On the K-theory of linear groups

We will show that for every ring $R$ the assembly map in algebraic $K$-theory
\[ H_n^G(EG; \mathbb{K}_R) \to K_n(R[G]) \]
is split injective for every finitely generated subgroup $G$ of a linear group which admits a finite dimensional model for the classifying space $EG$. For this we will use the concept of finite decomposition complexity, first introduced by Guentner, Tessera and Yu. It is a coarse invariant of metric spaces and generalizes the notation of finite asymptotic dimension.