

## Hopf algebras and Hopf algebroids in Number Theory and Algebraic Topology

Commutative *Hopf algebras* over fields are nowadays central objects in Mathematics and can be viewed as defining group schemes (group valued functors on commutative algebras).

I will begin by discussing a Hopf defined over the integers, namely the ring of stably numerical polynomials which is a much more subtle object in many ways since it captures both algebraic and number theoretic information. I will then explain how to enlarge this to a *Hopf algebroid*, a commutative ring defining a *groupoid scheme* (a groupoid valued functor).

Hopf algebroids are very important in Algebraic Topology and I will describe some examples which have very strong number theoretic aspects. The stably numerical polynomials turn out to be part of an infinite family indexed on prime numbers and natural numbers, all of which play a central part in the chromatic picture of stable homotopy, but also have connections with the theory of formal groups and local class field theory.