INTRODUCTION TO COARSE GEOMETRY
EXERCISE SHEET 1

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Exercise 1 (Example 1.8, 2+2+1)**.** *Let X be a topological space.*

(1) *Show that the collection $\mathcal{B}_{qc}$ of all subsets of X which are subsets of quasi-compact subspaces forms a bornology on X.*
(2) *Show that the collection of $\mathcal{B}_{rc}$ of relatively quasi-compact subsets of X forms a generalized bornology.*
(3) *Give an example where $\mathcal{B}_{rc}$ does not define a bornology.*

Exercise 2 (Example 1.10, 2+3)**.** *Let $\overline{X}$ be a topological space and A be any subset of X. Set $X := \overline{X} \setminus A$.*

(1) *Show that the collection*

$$\mathcal{B} := \{Y \subseteq X \mid \overline{Y} \cap A = \emptyset\}$$

is a generalized bornology which is a bornology whenever X is Hausdorff.
(2) *Show that the set of continuous maps $X \to \mathbb{R}$ whose support is disjoint from A forms a subgroup of $C(X)$, the set of all continuous maps $X \to \mathbb{R}$. Warning: For continuous maps, the notion of support is slightly different from the support of a function defined in the lecture.*

In the following, we consider the category $\mathbf{Born}^{\mathrm{b}}$ of bornological spaces and bornological maps.

Exercise 3 (2+2+1)**.** *Let $S\colon \mathbf{Born}^{\mathrm{b}} \to \mathbf{Set}$ denote the forgetful functor.*

(1) *Does S have a left adjoint?*
(2) *Does S have a right adjoint?*
(3) *Does the forgetful functor $\mathbf{Born} \to \mathbf{Set}$ from the lecture have a right adjoint?*

Exercise 4 (2.5+2.5)**.**

(1) *Show that $\mathbf{Born}^{\mathrm{b}}$ is complete.*
(2) *Show that $\mathbf{Born}^{\mathrm{b}}$ is cocomplete.*

Due on Tuesday, 10th November 2020, 6 pm. Please send solutions to winges at math dot uni-bonn dot de.