Rationality (and irrationality) of $L$-values.

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Euler’s beautiful formula

$$\zeta(2n) = -\frac{(2\pi i)^{2n}}{2(2n)!}B_{2n}.$$  

can be seen as the starting point of the investigation of special values of $L$-functions. In particular, Euler’s result shows that all critical zeta values are rational up to multiplication with a particular period, here the period is a power of $(2\pi i)$. Conjecturally this is expected to hold for all critical $L$-values of motives. In this talk, we will focus on $L$-functions of number fields. In the first part of the talk, we will discuss the ‘critical’ and ‘non-critical’ $L$-values exemplary for the Riemann zeta function. Afterwards, we will head on to more general number fields and explain our recent joint result with Guido Kings on the algebraicity of general Hecke $L$-values.