Dyson series and short time asymptotics for the Green function of stochastic volatility models

The numerical solution of certain stochastic ordinary differential equations may be obtained by approximating the solution of certain parabolic partial differential equations. We are interested in the short time asymptotics of the solutions of these parabolic equations. We use parabolic rescaling and a perturbation argument (Dyson series) common in physics to devise an algorithm that will compute the Green functions (or fundamental solutions, also called “heat kernels” in this setting) of these parabolic equations to arbitrary precision. Numerical tests for some problems motivated by the pricing of options is presented. The results are joint work with Cheng, Constantinescu, Costanzino, Mazzucato, and Liechty. Our results are for uniformly elliptic generators. I will indicate also how the framework of Lie manifolds (introduced jointly with Ammann and Lauter) can be used to extend our results to certain degenerate equations that arise in practice.