

Differential Geometry I: Week 14

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CAT(2021). Commander Blorx is invited as a special guest to the **Congress of Astro-Topologists 2021**. Blorx feels quite honoured, but is also a bit afraid that there might be too many too uninspiring talks and not enough chocolate cookies. This year's hot topic of the congress is planetary design products. Help Blorx to get through this event!



Problem 13.1 (the opening anthem). Same procedure as every year: The congress begins with the opening anthem. The exact key of the opening anthem is a well-kept secret. However, there are some rumours. Which of the following keys are musically geometrically sound?

$R^\sharp = Rm$	BL
$R^b = Rm$	JU
$Rm^\sharp = R$	PI
$Rm^b = R$	OR

Problem 13.2 (CAT-prize). Commander Blorx is awarded the prestigious *CAT-prize for non-felines* for his contributions to inter-planetary roguery. During the ceremony, Blorx is handed the ornate CAT cup, bearing his name. Unfortunately, the committee misspelled Blorx's name as R_{BLOX} . Which of the following inscriptions would be equivalent to R_{BLOX} ?

$-R_{BOLX}$	XI
R_{BBOX}	TER
R_{OXBL}	TE
$-R_{LBOX}$	RI
R_{LOXB}	XIS
$-R_{LBXO}$	R
R_{XOOX}	ST

Problem 13.3 (disc-shaped planets). After these preliminaries, the actual scientific part of the congress begins. The first session is about disc-shaped planets (which used to be the state of the art for a long time!). Which of the following properties can a complete Riemannian metric have on an open disc in the plane?

$sec < 0$	SFL
$sec = 0$	ATT
$sec > 0$	NOT

Please turn over

Problem 13.4 (planets of constant curvature). Currently it is fashionable to create planets of constant sectional curvature. Which of the following smooth manifolds admit a Riemannian metric of constant sectional curvature?

- | | |
|------------------------------------|------------|
| $\mathbb{R}^2 \times \mathbb{R}^2$ | ER |
| $\mathbb{S}^2 \times \mathbb{R}^2$ | APL |
| $\mathbb{S}^2 \times \mathbb{S}^2$ | A |

Problem 13.5 (doubly hyperbolic cookies). The congress then meanders into over-specialised areas. While others attend the talks, Blorx manages to find the cookie stash. The cookies are doubly hyperbolic, i.e., cookies that look like $\mathbb{H}^2 \times \mathbb{H}^2$ with the product Riemannian metric of the hyperbolic Riemannian metrics of sectional curvature -1 . Which properties do such cookies have?

- | | |
|---------------------------|------------|
| sec < 0 | NET |
| scal < 0 | TH |
| constant scalar curvature | AN |

Problem 13.6 (mixed-type planets). The top 1% indulge life on mixed-type planets diffeomorphic to $\mathbb{H}^2 \times \mathbb{S}^2$. Which of the following luxurious properties can a complete Riemannian metric on such a planet have?

- | | |
|------------|------------|
| flatness | NOR |
| sec > 0 | AMO |
| scal > 0 | EAR |

Problem 13.7 (excursion). The excursion consists of a visit to the *Metroplanetary Museum of Art*. Which famous planet design principle is depicted in the painting below?

- | | |
|-----------------|-----------|
| Cartan-Hadamard | ON |
| Bonnet-Myers | S |
| Hopf-Rhino | TH |



Blorx takes advantage of the excursion to slip away from the congress and to start his well-deserved break ...

Solution self-check:

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No submission!