AG Seminar, February 3rd 2022 - **Michael Batanin**

Polynomial monads, Grothendieck homotopy theory and delooping of spaces of long knots

Abstract: The category of polynomial monad (in Set) is a particular nice sort of monads well adapted for combinatorial calculations. The category of small categories is a full subcategory of the category of polynomial monads consisting of 'linear' polynomial monads. But other well known monads are also polynomial. For example, free monoid monad, free operad monad, free cyclic or modular operad and many more.

Grothendieck homotopy theory is the homotopy theory of small categories and presheaves on them. Beside being beautiful it is also immensely practical as it allows to develop many tools for calculations in homotopy theory.

In this talk I am going to show that many fundamental constructions and results of Grothendieck theory can be expanded to the word of polynomial monads and their algebras which replace categories of presheaves in this context. This includes: Quillen Theorem A, Thomason theorems, theory of (algebraic) homotopy Kan extensions, theory of aspherical morphisms, locally constant algebras etc.. Moreover, there are new results which can not be considered in classical setting, for example, a theory of delooping of mapping spaces between algebras of polynomial monads.

As an illustration of these methods I will provide a new and more conceptual proof of a seminal theorem of Turchin and Dwyer-Hess on double delooping of space of long knots.