Seminar on Wave equations on Lorentzian manifolds and Quantization

Summer term 2022

Prof. Bernd Ammann

Thursday 14-16, M009+Zoom

Number of sessions: **13**

Available Dates: 28.4., 12.5., 19.5., 2.6., 9.6., 23.6., 30.6. (Bernd in Cortona), 7.7. (Bernd in Freiburg), 14.7., 21.7., 28.7. (Bavarian Geometry and Topology Meeting), 29.9. (morning), 29.9. (afternoon)

Special obstruction:

- May 5: Opening of an exhibition about women in mathematics by Sylvie Paycha in parallel, thus no seminar
- May 26: Ascension (Himmelfahrt)
- June 16: Corpus Christi (Fronleichnam)
- June 30: Bernd in Cortona
- July 28: Bavarian Geometry and Topology Meeting in Augsburg, where several participants will got to or talk at

In the seminar we mainly follow [5].

Talk no. 1: Riesz distributions on Minkowski space. *28.4.*
Introduce Riesz distributions as a tool to solve hyperbolic equations [5, Sec. 1.2]

Supplementary Talk no. 1: Foundations of Lorentzian manifolds *12.5.*
Whether this talk will be included depends on the audience. It even might be extended two 2 sessions if needed.

Talk no. 2: Riesz distributions on Lorentzian domains and normally hyperbolic operatrors. *19.5.*

The subject of the talk is [5, Sec. 1.4 and 1.5]. In the talk, some parts of Sec. 1.3 shall be recalled, but the speaker may assume that the audience is either familiar with basic facts about Lorentzian manifold or willing to read it, e. g., in [1].

Talk no. 3: Local theory, part 1. *2.6.*
Subtitle: Local solutions of normally hyperbolic pdes on Lorentzian manifolds[5, Sec. 2.1 to 2.3]

Talk no. 4: Local theory, part 2. *9.6.*
[5, Sec. 2.4]

Talk no. 5: Local theory, part 2. *23.6.*
[5, Sec. 2.5 to 2.6]

Talk no. 6: Global theory, part 1. *30.6.* **(Bernd in Cortona)**
Uniqueness of the fundamental solution and the Cauchy problem [5, Sec. 3.1 and 3.2]

Talk no. 7: Global theory, part 2. *7.7.* **(Bernd in Freiburg)**
Fundamental solutions and Green's operators
[5, Sec. 3.3 to 3.5]

Talk no. 8: Quantization, part 1. *14.7.*
C^*-algebras, the canonical commutation relations, CCR algebras [5, Sec. 4.1 and 4.2, Part 1]

Talk no. 9: Quantization, part 2. *21.7.*
Quantization functors [5, Sec. 4.2 Part 2 and 4.3]

No talk on *28.7.* **(Bavarian Geometry and Topology Meeting)** .

Talk no. 10: Quantization, part 3. *29.9. (morning)*
Quasi-local C^*-algebras, and the Haag-Kastler axioms from quantum field theory on curved spaces [5, Sec. 4.4 and 4.5]

Talk no. 11: Quantization, part 4. *29.9. (afternoon)*
Fock space and the quantum field defined by a Cauchy hypersurface [5, Sec. 4.6 and 4.7]

Further Literature

All kind of further literature, partially to motivate the subject from the physical side, partially for historic reasons.
[3] [4] [2] [8] [7] [6] [12] [9, 10] [11]

Seminar-Homepage

http://www.mathematik.uni-regensburg.de/ammann/wave-equations

Literatur

[1] Bär, C. Lorentzian geometry. Lecture Notes, University of Potsdam, Summer term 2004. Version of Jan 7, 2020, direct link.

[2] Bär, C., and Fredenhagen, K., Eds. Quantum field theory on curved spacetimes, vol. 786 of Lecture Notes in Physics. Springer-Verlag, Berlin, 2009. Concepts and mathematical foundations, Lecture notes from the course held at the University of Potsdam, Potsdam, October 2007.

[3] Bär, C., and Ginoux, N. CCR- versus CAR-quantization on curved spacetimes. In Quantum field theory and gravity. Birkhäuser/Springer Basel AG, Basel, 2012, pp. 183–206. arXiv: 1104.1158.

[4] Bär, C., and Ginoux, N. Classical and quantum fields on Lorentzian manifolds. In Global differential geometry, C. Bär, J. Lohkamp, and M. Schwarz, Eds., vol. 17 of Springer Proc. Math. Springer, Heidelberg, 2012, pp. 359–400.

[5] Bär, C., Ginoux, N., and Pfäffle, F. Wave equations on Lorentzian manifolds and quantization. ESI Lectures in Mathematics and Physics. European Mathematical Society (EMS), Zürich, 2007. arXiv: 0806.1036.

[6] Brunetti, R., and Fredenhagen, K. Interacting quantum fields in curved space: renormalizability of ϕ^4. In Operator algebras and quantum field theory (Rome, 1996). Int. Press, Cambridge, MA, 1997, pp. 546–563.

[7] Brunetti, R., Fredenhagen, K., and Köhler, M. The microlocal spectrum condition and Wick polynomials of free fields on curved spacetimes. Comm. Math. Phys. 180, 3 (1996), 633–652.

[8] Brunetti, R., Fredenhagen, K., and Verch, R. The generally covariant locality principle—a new paradigm for local quantum field theory. vol. 237. 2003, pp. 31–68. Dedicated to Rudolf Haag.

[9] Dimock, J. Algebras of local observables on a manifold. Comm. Math. Phys. 77, 3 (1980), 219–228.

[10] Dimock, J. Dirac quantum fields on a manifold. Trans. Amer. Math. Soc. 269, 1 (1982), 133–147.

[11] Dimock, J. Quantized electromagnetic field on a manifold. Rev. Math. Phys. 4, 2 (1992), 223–233.

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