Positive scalar curvature on manifolds with odd order abelian fundamental groups

We introduce Riemannian metrics of positive scalar curvature on manifolds with Baas-Sullivan singularities, prove a corresponding homology invariance principle and discuss admissible products. Using this theory we construct positive scalar curvature metrics on closed smooth manifolds of dimension at least five which have odd order abelian fundamental groups, are non-spin and atoral. This solves the Gromov-Lawson-Rosenberg conjecture for a new class of manifolds with finite fundamental groups.