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Evolution and long-time behaviour of the free boundary in nonlinear Stefan problems

We consider the following free boundary problem

$$\begin{cases} u_t - d\Delta u = f(u) & \text{for } \ x \in \Omega(t), t > 0, \\ u = 0 \text{ and } u_t = \mu|\nabla_x u|^2 & \text{for } \ x \in \Gamma(t), t > 0, \\ u(0,x) = u_0(x) & \text{for } \ x \in \Omega_0, \end{cases} \tag{1}$$

where $\Omega(t) \subset \mathbb{R}^n$ $(n \geq 2)$ is bounded by the free boundary $\Gamma(t)$, with $\Omega(0) = \Omega_0$, μ and d are given positive constants. Our assumptions on $f(u)$ include monostable, bistable and combustion type nonlinearities.
We show that the free boundary $\Gamma(t)$ is C^1 outside the convex hull of Ω_0, and as $t \to \infty$, either $\Gamma(t)$ remains bounded and $u(t,.) \to 0$ in the L^∞ norm, or $\Gamma(t)$ goes to infinity in the sense that it is contained in an annulus of the form $\{R(t) - C_0 \leq |x| \leq R(t)\}$, with $R(t) \to \infty$ as $t \to \infty$. Moreover, $R(t)/t \to k_0 > 0$ as $t \to \infty$.
This is joint work with Hiroshi Matano (Tokyo) and Kelei Wang (Sydney).