Title:
Decomposition spaces: basic theory and examples

Abstract:
We introduce the notion of decomposition space, generalising that of Segal space, as a simplicial object (in infinity-groupoids) satisfying a certain exactness condition. We show that decomposition spaces are precisely the objects to which one may associate a coherently coassociative "incidence coalgebra" with infinity-groupoid coefficients, using the language of homotopy linear algebra. Duality then gives us the associated convolution algebras, and via homotopy cardinality one recovers classical incidence coalgebras and convolution algebras of posets, monoids and categories. Important examples of decomposition spaces are the Waldhausen S-construction of an abelian (or stable infinity-) category (cf. also Dyckerhoff-Kapranov), whose incidence algebra is a derived Hall algebra. Many convolution algebras that arise classically as ad hoc quotients of incidence algebras of posets can be obtained more canonically from decomposition spaces. We'll present some relevant examples.

[Joint work with J Kock (Universitat Autònoma de Barcelona) and A Tonks (University of Leicester)]