Lifts of Cycle Classes of Sections of $p$-adic Curves

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Abstract

Let $X/k$ be a smooth projective curve of genus $> 1$ over a $p$-adic field $k$ together with a continuous section of the canonical map from the étale fundamental group of $X$ to the absolute Galois group of $k$. Esnault and Wittenberg associated to each such section a cycle class in the étale cohomology of $X$ and showed that it is algebraic in $\ell$-adic cohomology for $\ell \neq p$. The $p$-adic section conjecture predicts that this cycle class admits a canonical lift to the $\ell$-adic cohomology of any reduction of the curve $X$. Studying the étale homotopy type of a sufficiently nice reduction, we give a construction for such a canonical lift of the cycle class, giving a partial answer to a question of Esnault and Wittenberg.