EXOTIC FINITE FUNCTORIAL SEMI-NORMS ON SINGULAR HOMOLOGY

DANIEL FAUSER AND CLARA LÖH

ABSTRACT. Functorial semi-norms on singular homology give refined "size" information on singular homology classes. A fundamental example is the ℓ^1-semi-norm. We show that there exist finite functorial semi-norms on singular homology that are exotic in the sense that they are *not* carried by the ℓ^1-semi-norm.

1. INTRODUCTION

Functorial semi-norms on singular homology give refined "size" information on singular homology classes. On the one hand, functorial semi-norms lead to obstructions for mapping degrees. On the other hand, mapping degrees allow to construct functorial semi-norms on singular homology. A fundamental example of a finite functorial semi-norm on singular homology is the ℓ^1-semi-norm underlying the definition of simplicial volume (see Section 2 for the definitions).

While the general classification of functorial semi-norms on singular homology is out of reach, one can ask for the role of the ℓ^1-semi-norm among all finite functorial semi-norms [1, Question 5.8]. A simple rescaling manipulation shows that not all finite functorial semi-norms on singular homology are dominated by a multiple of the ℓ^1-semi-norm [1, Section 5]. Relaxing the domination condition, we introduce the following relation between functorial semi-norms:

Definition 1.1 (carriers of functorial semi-norms)**.** Let $d \in \mathbb{N}$. A functorial semi-norm $|\cdot|$ on $H_d(\,\cdot\,;\mathbb{R})$ *carries* a functorial semi-norm $|\cdot|'$ if for all topological spaces X and all $\alpha \in H_d(X;\mathbb{R})$ we have

$$|\alpha| = 0 \Longrightarrow |\alpha|' = 0.$$

In these terms, the current paper is concerned with the question whether every finite functorial semi-norm on singular homology is carried by the ℓ^1-semi-norm. All finite functorial semi-norms on $H_d(\,\cdot\,;\mathbb{R})$ that are multiplicative under finite coverings are carried by the ℓ^1-semi-norm in a strong sense [1, Proposition 7.11]. However, if the multiplicativity condition is dropped, then exotic finite functorial semi-norms appear:

Theorem 1.2. *Let $d \in \{3\} \cup \mathbb{N}_{\geq 5}$. Then there exists a finite functorial semi-norm on $H_d(\,\cdot\,;\mathbb{R})$ that is* not *carried by the ℓ^1-semi-norm.*

Date: February 12, 2018. This work was supported by the CRC 1085 *Higher Invariants* (Universität Regensburg, funded by the DFG).
MSC 2010 classification: 55N10, 57N65.

In particular, this answers a question by Crowley and Löh on "maximality" of the ℓ^1-semi-norm in the negative [1, Question 5.8].

Our construction of exotic finite functorial semi-norms is based on the parallel observation about mapping degrees of manifolds:

Theorem 1.3. *Let $d \in \{3\} \cup \mathbb{N}_{\geq 5}$. Then there exists a strongly inflexible oriented closed connected d-manifold M with $\|M\| = 0$. Moreover, we can choose M to be aspherical.*

In contrast, Theorem 1.3 is clearly wrong in dimension 2. However, it is an open problem to decide whether all finite functorial semi-norms on $H_2(\,\cdot\,;\mathbb{R})$ are carried by the ℓ^1-semi-norm. In the case of dimension 4, both Theorem 1.2 and 1.3 remain open.

Organisation of this article. We recall the basic terminology for functorial semi-norms in Section 2. Strongly inflexible manifolds are discussed in Section 3. The proof of Theorem 1.3 is given in Section 3.3 and Theorem 1.2 is then derived in Section 4. Moreover, we briefly we explain the relation with secondary simplicial volume in Section 3.4.

2. Functorial semi-norms

We begin by recalling the terminology for functorial semi-norms and the ℓ^1-semi-norm in particular.

2.1. **Terminology.** In the following, semi-norms are allowed to have values in $\mathbb{R}_{\geq 0} \cup \{\infty\}$, where we use the usual conventions that $a + \infty = \infty$ and $b \cdot \infty = \infty$ holds for all $a \in \mathbb{R}_{\geq 0}$ and all $b \in \mathbb{R}_{>0}$.

Definition 2.1 (functorial semi-norm)**.** Let $d \in \mathbb{N}$. A *functorial semi-norm* on $H_d(\,\cdot\,;\mathbb{R})$ is a lift of the functor $H_d(\,\cdot\,;\mathbb{R})\colon \mathsf{Top} \longrightarrow \mathsf{Vect}_{\mathbb{R}}$ to a functor $\mathsf{Top} \longrightarrow \mathsf{Vect}^{\mathsf{sn}}_{\mathbb{R}}$, where $\mathsf{Vect}^{\mathsf{sn}}_{\mathbb{R}}$ denotes the category of semi-normed $\mathbb{R}$-vector spaces with norm non-increasing $\mathbb{R}$-linear maps. More concretely, a functorial semi-norm on $H_d(\,\cdot\,;\mathbb{R})$ consists of a choice of a semi-norm $|\cdot|$ on $H_d(X;\mathbb{R})$ for every topological space such that the following compatibility holds: If $f\colon X \longrightarrow Y$ is a continuous map, then

$$\forall_{\alpha \in H_d(X;\mathbb{R})} \ \bigl|H_d(f;\mathbb{R})(\alpha)\bigr| \leq |\alpha|.$$

A functorial semi-norm on $H_d(\,\cdot\,;\mathbb{R})$ is *finite* if $|\alpha| < \infty$ for all singular homology classes α in degree d.

Remark 2.2. If $|\cdot|$ is a functorial semi-norm on $H_d(\,\cdot\,;\mathbb{R})$ and if $f\colon N \longrightarrow M$ is a continuous map between oriented closed connected d-manifolds, then

$$|\deg f| \cdot \bigl|[M]_{\mathbb{R}}\bigr| \leq \bigl|[N]_{\mathbb{R}}\bigr|.$$

In particular: If $0 < |[M]_{\mathbb{R}}| < \infty$, then M is strongly inflexible (Definition 3.1).

The classical example of a finite functorial semi-norm on $H_d(\,\cdot\,;\mathbb{R})$ is the ℓ^1-semi-norm (see Section 2.2 below). Other examples of functorial semi-norms can be constructed by means of manifold topology, e.g., the products-of-surfaces semi-norm [1, Sections 2, 7] or infinite functorial semi-norms that exhibit exotic behaviour on certain classes of simply connected spaces of high dimension [1, Theorem 1.2]; such a construction principle via manifold topology will be recalled in Section 4.1.

2.2. **The ℓ^1-semi-norm and simplicial volume.** For the sake of completeness we include the definition of the ℓ^1-semi-norm on singular homology and simplicial volume.

Definition 2.3 (ℓ^1-semi-norm)**.** Let $d \in \mathbb{N}$ and let X be a topological space. For a singular chain $c = \sum_{j=1}^{m} a_j \cdot \sigma_j \in C_d(X;\mathbb{R})$ (in reduced form) we define

$$|c|_1 := \sum_{j=1}^{m} |a_j|.$$

I.e., $|\cdot|_1$ is the ℓ^1-norm on $C_d(X;\mathbb{R})$ associated with the basis given by all singular d-simplices in X. The semi-norm $\|\cdot\|_1$ on $H_d(X;\mathbb{R})$ induced by the norm $|\cdot|_1$ via

$$\begin{aligned} \|\cdot\|_1 \colon H_d(X;\mathbb{R}) &\longrightarrow \mathbb{R}_{\geq 0} \\ \alpha &\longmapsto \inf\bigl\{|c|_1 \bigm| c \in C_d(X;\mathbb{R}),\ \partial c = 0,\ [c] = \alpha\bigr\} \end{aligned}$$

is the *ℓ^1-semi-norm on $H_d(X;\mathbb{R})$*.

A straightforward calculation shows that the ℓ^1-semi-norm indeed is a functorial semi-norm on $H_d(\,\cdot\,;\mathbb{R})$. Applying this semi-norm to fundamental classes of manifolds gives rise to Gromov's simplicial volume [4]:

Definition 2.4 (simplicial volume/Gromov norm)**.** The *simplicial volume* of an oriented closed connected manifold M is defined by

$$\|M\| := \bigl\|[M]_{\mathbb{R}}\bigr\|_1 \in \mathbb{R}_{\geq 0}.$$

Geometrically speaking, simplicial volume measures how many singular simplices are needed to reconstruct (the real fundamental class of) the given manifold. Simplicial volume allows for interesting applications, linking topology and geometry of manifolds [4, 9]. Useful algebraic tools in the context of simplicial volume are so-called bounded cohomology [4, 5] and ℓ^1-homology [8].

3. Strongly inflexible manifolds

We will now focus on the manifold aspects, discussing the basic terminology of (strong) inflexibility and proving Theorem 1.3. Moreover, we will discuss the meaning of this result in terms of secondary simplicial volume (Section 3.4).

3.1. **Terminology.** We first recall the definition of (strong) inflexibility [1].

Definition 3.1 (inflexibility)**.** Let M be an oriented closed connected manifold of dimension $n \in \mathbb{N}$. For oriented closed connected n-manifolds N we write

$$D(N,M) := \bigl\{\deg f \bigm| f \in \operatorname{map}(N,M)\bigr\}.$$

We call M *inflexible* if $D(M,M)$ is finite. We call M *strongly inflexible* if for every oriented closed connected n-manifold N the set $D(N,M)$ is finite. Conversely, we call M *weakly flexible* if it is not strongly inflexible, i.e., if there exists an oriented closed connected n-manifold N such that $D(N, M)$ is infinite.

For example, spheres and tori are flexible (i.e., not inflexible). Oriented closed connected hyperbolic manifolds are strongly inflexible [4, 1]. In fact, all oriented closed connected manifolds with non-zero simplicial volume are strongly inflexible. More conceptually, all finite functorial semi-norms provide obstructions to strong inflexibility and vice versa [1, 10]. We will discuss this relation in more detail in Section 4.1.

3.2. **Products of strongly inflexible manifolds.** In many examples it is known that products of inflexible manifolds are inflexible [1]. However, it is not clear that this holds in general. In the case of strongly inflexible manifolds, the situation simplifies as follows:

Proposition 3.2 (products of strongly inflexible manifolds)**.** *Let M_1 and M_2 be oriented closed connected strongly inflexible manifolds. Then also $M_1 \times M_2$ is strongly inflexible.*

The proof is based on Thom's representation of homology classes by manifolds and straightforward calculations in singular homology and cohomology; similar arguments appear in related work on domination of/by product manifolds [6, 7, 12].

Proof. We abbreviate $n_1 := \dim M_1$, $n_2 := \dim M_2$ and $n := \dim M_1 \times M_2 = n_1 + n_2$. Let N be an oriented closed connected n-manifold. We need to show that $D(N, M_1 \times M_2)$ is finite.

As first step, we will prove that the sets

$$\begin{aligned} F_1 &:= \bigl\{ H_{n_1}(f_1;\mathbb{Q}) \bigm| f_1 \in \operatorname{map}(N, M_1) \bigr\} \subset \operatorname{Hom}_{\mathbb{Q}}\bigl(H_{n_1}(N;\mathbb{Q}), H_{n_1}(M_1;\mathbb{Q})\bigr) \\ F_2 &:= \bigl\{ H_{n_2}(f_2;\mathbb{Q}) \bigm| f_2 \in \operatorname{map}(N, M_2) \bigr\} \subset \operatorname{Hom}_{\mathbb{Q}}\bigl(H_{n_2}(N;\mathbb{Q}), H_{n_2}(M_2;\mathbb{Q})\bigr) \end{aligned}$$

are finite.

Because N is a closed manifold we know that $H_{n_1}(N;\mathbb{Q})$ is finite dimensional. Let $B_1 \subset H_{n_1}(N;\mathbb{Q})$ be a $\mathbb{Q}$-basis. Let $\beta \in B_1$. In view of Thom's representation of homology classes by manifolds [13] there exists an oriented closed connected n_1-manifold N_β, a continuous map $g_\beta \colon N_\beta \longrightarrow N$ and a $k_\beta \in \mathbb{Q} \setminus \{0\}$ with

$$H_{n_1}(g_\beta;\mathbb{Q})[N_\beta]_{\mathbb{Q}} = k_\beta \cdot \beta \in H_{n_1}(N;\mathbb{Q}).$$

Because M_1 is strongly inflexible, the set $D(N_\beta, M_1)$ is finite. Composition with g_β shows therefore that also the set

$$\bigl\{ d \in \mathbb{Z} \bigm| \exists_{f_1 \in \operatorname{map}(N, M_1)}\ H_{n_1}(f_1;\mathbb{Q})(\beta) = d \cdot [M_1]_{\mathbb{Q}} \bigr\}$$

is finite. Because B_1 is a finite $\mathbb{Q}$-basis of $H_{n_1}(N;\mathbb{Q})$ we hence obtain that the set F_1 is finite. For the same reason also F_2 is finite.

As second step, we will now combine the finiteness of F_1 and F_2 with the cohomological cross-product to derive finiteness of $D(N, M_1 \times M_2)$.

To this end let $f \in \operatorname{map}(N, M_1 \times M_2)$. We write

$$f_1 := p_1 \circ f \in \operatorname{map}(N, M_1) \qquad \text{and} \qquad f_2 := p_2 \circ f \in \operatorname{map}(N, M_2),$$

where $p_1 \colon M_1 \times M_2 \longrightarrow M_1$ and $p_2 \colon M_1 \times M_2 \longrightarrow M_2$ are the projections onto the factors. We then obtain for the cohomological fundamental classes with $\mathbb{Q}$-coefficients that

$$\begin{aligned}
\deg f \cdot [M_1 \times M_2]^*_{\mathbb{Q}} &= H^n(f;\mathbb{Q})[M_1 \times M_2]^*_{\mathbb{Q}} \\
&= \varepsilon \cdot H^n(f;\mathbb{Q})\big([M_1]^*_{\mathbb{Q}} \times [M_2]^*_{\mathbb{Q}}\big) \\
&= \varepsilon \cdot H^n(f;\mathbb{Q})\big(H^{n_1}(p_1;\mathbb{Q})[M_1]^*_{\mathbb{Q}} \cup H^{n_2}(p_2;\mathbb{Q})[M_2]^*_{\mathbb{Q}}\big) \\
&= \varepsilon \cdot H^{n_1}(f_1;\mathbb{Q})[M_1]^*_{\mathbb{Q}} \cup H^{n_2}(f_2;\mathbb{Q})[M_2]^*_{\mathbb{Q}},
\end{aligned}$$

where $\varepsilon := (-1)^{n_1 \cdot n_2}$. In particular, the degree $\deg f$ is uniquely determined by $H^{n_1}(f_1;\mathbb{Q})$ and $H^{n_2}(f_2;\mathbb{Q})$, which in turn (by the universal coefficient theorem) are uniquely determined by $H_{n_1}(f_1;\mathbb{Q}) \in F_1$ and $H_{n_2}(f_2;\mathbb{Q}) \in F_2$. Because the sets F_1 and F_2 are finite by the first step, we obtain that also $D(N, M_1 \times M_2)$ is finite. □

3.3. **Strongly inflexible manifolds with trivial simplicial volume.** We will now give examples of stronlgy inflexible aspherical manifolds whose simplicial volume is zero. We start with the case of dimension 3 and then use inheritance of strong inflexibility under products to deal with the general case.

Example 3.3 (dimension 3). Let M be the total space of an orientable non-trivial S^1-bundle over an oriented closed connected surface of genus at least 2. Then M is strongly inflexible (see Corollary 3.9 below) and aspherical. On the other hand, it is known that $\|M\| = 0$ [4, 5].

In combination with Proposition 3.2 we can now prove Theorem 1.3:

Proof of Theorem 1.3. Let M_1 be an oriented closed connected aspherical 3-manifold that is strongly inflexible and satisfies $\|M_1\| = 0$; such manifolds exist by Example 3.3.

If $d \geq 5$, we take an oriented closed connected hyperbolic manifold M_2 of dimension $d - 3$. Then M_2 is aspherical and M_2 is strongly inflexible.

In view of Proposition 3.2 also the product $M_1 \times M_2$ is strongly inflexible. Moreover, by construction, $M_1 \times M_2$ is an oriented closed connected aspherical d-manifold and we have [4]

$$\|M_1 \times M_2\| \leq \binom{d}{3} \cdot \|M_1\| \cdot \|M_2\| = 0,$$

as desired. □

Remark 3.4. Using fundamental properties of ℓ^1-homology, one can see that the examples of strongly inflexible manifolds M constructed in the proof of Theorem 1.3 do not only have trivial simplicial volume but also are ℓ^1-*invisible*, i.e., the image $[M]^{\ell^1} \in H_*^{\ell^1}(M;\mathbb{R})$ of $[M]_{\mathbb{R}}$ in ℓ^1-homology is the zero class [8, Example 6.7]. In other words, ℓ^1-invisibility of manifolds does not imply weak flexiblity. Notice that it is an open problem whether all manifolds with trivial simplicial volume are ℓ^1-invisible.

We conclude this section with some open problems on strong inflexibility. Generalising Example 3.3, one could ask:

Question 3.5. *Let B be an oriented closed connected hyperbolic manifold and let $M \longrightarrow B$ be a non-trivial circle bundle over B. Under which conditions will M be strongly inflexible?*

In general, the fundamental class of B cannot be lifted to M and hence does not provide a useful obstruction on M. However, one could try to use a lift in ℓ^1-homology. More concretely: Let $p\colon M \longrightarrow B$ be a circle bundle. Then the induced map $H_*^{\ell^1}(p;\mathbb{R})\colon H_*^{\ell^1}(M;\mathbb{R}) \longrightarrow H_*^{\ell^1}(B;\mathbb{R})$ is an isometric isomorphism [8]. In particular, if M and B are oriented closed connected manifolds of dimension n and $n-1$ respectively, we obtain the codimension 1 class

$$\alpha := H_{n-1}^{\ell^1}(p;\mathbb{R})^{-1}([B]^{\ell^1}) \in H_{n-1}^{\ell^1}(M;\mathbb{R})$$

in ℓ^1-homology of M. For example, in the case that B is a hyperbolic surface and p is a non-trivial bundle, this class was considered by Derbez in the study of local rigidity of aspherical 3-manifolds [2]. More generally, if B is an oriented closed connected hyperbolic manifold, the class α will be non-trivial – it will even have non-zero ℓ^1-semi-norm. Can this class be used as an obstruction to prove strong inflexibility of M ?

At the other extreme, it also remains an open problem to determine whether there exist simply connected strongly inflexible manifolds (of non-zero dimension).

3.4. **Secondary simplicial volume.** Secondary simplicial volume is a refinement of simplicial volume that allows to give refined information about vanishing of simplicial volume.

Definition 3.6 (secondary simplicial volume)**.** Let M be an oriented closed connected manifold of dimension $n \in \mathbb{N}$. The *secondary simplicial volume of M* is defined to be the integral sequence

$$\Sigma(M) := \bigl(\bigl\|k \cdot [M]_{\mathbb{Z}}\bigr\|_{1,\mathbb{Z}}\bigr)_{k\in\mathbb{N}},$$

where $\|\cdot\|_{1,\mathbb{Z}}$ is the semi-norm on $H_n(\;\cdot\;;\mathbb{Z})$ induced by the $\mathbb{Z}$-valued ℓ^1-norm on $C_n(\;\cdot\;;\mathbb{Z})$.

Remark 3.7. For all oriented closed connected manifolds M the following holds [10, Remark 5.4]:

$$||M|| = \inf_{k\in\mathbb{N}_{>0}} \frac{1}{k} \cdot \bigl\|k \cdot [M]_{\mathbb{Z}}\bigr\|_{1,\mathbb{Z}}.$$

In particular, vanishing of the simplicial volume of M can be expressed in terms of the growth behaviour of the sequence $\Sigma(M)$.

The strongest vanishing of simplicial volume occurs if the secondary simplicial volume contains a bounded subsequence. We recall the complete geometric characterisation of such manifolds in Propositions 3.8 and 3.11 in terms of flexibility.

Proposition 3.8 ([10, Corollary 5.5])**.** *Let M be an oriented closed connected manifold of dimension $n \in \mathbb{N}$. Then the following are equivalent:*

(1) *The manifold M is weakly flexible.*
(2) *The secondary simplicial volume $\Sigma(M)$ contains a bounded subsequence.*

(3) *All finite functorial semi-norms on* $H_n(\,\cdot\,;\mathbb{R})$ *vanish on* $[M]_\mathbb{R}$.

We will now focus on the 3-dimensional case. The classification of weakly flexible 3-manifolds by Derbez, Sun, and Wang [3] translates into the following result:

Corollary 3.9. *Let M be an oriented closed connected 3-manifold. Then the following are equivalent:*

(1) *The secondary simplicial volume $\Sigma(M)$ contains a bounded subsequence.*
(2) *Each prime summand of M is covered by a torus bundle over S^1, by a trivial S^1-bundle or by S^3.*

In particular, the 3-manifolds from Example 3.3 are strongly inflexible.

Remark 3.10. In the previous section we observed that the 3-manifolds from Example 3.3 even give examples for ℓ^1-invisible manifolds with non-trivial secondary simplicial volume. In contrast, we asked whether all manifolds with trivial secondary simplicial volume are ℓ^1-invisible. In dimension 3, this easily follows from the characterization above and the following two facts [8, Example 6.7]:

– Total spaces of fibrations of oriented closed connected manifolds with fibre an oriented closed connected manifold that has amenable fundamental group are ℓ^1-invisible.
– The connected sum of two ℓ^1-invisible 3-manifolds is ℓ^1-invisible.

Proposition 3.11 ([11, Theorem 3.2])**.** *Let M be an oriented closed connected manifold of dimension $n \in \mathbb{N}_{>0}$. Then the following are equivalent:*

(1) *The secondary simplicial volume $\Sigma(M)$ contains a bounded subsequence with bound 1.*
(2) *The manifold M is dominated by S^n (i.e., there exists a map $S^n \longrightarrow M$ of non-zero degree) and n is odd.*

Corollary 3.12. *Let M be an oriented closed connected 3-manifold. Then the following are equivalent:*

(1) *The secondary simplicial volume $\Sigma(M)$ contains a bounded subsequence with bound 1.*
(2) *The manifold M is spherical, i.e., finitely covered by S^3.*

Proof. If $f\colon N \longrightarrow M$ is a map of non-trivial degree between oriented closed connected manifolds of the same dimension, then the image of $\pi_1(f)$ is a finite index subgroup in $\pi_1(M)$. Therefore, it follows by the Elliptization Theorem, that domination by S^3 and being finitely covered by S^3 is equivalent for oriented closed connected 3-manifolds. □

4. Exotic finite functorial semi-norms

We will first recall how strongly inflexible manifolds generate interesting functorial semi-norms (Section 4.1). Combining this construction with Theorem 1.3 will then complete the proof of Theorem 1.2 (Section 4.2).

4.1. **Generating functorial semi-norms via manifolds.** We will apply the following principle to generate exotic functorial semi-norms:

Proposition 4.1 (domination semi-norm associated with a manifold [1, Section 7.1]). *Let $d \in \mathbb{N}$ and let M be an oriented closed connected d-manifold. Then there exists a functorial semi-norm $|\cdot|_M$ on $H_d(\,\cdot\,;\mathbb{R})$ satisfying*

$$\big|[N]_{\mathbb{R}}\big|_M = \sup\big\{|D| \bigm| D \in D(N,M)\big\} \in \mathbb{R}_{\geq 0} \cup \{\infty\}$$

for all oriented closed connected d-manifolds N. The functorial semi-norm $|\cdot|_M$ is finite if and only if M is strongly inflexible.

For instance, in this way, functorial semi-norms on $H_{64}(\,\cdot\,;\mathbb{R})$ have been constructed that are non-trivial on certain classes of simply connected spaces [1, Theorem 1.2]. However, it is not known whether these are examples of finite functorial semi-norms.

4.2. **Construction of exotic finite functorial semi-norms.** We finally complete the proof of Theorem 1.2.

Proof of Theorem 1.2. Let $d \in \{3\} \cup \mathbb{N}_{\geq 5}$. By Theorem 1.3 there exists a strongly inflexible oriented closed connected d-manifold M with $\|M\| = 0$. Let $|\cdot|_M$ be the associated domination semi-norm on $H_d(\,\cdot\,;\mathbb{R})$; because M is strongly inflexible, this functorial semi-norm is indeed finite. By construction, we have

$$\big|[M]_{\mathbb{R}}\big|_M = \sup\big\{|D| \bigm| D \in D(M,M)\big\} = 1,$$

but $\big\|[M]_{\mathbb{R}}\big\|_1 = \|M\| = 0$. In particular, $|\cdot|_M$ is *not* carried by the ℓ^1-semi-norm in the sense of Definition 1.1 (this holds even on fundamental classes of aspherical manifolds). □

Question 4.2. *Does there exist a finite functorial semi-norm on $H_d(\,\cdot\,;\mathbb{R})$ that carries all other finite functorial semi-norms on $H_d(\,\cdot\,;\mathbb{R})$?*

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Clara Löh
Daniel Fauser

Fakultät für Mathematik, Universität Regensburg, 93040 Regensburg
clara.loeh@mathematik.uni-r.de, http://www.mathematik.uni-r.de/loeh
daniel.fauser@mathematik.uni-regensburg.de, http://www.mathematik.uni-r.de/fauser