Toric locally conformally Kähler manifolds

A hermitian metric on a complex manifold is called locally conformally Kähler (LCK) if, around every point of the manifold, it is conformal to a local Kähler metric. There exists a natural notion of hamiltonian action of a Lie group with respect to it, generalising the one from symplectic or Kähler geometry.

I will introduce and motivate these notions, focusing on toric LCK manifolds. I will then give a sketch of the proof that any compact toric LCK manifold admits a Vaisman metric. This is a particularly well behaved metric, whose existence translates into important features of the geometry of the manifold. Finally, I will explain how this, together with other known results, leads to a classification of toric LCK manifolds.