Theoretical evidence for the extended abelian rank one Stark conjecture for the base field \( \mathbb{Q} \)

In this talk, we will explain how a generalized Brumer-Stark type statement has consequences for the extended abelian rank one Stark conjecture. We will then exhibit an infinite family of abelian extensions of \( \mathbb{Q} \) with Galois group of type \((2,p,p)\), \(p\) being an odd prime, where this generalized Brumer-Stark type statement can be proven building on previous work of Greither and Kucera.