

# Ergodic Theory of Groups: Week 15

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**The absolutely fabulous programming (AFP) incident.** Commander Blorx decided to professionalise the workflow on his galaxy-hopping starship ISA (Incredibly Speedy Arrow) by hiring a fashionable PA (proof assistant). As soon as the very committed PA arrived on board, to ensure safety of everyone and everything, the PA enforced the implementation of drastic measures: Every single step now required valid formal verification. It didn't take long and Blorx was trapped in a dungeon of locales, states, and goals. And all of this on his very own, beloved, starship! This was not the paradise of relaxed rapid rigid proving that Blorx envisioned.

Help Blorx to escape! Give ~~formal~~ pen-and-paper arguments for your answers!

**Problem 14.1** (step 0: negotiating with the replicator (2 credits)). As always, Blorx needs chocolate from the replicator to accelerate his brain functions.

- Replicator: How can I be of assistance?
- Blorx: I urgently need chocolate.
- Replicator: Do you have proof?
- Blorx: Of course:

A: show   D: auto   T: qed   E: by   L: thesis   F: proof   I: ?

Arrange Blorx's answer into the correct order!

**Problem 14.2** (step 1: leaving quarters (4 credits)). Empowered by the dark force of chocolate, Blorx is convinced that even the bestest formal system might succumb to corruption. Thus, he decides to offer his prized collection of groups to the automatic (but pedantic) door of his quarters.

$(F_{2020})^{2020}$	EA: of fractional value bigger than 1
$\mathbb{Z}/2020$	OR: as cheap as $\mathbb{Z}/2 * \mathbb{Z}/2$
$F_{2020}$	EF: cheaper than cheap
$\mathbb{Z}/2020 * \mathbb{Z}/2021$	IN : the most expensive one

Find the correct price for each of these groups!

**Problem 14.3** (step 2: finding the main hatch (4 credits)). Having successfully bribed the door, Blorx sets out for the main hatch of the ISA. Unfortunately, the PA's reorganisation resulted in a random rewiring of the pathways on the starship. Therefore, Blorx evaluates more dynamical options.

The translation action $\mathbb{Z}/2020 \curvearrowright \mathbb{Z}/2020$	PE: ergodic and free, but not transitive
The canonical action $F_{2020} \curvearrowright \widehat{F_{2020}}$	IN : mixing, but not free
The diagonal translation action $\mathbb{Z}/2020 \curvearrowright \mathbb{Z}/2020 \times \mathbb{Z}/2020$	NY: ergodic and free and transitive
The standard Bernoulli shift of $F_{2020}$	ND: free but not ergodic

Which action has which consequences?

*Please turn over*

