Seminar on Ricci flow, part IIb

Winter term 2010/11

Prof. Bernd Ammann (in collaboration with Prof. F. Finster)

Time: Wed 8-10, M103

Dates:

22.12., 12.1., 19.1., 26.1., 2.2., 9.2. (6 Sessions), possibly one spring-break-session would not be bad

Planing obstructions:

Bernd is not present on Jan 26.

Vortrag 1. *Non-negative isotropic curvature.*
Stefan Suhr, 22.12. Follow [5, Sections 7.1–7.3,7.5,7.7]. Additional literature (optional) [6] and the literature by Micallef, Wang and Moore cited therein.

Non-negative isotropic curvature (wPIC) and positive isotropic curvature (PIC) is preserved under the Ricci flow. Discuss geometric interpretation of (w)PIC, and — if possible — indicate some comments on how to prove it. Then discuss (w)PIC on $M \times \mathbb{R}^2$. Also introduce the cones C and $\hat{C}$ of [5]. Mention (w)PIC on $M \times \mathbb{R}$ and on $M \times S^2$.

Vortrag 2. *The Böhm-Wilking curvature identity.*
Farid Madani, 12.1. We follow [5, Sections 8.1 and 8.2]. The original article [3] might also be helpful, but aims in slightly different direction.

Explain the Böhm-Wilking curvature identity [**?**][brendle], and check to what extend its proof can be presented. The most important part of the talk is that we understand well [5, Section 8.2]. If time remains, the cones $\hat{C}(s)$ from [5, Section 8.3] can already be introduced and discussed (until Prop 8.8).

Vortrag 3. *The differentiable sphere theorem.*
Carolina Neira Jimenez, 19.1. We follow [5, Sections 8.3], but it might be wise to read also in [6].

Prove Theorem 8.12 in [5], as well as Prop. 8.13 and Cor 8.14. Some few notions that have not been discussed in the seminar before (such as the pinching set) should be briefly introduced.

If time remains, discuss the improved version [5, Sections 8.4].

Vortrag 4. *Preliminaries for the rigidity discussion.*
Olaf Müller, 26.1. The talk has two parts which are independent from each others and they could be given by different speakers. Part (b) might be pushed to later, depending on how fast we advance.
(a) Bony's strict maximum principle for degenerate elliptic equations, [5, Section 9.3]

(b) Summary of Berger's classification of special holonomy, [5, Section 9.2]

Vortrag 5. *Invariance of a set of frames.*
Manuel Streil, 2.2. [5, Sections (9.4 and) 9.5]
The main goal is to prove [5, Theorem 9.13], Cor. 9.14 should be deduced. Section 9.4 mainly serves as a motivation for the techniques used here.

Vortrag 6. *Kähler-Einstein and quaternionic-Kähler manifolds.*
Mihaela Pilca, 9.2. [5, Section 9.6]
The rigidity discussion before naturally leads to such manifolds. We have to investigate when they carry non-negative isotropic curvature.

Vortrag 7. *Applications and classification results.*
Andreas Hermann, ??.2. [5, Sections 9.7 and 9.8]

Homepage:

`http://www.mathematik.uni-r.de/ammann/lehre/2010w_af_seminar`

Central Literature: [5], [6], [7], [3]
Good overview article: [8]

References

[1] P. Topping; Lectures on the Ricci flow, LMS Lecture Note series 325, Cambridge

[2] Besson, Gérard; Preuve de la conjecture de Poincaré en déformant la métrique par la courbure de Ricci (d'après G. Perelman). Séminaire Bourbaki. Vol. 2004/2005. Astérisque No. 307 (2006), Exp. No. 947, ix, 309–347.

[3] Böhm, Christoph; Wilking, Burkhard; Manifolds with positive curvature operators are space forms. Ann. of Math. (2) 167 (2008), no. 3, 1079–1097.

[4] M. Boileau, Geometrization of 3-manifolds with symmetries. Link siehe Homepage.

[5] S. Brendle; Ricci Flow and the Sphere Theorem, AMS Graduate Studies Vol. 111, 2010

[6] Brendle, Simon; Schoen, Richard; Manifolds with 1/4-pinched curvature are space forms. J. Amer. Math. Soc. 22 (2009), no. 1, 287–307.

[7] Brendle, Simon; Schoen, Richard; Classification of manifolds with weakly 1/4-pinched curvatures, Acta Math. 200, 1–13 (2008) See also ArXiv: 0705.3963

[8] Brendle, Simon; Schoen, Richard; Curvature, sphere theorems, and the Ricci flow; to appear at Bull. AMS, ArXiv: 1001.2278

[9] Chow, B.; Lu P.; Ni, L.; Hamilton's Ricci flow; Graduate Studies in Mathematics, 77; American Mathematical Society, Providence, RI; Science Press, New York, 2006

[10] H.D. Cao, B. Chow, S.C. Chu, S.T. Yau (editeurs), Collected Papers on Ricci Flow, International Press, Series in Geometry and Topology, Volume 37

[11] B. Chow, J. Diff. Geom., **33**, (1991), pp. 325–334

[12] B. Chow, D. Knopf, The Ricci Flow: an introduction, AMS 2004

[13] B. Chow, S.-C. Chu, D. Glickenstein, C. Guenther, J. Isenberg, T. Ivey, D. Knopf, P. Lu, F. Luo, L. Ni, The Ricci flow, techniques and applications, PartI Geometric Aspects, AMS Math. surv. and Monographs, Vol. 135.

[14] DeTurck, D. M.; Deforming metrics in the direction of their Ricci tensors; J. Differential Geom. 18 (1983), no. 1, 157–162.

[15] R. Hamilton, The Ricci flow on surfaces

[16] A. Hatcher. Notes on basic 3-manifold topology, Link siehe Homepage.

[17] B. Kleiner, J. Lott, Notes on Perelman's Paper

[18] J. Lott, www-Seite, http://math.berkeley.edu/ lott/ricciflow/perelman.html

[19] J. Morgan, G. Tian, Ricci Flow and the Poincaré Conjecture, ArXiv: `math/0607607`

[20] J. Morgan, Recent progress on the Poincaré Conjecture and the classification of 3-manifolds. Bull. Amer. Math. Soc. 42 (2005), 57-78. Link siehe Homepage

[21] J. Milnor, Towards the Poincaré Conjecture and the Classification of 3-Manifolds, Notices Amer. Math. Soc. 50 (2003), no. 10, 1226–1233.

[22] G. Perelman, The entropy formula for the Ricci flow and its geometric applications, ArXiv: `math.DG/0211159`

[23] G. Perelman, Ricci flow with surgery on three-manifolds, ArXiv: `math.DG/0303109`

[24] G.Perelman, Finite extinction time for the solutions to the Ricci flow on certain three-manifolds, ArXiv: `math.DG/0307245`

[25] T. Shioya, T. Yamaguchi, Collapsing three-manifolds under a lower curvature bound. J. Differential Geom. 56 (2000), no. 1, 1–66.

[26] T. Shioya, T. Yamaguchi, Volume collapsed three-manifolds with a lower curvature bound, Math. Ann. 333 (2005), no. 1, 131–155.

Alternative literature list on `web site of the seminar`.