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Optimal closed-loop controls via finite system of control devices for reaction-diffusion processes

The basic concept in view of control theory for partial differential equations is so-called open-loop controls approach consisting in prescribing a priori the strain of the control influencing the process, in a way such that the process would evolve in the expected manner. However, in real applications the attainable accuracy of utilized controls is limited and hence the open-loop approach may occur to be not sufficient in case of processes which exhibit unstable behaviour, as reaction-diffusion processes with nonmonotone reaction term. This leads to the idea of closed-loop approach in which the strain of the control is established during the experiment, in real-time, basing on some available observation data about the actual state of the process. We consider an implementation of a closed-loop control system for nonlinear reaction-diffusion processes as a system with a finite number of control devices which actions are based on the observation data collected by a finite set of measurement units compared with given reference pattern. Every measurement unit is characterized with some density function and provides the observation data computed as a scalar product of this density and the state of the process. This results in a mathematical model in which the observation operators are nonlocal in space. The problem we focus on is a problem of optimal localization and shape of control devices and measurement units. The criterion of optimality under investigation is to minimize the cost functional computed as the L^2 distance between the real evolution of the controlled process and the reference state, modified by adding suitable regularizing term. The latter term is a norm of optimized elements in a suitable function space. During the talk, we will present present the result which says that the above described optimization problem has at least one solution. The proof of the result bases on the standard concept of selecting a minimizing sequence for analyzed cost functional and justifying the limit passage in this sequence. Next, we will focus on characterization of the optimal control devices and measurement units by formulating the suitable optimality system. Deriving the optimality system involves calculating the Gateaux differential of the state operator (which assigns the realization of the process to given choice of the devices) and calculating its adjoint in suitable sense, together with formulating the Banach space equivalent of the Fermat's necessary condition for minimizing the given functional. We also intend to present numerical results concerning the presented problem. The above problem is investigated in frames of a Ph.D. project supervised by Professor Marek Niezgdka (ICM, Warsaw Univerity).