Very singular diffusion equations: second and fourth order models for crystal growth phenomena

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Abstract

In crystal growth phenomena it frequently observed a flat surface called a facet especially in low temperature. From macroscopic thermodynamical point of view it is sometimes explained by singular interfacial energy. However, a model equation describing motion of a crystal surface is a very singular diffusion equation in the sense that the speed is determined by a nonlocal quantity like area of a facet. Typical examples include the total variation flow which is popular in image processing and crystalline flow in materials science. Mathematical analysis on these equations is highly nontrivial. There are several approaches including viscosity and variational approach. There are also fourth order models describing relaxation dynamics of a crystal surface. Compared with second order problems its analysis becomes much tougher because comparison principle does not apply. In this talk we survey mathematical analysis of both second and fourth models with emphasis on recent developments. This lecture is based on my joint work with M.-H. Giga, R. V. Kohn, P. Rybka.

References

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