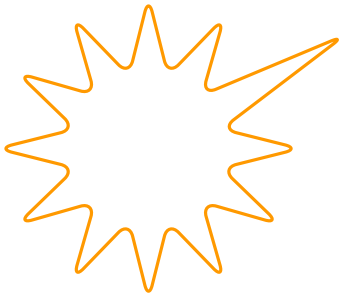
Differential Geometry I: Week $8\frac{1}{2}$

Prof. Dr. C. Löh/AG Ammann December 17, 2020

Bonus problem (one-dimensional Riemannian geometry is boring)**.** Show that every Riemannian manifold of dimension 1 is locally isometric to $\mathbb{R}$ (equipped with the Euclidean Riemannian metric).



Bonus problem (rescaling maximal geodesics)**.** Let M be a smooth manifold with a linear connection ∇, let $x \in M$, and let $v \in \mathrm{T}_x M$. Show that for all $c \in \mathbb{R}$ and all $t \in \mathbb{R}$, we have

$$\mathrm{geod}_{x,c\cdot v}(t) = \mathrm{geod}_{x,v}(c \cdot t),$$

whenever either side is defined. Here, we consider the maximal geodesics with respect to ∇.

Bonus problem (covariant derivatives along curves on submanifolds)**.** Let $N \in \mathbb{N}$ and let $M \subset \mathbb{R}^N$ be a smooth submanifold. We equip M with the linear connection $\nabla^\top$ induced (via the orthogonal projection $p \colon \mathrm{T}\,\mathbb{R}^N \longrightarrow \mathrm{T}\,M$) from the Euclidean linear connection $\overline{\nabla}$ on $\mathbb{R}^N$. Moreover, we denote the corresponding covariant derivatives along curves by $D^\top$ and $\overline{D}$, respectively.

Let $\gamma \colon I \longrightarrow M$ be a smooth curve. Show that

$$\forall_{X \in \Gamma(\mathrm{T}\,M|_\gamma)} \quad \forall_{t \in I} \quad (D^\top_\gamma X)(t) = p\bigl((\overline{D}_\gamma X)(t)\bigr).$$

Hints. If you like local coordinates, you can just perform this computation in local coordinates. If you want to avoid local coordinates, you can work with the defining properties of covariant derivatives along curves.

Bonus problem (curves)**.** Explain as many terms as possible related to curves and vector fields using rollercoasters. Draw pictures!
Hints. Don't waste too much time on watching videos on crazy rollercoasters!

Bonus problem (lecture notes)**.** Find typos in the lecture notes!

Optional submission before January 7, 2021, 10:00, via email to your tutor.