The singularities of the invariant metric on the Poincaré bundle I & II

In this talks we will recall the definition of the Poincaré bundle on a family of abelian varieties and their dual as well as the definition of the invariant metric on it. Following Hain we will also explain the two interpretations of the Poincaré bundle and its metric, as the bundle of biextensions of Hodge structures and as the Archimedean height pairing between homologically trivial zero cycles and divisors.

Using Pearlstein nilpotent orbit theorem for mixed Hodge structures, will study the singularities of the invariant metric when one approaches the boundary of a toroidal compatification of the family of abelian varieties. We will give two applications of this study.

First we will show that the asymptotic behaviour of the height pairing on a family of curves when one approaches a singular curve, is governed by the Symanzic polynomials of the dual graph of the singular curve. This fact is related to the physical intuition that the asymptotic behaviour of string theory when the length parameter goes to zero is governed by quantum field theory of particles.

Finally we will show that the singularities of the invariant metric are an avatar of the height jump phenomenon discovered by Hain. As an application we will prove a particular case of a conjecture by Hain on the positivity of the height jump.

The applications are joint work with O. Amini, S. Bloch and J. Fresán on one hand and R. de Jong and D. Holmes on the other.