. (It is difficult to assess where these revenues will go but they will likely enhance support.) Moreover, this also ignores the fact that the increase in $t$ raises the urban cost-of-living $t\tilde{r}$, and so makes all tenants worse off.

Tenants are against an ordinary tax on commuting. Yet if the commuting tax is meant to fight carbon emissions it can be expected to compactify the city. Rising rent will increase the incentive to build higher near the center, inducing the monocentric city’s residents to move further in. Both average commuting distance and urban boundary recede (e.g., Borck (2016)). Smaller average commuting distance forces the city’s emissions down. Also, compactification will tend to drive urban cost-of-living down again (though not to the level seen before).

**Proposition 6: (Climate Change/Voting on a Commuting Tax)**

*The more compact the city is, the likelier introducing a carbon/commuting tax becomes.*

Let the distribution of (public good) benefits from emissions abatement be independent of the distribution of housing tenure. Suppose the climate benefit has a share of $\alpha$ of

21 A city that decentralizes (exhibits a majority of resident landlords in favor of decentralization) will see its peripheral rents go up. This has investors build higher there. Over time this city observes its decentrist share rise even further. Alternatively, a city that does not decentralize (a city where a majority of voters oppose decentralization) maintains its initial rent gradient. Centrists’ numbers will not erode, and the decision to not decentralize is upheld for good. To summarize, the interaction of economic with political forces gives rise to two very different (i.e. multiple) equilibria.
all tenants and landlords vote for the tax, notwithstanding the immediate damage to the urban cost of living $t\tilde{r}$. Then $\alpha(1-\lambda^c) + \lambda^c$ or $\Lambda^c(1-\alpha) + \alpha$ is a lower bound on those who vote for the tax. Now consider two cities, such that $\lambda^c_2 < \lambda^c_1$. Holding everything else, both cities exhibit an identical share of tenants for the commuting tax. And so introducing the commuting tax is likelier in city 1. This may explain why the US appears to fight climate change less than Europe does (where cities generally are more compact).

**Development in the Growth-Controlled City.** Suppose the city has suddenly become more attractive (a wage increase, say, or deteriorating conditions elsewhere), transiting from being closed to being open. Only, existing zoning (coinciding with the city’s initial equilibrium) prevents housing construction except for in one particular ring. So consider an exogenous population (and apartment) increase in ring $i$, where $i \leq i^*$, equal to $\gamma > 0$. Making use of our formula (12), this swells the minimum number of centrists $s\lambda^c$ by just:

$$\gamma \cdot (\Delta(i^*)/2 + \gamma)/(s + \gamma)$$

the post-shock minimum centrist share in overall voters, the exogenous rise in $\gamma$ from 0 initially to some positive post-shock value clearly increases minimum centrists’ share in urban voters, as the first derivative

$$\frac{d\lambda^c}{d\gamma} = \frac{d(\Delta(i^*)/2 + \gamma)}{d\gamma} = \frac{s - \Delta(i^*)/2}{(s + \gamma)^2} > 0$$

reveals.\(^2\)

The introduction’s example on Berlin’s Tempelhof airfield could be told along these lines (section 1). Of course, a largely similar analysis might in reverse apply to the establishment of growth controls in peripheral areas. Such an intervention is highly asymmetric, keeping the number of decentrists in the city in check. It is prone to prevent the upsetting of the city’s initial centrist/decentrist-balance. Policy makers, so we suggest, may be enthusiastic to undertake particular policies in anticipation of their impact on the city’s voter composition.

**Proposition 7: (A Decentralization Constraint)**

If a sitting majority of centrists (decentrists) anticipates a current policy to benefit the future minimum decentrist (centrist) share, $\lambda^c$ ($\lambda^d$), it may reject this (otherwise sound) policy for fear of future decentralization (centralization).

6 **Conclusions**

This paper parts with a long tradition of assuming landlords to be absentee or assuming landlords to be resident yet equally invested into urban housing. In this paper, realistically and appropriately, landlords are resident yet heterogeneously invested in the city’s housing stock. Given this framework, we first trace important policy decisions back to a centrist/decentrist-distinction. We then trace back this centrist/decentrist-balance to the

\(^{22}\)Note that a similar analysis applies to understanding the effect of an exogenous housing change in rings $i \leq i^{**}$, where $i^{**}$ is the maximizer of the program on the very r.h.s. of the last line in Proposition 1. The exogenous increase $d\gamma > 0$ has an ambiguous effect on $\Lambda^c$ if $i > i^*$ (or on $\Delta^d$ if $i > i^{**}$).
greatest cumulative ring difference, as a useful bound on centrist numbers that derives from the city’s commuting distribution $s$ and hence is related to the city’s physical shape. The functional relationships between the city’s greatest cumulative ring difference on the one hand and urban outcomes are empirically testable (for an empirical assessment see complete version of the paper).
7 Literature


