Daylight Developers
(and the Inexorable Rise of the Chess Block)

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Abstract: The "chess block" is an intuitive design for the urban block. The chess block simply alternates houses with yards. We here wish to investigate the chess block because it appears to dominate contemporary construction. We first show that the chess block maximizes the block's daylit houses, daylit windows, and even profit. The chess block is the built manifestation of a large developer's optimizing plan, elegantly negotiating all within-block daylight externalities. In this, so we show then, developers of subdivisions of the block may fail. When households spend more time at home (so that daylighting is valued more), that failure becomes more acute. Then developers consolidate, build chess blocks, and so "develop the daylight" inherent to the block. But the chess block is not dense. Its proliferation pushes the marginal resident out. The price of housing rises. Today's more time-at-home offers a novel, parsimonious and fundamental explanation of: growing concentration in construction, chess block dominance in contemporary design, and a rising price of residential real estate globally.

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[1]Stimulating discussion with Jason Barr and Gabe Lee is much appreciated. Remaining errors are our own.

1 Introduction

Consider the "chess block", i.e. the simple, intuitive design that alternates houses with yards on the block. (A stylized illustration of the chess block on a 6×5-block follows later (Fig. (3a)).) The chess block is exceptionally bright. The chess block provides each of its houses not just with daylight from all around. It also provides its houses with the numerous amenities that often come along with the availability of daylight. Houses that are daylit better also are more likely to: enjoy a view, be calmer, benefit from more privacy, extract photovoltaic energy, enable its residents to avoid air-borne infectious diseases, etc. From a vast number of examples surrounding us, Fig. (1) shows one large developer's suggestive layout for a block currently under construction. Fig. (2) adds four more examples. Chess blocks have been built long ago (Figs. (2a and (2b))), or more recently; they can be central (Figs. (2a) and (2d)), or not so central.

While the chess block has always been built, historically the configuration of the urban block has followed very different plans (if it has followed any central plan at all). Europe's medieval cities had little concern for open urban space, and Haussmann's designs for central Paris or Cerdà's plan for Barcelona's Eixample favored the perimeter block. Contemporary construction could not be more different. Construction appears to increasingly align with the chess block. Our interest in the chess block begins with our personal perception of a strong increase in its spread. Travel by car, train or plane illustrates the growing dominance of the chess block design. Browsing the globe by cartographic software (focusing on suburban and hence likely more recent development) illustrates it, too. Anecdotally at least, the chess block proliferates around the world.

We first offer a parsimonious model of the chess block. As we will see, the chess block makes the most efficient use of block daylight. Not all developers are equally well positioned to impose the chess block, however. The large developer is able to choose the chess block design. But two or more sub-developers' designs frequently fail to add up to the chess block. This is why developer profit is superadditive, and why smaller developers will want to consolidate. In the presence of transaction costs, only some of this consolidation will proceed. But now combine the chess block with a secular reduction in the household's *time-at-work*. When the household spends less and less time at work, the household's time-at-home likely increases. Then the household likely values daylight more. Greater daylight valuation reinforces the superadditivity of profit.

This triggers four important, even "inexorable", long-run adjustments at both industry and city level. Those are adjustments in (i) real estate industry organization, (ii) urban block design, (iii) population density, and (iv) the price of residential real estate. As developers consolidate, they increasingly succeed in implementing the chess block. On the one hand, the chess block is brighter than the block that would have been built by competing sub-developers. On the other hand, the chess block also is less dense (on the assumption of given building height). Fewer households live in it. Greater daylight valuation, and the real estate industry consolidation implied, drive a decrease in block density that pushes the representative city's marginal resident further out. By the standard Ricardian argument, the price of housing is higher throughout the city.

Incidentally, Kwon/Ma/Zimmermann (2024) recently observe a long run trend towards increasing concentration in the US construction industry. For example, these authors find that the largest 1% of firms in construction today own 63% of the industry's assets,

The figure shows a block currently under construction (not far from the center of the medium sized city of Dortmund, Germany). This block resembles, and hence may be counted as an example of, the paper's generic "chess block" (shown later, in Fig. (3a)). The block is built by a single developer, "Assmann Group". A number of pictures on construction progress are made available, by the developer, on the developer's website.

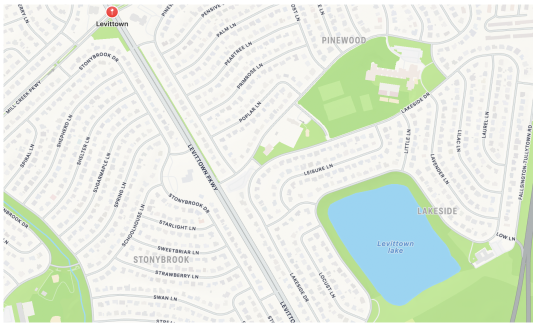
Figure 1: Contemporary residential real estate development: an example

up from 32% in the 1940s. Clearly the largest 1% percent of firms have increased in size, then. Larger developers are well-positioned to implement the chess block design. Quintero (2023) recently provides an empirical analysis of the number of local home-builders in US regional housing markets. Quintero (2024) finds that this number has significantly declined since 2007. Neither of these papers is at odds with the growth in developer size predicted in this paper. These papers are careful not to suggest that fewer and larger developers may be able to set higher prices. Yet even if fewer and larger developers have no market power, in our context they drive up city wide real estate prices up nonetheless as they make marginal residents commute longer distances.

The microeconomic analysis of urban daylight is at the heart of this paper. We depart from a simple "axiomatic" approach to daylighting technology. Equilibrium specifies the design of the block, for any context of space and time one may encounter within the city and for a wide variety of land ownership constellations. A fine-grained map of daylightings and occlusions emerges. This map assigns every house its individual daylighting quality. We then pursue the (dual) graph interpretation of the urban block. We translate the parcels of the block into vertices, and adjacencies between parcels into edges, of a grid graph. This graph highlights the complementarity of the house and the yard, for real estate daylight to succeed. We then are able to redress the block's "daylightability" in terms of the graph's "minimum cover" and "maximum matching".

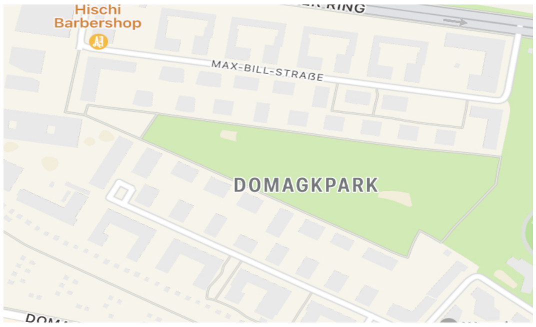
We will see that real estate developers best negotiate the block's daylighting externalities when building the chess block. Then real estate developers really turn into the "daylight developers" of this paper's title. We highlight and acknowledge the chess block's many admirable properties. The chess block (1) maximizes the number of fully daylit houses on the block, (2) maximizes the total number of block's daylit windows, and (for a reasonable restriction on prices) (3) even maximizes block profit. The chess block is

(a) Stuyvesant Town



(b) Levittown, Pennsylvania

(c) Guangzhou (China)



(d) Domagkpark (Munich/Germany)

The figure shows the figure ground plans of four blocks that approximate the stylized chess block in Fig. (3a), and hence in turn benefit from an understanding of the generic chess block. The blocks “Stuyvesant Town” in New York and “Domagkpark” in Munich/Germany are centrally located, Levittown and Guangzhou are at their cities’ respective urban peripheries. Source: Google Earth/Google Maps.

Figure 2: Approximate chess blocks

the built manifestation of a large developer’s optimizing plan, elegantly maximizing all within-block daylight externalities (by minimizing the within-block occlusions). It is even interspersed by cool outdoor space. Those yards daylighting the block’s houses offer relief in a warming climate.

At the same time, the chess block also is cause for concern. This theme we develop in a companion paper of ours (Dascher/Haupt (2024)). Not only is the chess block a driver of real estate prices. The chess block design’s virtuous effects on within-block organization also contrast with its not-so-virtuous role for between-block space, i.e. for the street. The street abutted by any succession of chess blocks has less windows facing it. Fewer eyes look into it. The chess-block-aligned street is supervised less. Chess blocks reduce pedestrians’ trust in the street; they also reduce orientation. They thereby diminish street use. Chess blocks also provide fewer premises for urban retail. Ultimately not even the chess block developers, or the landlords who hire them, may be satisfied with their equilibrium profit if households value not just daylight but urban public space, too.

On closer inspection, also the chess block’s record on sustainability fails to convince. The chess block raises at least three questions. First, lower block density propels suburban land use, car ownership, and floor space consumption. This contributes to warming the planet. Second, private spaces between the chess block’s houses are cool. In doing so, the chess block fails to continuously cast its shadow into the public space it abuts, outside it. It is this latter open space which urban residents frequent on their daily commute to work, to school, and to shop. And third, the chess block's uniformity predicts little variation in rent. Prospective residents are bound to earn similar wages (large or small). Income in any Mincer regression correlates with education. Thus chess blocks are also likely to beget segregation by income and education.

No microeconomic theory of urban daylight has so far, to the best of our knowledge, been put forward. Nonetheless, five strands of the literature relate. One important first strand shares our interest in why housing prices keep rising on an almost global scale, and for almost half a century now. This strand suggests that local landlords increasingly restrict construction, in order to drive local real estate prices up (e.g. Glaeser/Ward (2008)). The landlord majority zones for low-density housing, minimum lot sizes, or excessive environmental regulations. Such zoning policies restrict local developer output. They reduce housing supply, and hence drive housing prices up. Compare this with the real estate developer consolidation at the center of this paper: This consolidation reduces housing supply, too. Yet large real estate developers happily embrace the design of the chess block's bright low density housing because it is more profitable, not because they are politically constrained to embrace it. Zoning does not bind large developers.

A second strand of the economics literature shares our interest in the value of daylight consumption. Fleming et al. (2018) have recently provided the very first hedonic estimates of households' valuation of *sun*-light. Following these authors, and on data for Wellington (New Zealand), every extra hour of sunlight exposure raises the value of local real estate by 2.4%. This result does not assert that daylight valuation is greater today than it was in the past. But it does suggest that daylight has significant economic value today. A third strand of the literature addresses land ownership in the context of the assembly of urban land. Brooks/Lutz (2016) and Yamasaki et al. (2023) both stress the benefits of assembling urban land when wishing to build up. Building up is motivated by the same desire for daylighting houses' windows as is the chess block's building out. Fourth, Ellickson (2013) and Libecap/Lueck (2011) also analyze the layout of the block. These authors argue for rectangular parcels to enhance parcels' tradability. Our paper agrees; with a rectangular grid, block daylighting can be achieved at minimum cost. Finally, and fifth, we note that occluding a neighbor's window is the quintessential negative externality. Any occlusion's cost to the occluded equals the cost to the occluder. A large developer-manager successfully navigates these externalities, while two or more smaller manager-developers may well fail to do so (akin to Coase (1960)).

The paper is organized as follows. (Remarks introduce auxiliary results, Propositions state main results.) Section 2 first defines the chess block, then identifies its core properties. The chess block maximizes daylit houses on the block (Proposition 1), and even maximizes daylit windows on the block (Proposition 2). In fact, the chess block maximizes a large developer's profit on the block (Proposition 3). Section 3 revisits these results in a context of two or more "sub-developers". Daylight maximizing designs on subdivisions identify sub-developers' profit maximizing sub-designs (Proposition 4). In Nash-equilibrium, occlusions become probable (but not inevitable). Then population density recedes (Proposition 5). Section 5 derives superadditivity of the block's profit. This establishes smaller developers' incentive to consolidate (Proposition 6). Consolidation in turn implies rising real estate prices (Proposition 7). Section 6 addresses potential extensions, and section 7 concludes.

2 Daylit City

We analyze the equilibrium of profit maximizing developers in a closed *daylit city*. This is, to the best of our knowledge (and with the aforementioned exception of companion paper Dascher/Haupt (2024)), a novel city model. An area of land is large enough for our city to unfold inside it. This area is partitioned into *blocks* by vertical and horizontal strips of land of unit width called *streets*. Each block is a rectangular array of mn parcels, also called *lots*. The city center is coincident with the intersection of two streets. All jobs, shops, and schools are in the center. Households must commute to the center to avail of them. These commutes unfold along the street grid. Commuting distance δ is of the Manhattan type, and commuting cost per unit of distance is t.

Depending on whether its four neighboring lots are built up on or not, each lot may be daylit not at all, or once, twice, three or even four times. Specifically, five assumptions govern any lot's daylight reception. First, only indirect daylight, deflected by the lot's neighboring yards, daylights that lot. Direct daylight, streaming in from the sky, does not. Second, daylight deflected into any given lot only streams in from as far as that lot's neighbors, and never from lots further afar. Third, any yard daylights all four neighboring lots just as well as it daylights a single built-up neighbor. Fourth, sunlight streaming into any lot from different directions "adds up". Alternatively justified, it has the lot enjoy sunlight longer. And fifth, facing south – or north on the southern hemisphere – is as good as facing any other direction. (We cast our discussion in terms of single-storey houses. But arguably these five assumptions appear reasonable for $4, 5$, or even 6 storey houses, too. This also explains why we ignore the possibility of skylights.)

Households value the daylight streaming through each daylit window at the constant premium of $v > 0$. A house featuring $\lambda \in \{0, \ldots, 4\}$ daylit windows rents out at $p(\delta) + \lambda v$, where $p(\delta)$ is "base rent" in ring δ. When a house is daylit from all around, we say it is *daylit*. At the closed daylit city's boundary, $\tilde{\delta}$, a daylit house is naturally within a step's reach (i.e. available just outside the city). So

$$p(\tilde{\delta}) \ + \ 4v \ = \ 0 \qquad \text{(boundary daylit house price)} \tag{1}$$

must hold in equilibrium. But then a "dark home", with no daylit windows at all, fetches negative rent at the city boundary; it is $p(\tilde{\delta})$ equal to zero. No one wants to live in a house that is both peripheral *and* dark.

Residing in ring δ instead of ring $\tilde{\delta}$ saves on commuting costs. In competitive equilibrium we expect daylit houses in a ring closer to the center, $\delta < \tilde{\delta}$, to be priced at a premium equal to the savings in commuting cost:

$$p(\delta) \ + \ 4v \ = \ t\big(\tilde{\delta} \ - \ \delta\big) \qquad \text{(daylit house price)} \tag{2}$$

Whenever a house has less than four windows, we find its price by subtracting $(4 - \lambda)v$ from eq. (2). The number of blocks δ blocks away from the center is $4(\delta + 1)$. Then $4N$ times is the number of apartments in ring δ. Housing market equilibrium requires the supply of apartments across all rings up to $\tilde{\delta}$ to equal exogenous city population L:

$$4N \sum_{\delta=0}^{\tilde{\delta}} (\delta + 1) \ = \ L \quad \text{(housing market equilibrium).} \tag{3}$$

We now make the following (innocuous, really)

Assumption: Households' valuation is sufficiently "large", i.e. $v > t(\tilde{\delta} - \delta)/4$, or, equivalently, "dark rent" $p(\delta)$ is negative throughout the city, $p(\delta) < 0$. Not only is it then not possible, by arbitrage condition (1), to rent out a dark house at the city periphery. In fact, in *no* ring, not even in the city center, is it possible to rent out a dark house. By housing equilibrium (3), our Assumption places a constraint on population L. Assumption will not hold for London or New York, nor for the urban systems of non-industrialized countries today. We drop ring index δ for now.

Label the block's *lots* by j and k, with j ranging from 1 to m and k from 1 to n. Let

$$\boldsymbol{x}' \;=\; (x_{11}, \ldots, x_{mn}) \qquad \text{(configuration)} \tag{4}$$

denote the $mn \times 1$ configuration vector of *houses* and *yards*, such that lot jk has a house if x_{jk} equals 1, and a yard if x_{jk} is 0. Further configurations we introduce below will be labeled $\boldsymbol{y}$ or $\boldsymbol{c}$, for example. Each house has 4 *windows*, one on each of its 4 *faces*. Adjacency matrix $\boldsymbol{A}$ has a 1 in row jk and column pr if lots jk and pr are adjacent, and a 0 else. Let $\boldsymbol{\iota}$ be an $mn \times 1$-vector of 1's. Then the block has $\boldsymbol{\iota}'\boldsymbol{\iota}$ (or ν) lots, $\boldsymbol{\iota}'\boldsymbol{x}$ (or N) houses, $4\boldsymbol{\iota}'\boldsymbol{x}$ windows, and $\boldsymbol{\iota}'(\boldsymbol{\iota} - \boldsymbol{x})$ yards. If two houses neighbor one another, we say they mutually *occlude* each other. So any given adjacency of houses produces *two* occlusions. There are $\boldsymbol{\iota}'\boldsymbol{A}\boldsymbol{\iota}/2$ (or ε) adjacencies altogether. Total occlusions on the block are $\boldsymbol{x}'\boldsymbol{A}\boldsymbol{x}$. To account for daylight streaming in from outside the block, we define $n \times 1$ streetfront vector $\boldsymbol{f}$, with f_{jk} equal to 2 (1) if lot jk is a corner (streetfront though not corner) lot. For remaining lots jk interior to the block, f_{jk} is 0. We connect these concepts via identity

$$4\boldsymbol{\iota}'\boldsymbol{x} \;=\; \boldsymbol{x}'\boldsymbol{f} \;+\; \boldsymbol{x}'\boldsymbol{A}\boldsymbol{\iota} \qquad \text{(window accounting)}, \tag{5}$$

which gives two equivalent ways of accounting for the grand total of windows on the block (daylit or not).

On the one hand, $\boldsymbol{x}'\boldsymbol{A}(\boldsymbol{\iota} - \boldsymbol{x})$ indicates the number of windows daylit from within the block. These are "internal daylightings" or "internally daylit windows", or Λ_i for short. On the other hand, $\boldsymbol{x}'\boldsymbol{f}$ is the number of windows daylit from outside the block. Those are "external daylightings" or "externally daylit windows", denoted Λ_o. For reference,

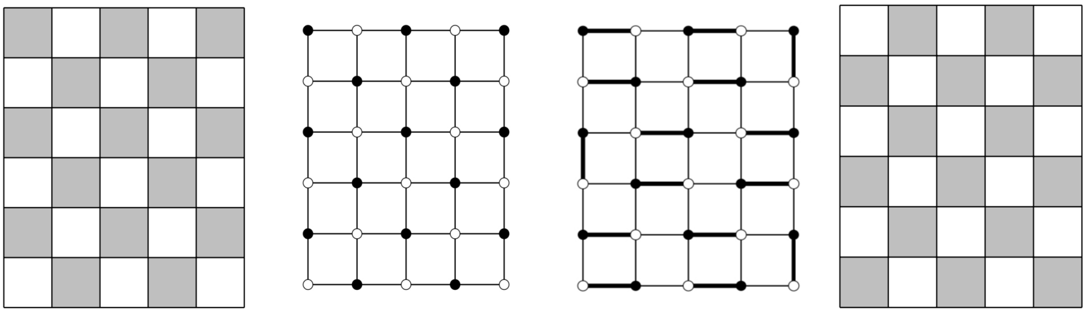
$$\Lambda_i \;=\; \boldsymbol{x}'\boldsymbol{A}(\boldsymbol{\iota} - \boldsymbol{x}) \qquad \text{(internally daylit windows)} \tag{6}$$
$$\Lambda_o \;=\; \boldsymbol{x}'\boldsymbol{f} \qquad \text{(externally daylit windows).} \tag{7}$$

There are also two different ways to account for the block's daylit windows, Λ. Either we decompose daylit windows Λ into those daylit from inside the block vs. those daylit from the outside, $\Lambda_i + \Lambda_o$ (first eq. in (8)). Or we may depart from the block's total houses, multiply by 4, then subtract occluded windows (second eq.). That is,

$$\Lambda \;=\; \begin{cases} \boldsymbol{x}'\boldsymbol{f} \;+\; \boldsymbol{x}'\boldsymbol{A}(\boldsymbol{\iota} - \boldsymbol{x}) \\ 4\boldsymbol{\iota}'\boldsymbol{x} \;-\; \boldsymbol{x}'\boldsymbol{A}\boldsymbol{x} \end{cases} \qquad \text{(daylit window accounting)}, \tag{8}$$

where one equation transforms into the other via window accounting identity (5). Either perspective in (8) will prove useful below.

Daylight is valuable, and hence profitable, only if it is consumed. A daylighting involves *two* lots: one lot that emits daylight, and another that receives it. So while "daylit



(a) **a block**
$\boldsymbol{\iota}'\overline{\boldsymbol{x}} = 15$
$\overline{\boldsymbol{x}}'\boldsymbol{A}(\boldsymbol{\iota} - \overline{\boldsymbol{x}}) = 49$
$\overline{\boldsymbol{x}}'\boldsymbol{A}\overline{\boldsymbol{x}} = 0$
$(\boldsymbol{\iota} - \overline{\boldsymbol{x}})'\boldsymbol{A}(\boldsymbol{\iota} - \overline{\boldsymbol{x}}) = 0$
$\overline{\boldsymbol{x}}'\boldsymbol{f} = 11$
$\Lambda = 60$

(b) **design $\overline{\boldsymbol{x}}$**
$\nu = 30$
$\varepsilon = 49$

(c) **min. cover:**
$|K| = \boldsymbol{\iota}'(\boldsymbol{\iota} - \overline{\boldsymbol{x}}) = 15$
as well as
max. matching:
$|M| = 15$

(d) **design $(\boldsymbol{\iota} - \overline{\boldsymbol{x}})$**
$\boldsymbol{\iota}'(\boldsymbol{\iota} - \overline{\boldsymbol{x}}) = 15$
$(\boldsymbol{\iota} - \overline{\boldsymbol{x}})'\boldsymbol{A}\overline{\boldsymbol{x}} = 49$
$(\boldsymbol{\iota} - \overline{\boldsymbol{x}})'\boldsymbol{A}(\boldsymbol{\iota} - \overline{\boldsymbol{x}}) = 0$
$\overline{\boldsymbol{x}}'\boldsymbol{A}\overline{\boldsymbol{x}} = 0$
$(\boldsymbol{\iota} - \overline{\boldsymbol{x}})'\boldsymbol{f} = 11$
$\Lambda = 60$

Notes: (i) Fig. (3a) and (3d), show the two bipartite designs, $\overline{\boldsymbol{x}}$ and $(\boldsymbol{\iota} - \overline{\boldsymbol{x}})$ for the 6×5-block, respectively. (ii) With blocks (3a) and (3d), houses are shown in black, yards in white. With graph representations (3b) and (3c) of the chess block in Fig. (3a), houses correspond to black vertices and yards to empty vertices. The graphs' number of edges is ε, the number of its vertices is ν. (iii) It is $\boldsymbol{x}$ the block, $\boldsymbol{\iota}'\boldsymbol{x}$ the number of houses, $\boldsymbol{x}'\boldsymbol{A}\boldsymbol{x}$ the block's aggregate occlusions and Λ the block's daylit windows. (iv) In Fig. (3c), empty vertices indicate a minimum (daylight) cover, while heavy edges indicate the edges in a (maximum) matching.

Figure 3: Chess block, grid graph, and minimum (daylight) cover

windows" and "daylightings" are used synonymously below, really only the term "daylighting" gives due credit to the complementarity of the daylit window with the yard it looks onto. This complementarity suggests a focus on adjacencies among pairs of lots, rather than on lots themselves.

Translating the block's adjacency matrix $\boldsymbol{A}$ into a *(grid) graph* provides for just that change in focus. We represent lot j by *vertex* k, then connect any two vertices j and k by an *edge* when they are neighbors according to $\boldsymbol{A}$. An edge is a daylighting if and only if it links a house to a yard. It is true that the grid graph fails to account for the daylightings external to the block, Λ_o. But the grid graph's edges do highlight the potential for daylightings internal to the block, Λ_i. For example, we immediately see that there can be no more daylightings within the block than there are edges to the graph, $\boldsymbol{\iota}'\boldsymbol{A}\boldsymbol{\iota}/2 = \varepsilon$. This simple yet important fact we will repeatedly use below.

Remark 1 comments on the grid graph's necessary feature. Consider any edge e of the grid graph. Now label its two vertices v_1 and v_2. Then both v_1 and v_2 are adjacent to two further vertices v_1' and v_2', respectively, linked up by another edge e'. Those two vertices v_1' and v_2' we call edge e's *pair of linked neighbors.* We exploit this property shortly.

Remark 1 (Grid Graph)

Each edge's two vertices have a pair of neighboring vertices that are linked.

Generally a graph is *bipartite* if there is a partition of its set of vertices into two subsets such that every edge links to a vertex from either subset. Incidentally, our block graph is bipartite. Define design vector $\overline{\boldsymbol{x}}$ as follows:

$$\overline{x}_{jk} = \begin{cases} 1 & \text{if} \quad j, k \text{ both odd or both even} \\ 0 & \text{if} \quad j, k \text{ neither both odd nor both even} \end{cases} \qquad \text{(design } \overline{\boldsymbol{x}}\text{)}, \qquad (9)$$

where j and k run through 1 to m and 1 to n, respectively. Design $\overline{\boldsymbol{x}}$ points, through its 1's, to one of the two vertex sets of the bipartition, while $(\boldsymbol{\iota} - \overline{\boldsymbol{x}})$, or $\overline{\boldsymbol{y}}$ for short, indicates the other. Intuitively, the two designs are complements of each other, with one design assigning a 1 whenever the other assigns a 0. By way of their "alternating character", either $\overline{\boldsymbol{x}}$ or $\overline{\boldsymbol{y}}$ define the bipartition (Remark 2). Whenever x_{jk} equals 1, we say that lot (or vertex) jk is built up (has a house), while if x_{jk} equals 0, lot (or vertex) jk is not built up (is a yard). In that sense, the first entry in $\overline{\boldsymbol{x}}$ always features a house, while the first entry in $\overline{\boldsymbol{y}}$ always has a yard. Fig. (3b) illustrates the graph corresponding to the block in Fig. (3a), and indicates $\overline{\boldsymbol{x}}$, while Fig. (3d) shows that graph joint with $\overline{\boldsymbol{y}}$.

Remark 2 (Bipartite Designs)

(i) The grid graph is bipartite, and $\overline{\boldsymbol{x}}$ and $(\boldsymbol{\iota} - \overline{\boldsymbol{x}}) = \overline{\boldsymbol{y}}$ are the bipartition's two designs.
(ii) Note that $\overline{\boldsymbol{x}}'\boldsymbol{A}(\boldsymbol{\iota} - \overline{\boldsymbol{x}}) = \overline{\boldsymbol{y}}'\boldsymbol{A}(\boldsymbol{\iota} - \overline{\boldsymbol{y}})$.
(iii) As regards numbers of houses N and externally daylit windows Λ_o, we observe:

$$\boldsymbol{\iota}'(\overline{\boldsymbol{x}} - \overline{\boldsymbol{y}}) = \begin{cases} 1 & \text{if } mn \text{ odd} \\ 0 & \text{if } mn \text{ even} \end{cases} \quad \text{and} \quad (\overline{\boldsymbol{x}} - \overline{\boldsymbol{y}})'\boldsymbol{f} = \begin{cases} 4 & \text{if } mn \text{ odd} \\ 0 & \text{if } mn \text{ even} \end{cases} . \tag{10}$$

(So if mn is odd, bipartite design $\overline{\boldsymbol{x}}$ has 1 house, and 4 windows, more than design $\overline{\boldsymbol{y}}$.)

Proof: (i) Label each vertex of the block graph by the address of the lot of the corresponding block. Consider vertex jk with j, k either both odd or both even (i.e. vertex jk is a house). Consider vertex jk's neighbors, of which there are 4 at most. Any neighboring vertex adds 1 to, or subtracts 1 from, one, and just one, of vertex jk's two indices. If both indices are odd initially, now one index is even; if both indices are even initially, now one is odd. So by definition (9), every neighbor of a house is a yard. Similar reasoning shows that each neighbor of a yard is a house. So the set of houses and the set of yards are indepdendent sets, i.e. no edge has both ends in the same set. Hence the graph is bipartite, and $\overline{\boldsymbol{x}}$ and $\overline{\boldsymbol{y}}$ indicate the bipartition. □

(ii) With either design $\overline{\boldsymbol{x}}$ and $\overline{\boldsymbol{y}}$, each edge links a house to a yard. So both designs have ε daylightings. □

(iii) On the one hand, if mn is odd then m must be odd. With $\overline{\boldsymbol{x}}$, every two successive rows have n houses, while the very last row (which exists because m is odd) has $(n+1)/2$ houses. With $\overline{\boldsymbol{y}}$, every two successive rows have n houses, while the very last row (again, which exists because m is odd) has $(n - 1)/2$ houses. So $\boldsymbol{\iota}'(\overline{\boldsymbol{x}} - \overline{\boldsymbol{y}})$ equals 1. On the other hand, if mn is even, then either m or n is even. Suppose m is. Successive pairs of rows exhaust m, and so $\boldsymbol{\iota}'(\overline{\boldsymbol{x}} - \overline{\boldsymbol{y}})$ equals 0. Finally we note that if $\overline{\boldsymbol{x}}$ has 1 more house than $\overline{\boldsymbol{y}}$, then it must also (since no window is occluded) have 4 daylit windows more. □

In a daylighting context, bipartiteness is naturally appealing. With bipartiteness, neither is there any need for some house to border on some other house, nor is there any need for some yard to border on some other yard. Instead, a bipartite design successfully activates each of the total ε edges as daylighting. The two bipartite designs $\overline{\boldsymbol{x}}$ and $\overline{\boldsymbol{y}}$ only disagree on the number of externally daylit windows and the number of houses, and only if mn is odd. In that case $\overline{\boldsymbol{x}}$ has 1 house, and 4 daylit windows, more than $\overline{\boldsymbol{y}}$ (Remark 2 (iii)).

We now define the *chess block* design. It coincides with the bipartite design $\overline{\boldsymbol{x}}$ if mn is odd, and with either $\overline{\boldsymbol{x}}$ or $\overline{\boldsymbol{y}}$ if mn is even (Definition 1). Figs. (3a) and (3d) illustrate the case of mn even. Both $\overline{\boldsymbol{x}}$ and $\overline{\boldsymbol{y}}$ produce $\Lambda_i = 49$ internal daylightings and $\Lambda_o = 11$ external daylightings imported from the street. Daylit windows are $\Lambda = 60$ with either design. In contrast, in the case of where mn is odd, $\overline{\boldsymbol{y}}$ has 1 house – as well as 4 externally daylit windows and hence 4 daylit windows – less than $\overline{\boldsymbol{x}}$.

Definition 1 (Chess Block)

The "chess block", $\overline{\boldsymbol{c}}$, refers to the bipartite design $\begin{cases} \overline{\boldsymbol{x}} & \textit{if } mn \textit{ is odd} \\ \overline{\boldsymbol{x}} \textit{ or } \overline{\boldsymbol{y}} & \textit{if } mn \textit{ is even} \end{cases}$.

Given our definition of $\overline{\boldsymbol{x}}$ (in eq. (9)), the chess block has $\overline{N}$ daylit houses,

$$\overline{N} \;=\; \begin{cases} (mn+1)/2 & \text{if } mn \text{ odd} \\ mn/2 & \text{if } mn \text{ even} \end{cases} \qquad \text{(chess block houses).} \qquad (11)$$

All $\overline{N}$ houses are daylit on all 4 faces, and so the chess block has $4\overline{N}$ daylit windows, or $\overline{\Lambda}$ for short.

3 Block Developer

Consider a developer who owns all lots of the block. Such a developer we henceforth call a *block developer*. The block developer wishes to maximize block profit, Π, as the price-weighted sum of dark houses, N, and the aggregate of daylit windows, $\Lambda(\boldsymbol{x})$. That is, she wishes to solve

$$\max_{\boldsymbol{x},N} \; \Pi(\boldsymbol{x}, N) \;=\; pN \;+ v\Lambda(\boldsymbol{x}) \qquad \text{(block profit).} \qquad (12)$$

A total of 2^{mn} different possible designs exists. Which one of these should the block developer choose? An intuitive candidate design is the chess block (Definition 1). In this section we successively, in the order of the list below, show that insisting on the chess block design $\overline{\boldsymbol{c}}$ provides the block developer with the maximum number of fully daylit houses (Proposition 1), maximum number of daylit windows, $\max_{\boldsymbol{x}} \Lambda(\boldsymbol{x})$ (Proposition 2) and maximum block profit, $\max_{\boldsymbol{x}} \Pi(\boldsymbol{x})$ (Proposition 3).

To start, recall the following two elementary concepts from graph theory. An *independent set* is a subset of vertices such that no two of its members are linked. In a daylighting context, if houses form an independent set then each house is daylit on all four of its faces. In this case we say a house is (fully) *daylit*. And a *cover* is a set of vertices such that every of $\boldsymbol{A}$'s edges links to a vertex in that set. For instance, suppose that a block's yards form a cover. Such a daylight cover (fully) daylights every house on the block. We now show that the chess block $\overline{\boldsymbol{c}}$ represents a – minimum – daylight cover (Proposition 1 (i)). The chess block has the bare number of vertices necessary to cover every edge of the grid graph. That number, as the grid graph's covering number β, is

$$\beta \;=\; \begin{cases} (mn-1)/2 & \text{if } mn \text{ odd} \\ mn/2 & \text{if } mn \text{ even} \end{cases} \qquad \text{(covering number).} \qquad (13)$$

It is not possible to daylight all houses within the block from all around with fewer than β yards. But then the chess block has the maximum number fully daylit houses (Proposition 1 (iii)). Proposition 1 (iv) adds that $4\overline{N}$ already also provides a lower bound on the maximum number of daylit windows, $\max_{\boldsymbol{x}} \Lambda$.

Proposition 1 (Chess Block and Maximum Daylit Houses)

(i) The chess block is a minimum daylight cover.

(ii) The grid graph's covering number is β*.*

(iii) The chess block yields the maximum number of daylit houses, equal to $\overline{N}$*.*

(iv) Maximum daylit windows, $\max_{\boldsymbol{x}} \Lambda$*, are* $4\overline{N}$ *at least.*

> **Proof**: (i) The proof follows Bondy/Murty (2018). Let every chess block yard be a vertex in the cover. Because the graph is bipartite, each edge is linked to a yard, and hence to a vertex in that cover. Match up every successive pair of vertices (alternately representing a house and a yard). Since each match links to a yard, and since each yard is a vertex in the cover, the number of matches equals the number of vertices in the daylight cover. Hence the matching is maximum and the cover minimum.
>
> (ii) The number of vertices in Proposition 1 (i)'s minimum cover equals $mn/2$ if mn is even, and $(mn-1)/2$ if mn is odd. By eq. (13), this is just β. □
>
> (iii) Because there are no occlusions from outside the chess block, each of the chess block's houses is daylit fully. Since the number of yards in the chess block's daylight cover is minimum (Proposition 1 (i)), the number of fully daylit houses is maximum.
>
> (iv) The chess block's daylit windows, $\overline{\Lambda}$, are feasible. An optimum design must provide at least as many daylit windows as $\overline{\Lambda}$. □

By virtue of being bipartite, the chess block $\overline{\boldsymbol{c}}$ extracts one daylighting from every edge. The chess block "turns every edge on"; it makes full use of the block's capacity for internal daylightings. Thus the chess block maximizes internally daylit windows, Λ_i (Proposition 2 (i)). Intuitively, importing more external daylight from the street sacrifices as much internal daylight at least. This is why the chess block $\overline{\boldsymbol{c}}$ even maximizes daylit windows on the block, Λ (Proposition 2 (ii)).

Proposition 2 (Chess Block and Maximum Daylit Windows)

(i) The chess block $\overline{\boldsymbol{c}}$ *provides maximum internally daylit windows,* $\Lambda_i(\overline{\boldsymbol{c}}) = \max_{\boldsymbol{x}} \Lambda_i(\boldsymbol{x})$*.*

(ii) The chess block $\overline{\boldsymbol{c}}$ *provides maximum daylit windows,* $\Lambda(\overline{\boldsymbol{c}}) = \max_{\boldsymbol{x}} \Lambda(\boldsymbol{x})$*.*

> **Proof**: (i) Since the chess block $\overline{\boldsymbol{c}}$ is a bipartite design, every edge is an internal daylighting. Hence $\Lambda_i = \overline{\boldsymbol{c}}'\boldsymbol{A}(\boldsymbol{\iota} - \overline{\boldsymbol{c}}) = \varepsilon$. Because no block has more than ε internally daylit windows, the chess block $\overline{\boldsymbol{c}}$ maximizes Λ_i.
>
> (ii) Consider a design change from $\overline{\boldsymbol{c}}$ to $\boldsymbol{z} \neq \overline{\boldsymbol{c}}$. There are three possibilities. (A) Suppose $\boldsymbol{z}$ features k houses less than $\overline{\boldsymbol{c}}$. Because the first term on the r.h.s. of (8) is smaller (by $4k$) while the second term cannot be smaller (than 0), we have $\Lambda(\boldsymbol{z}) < \Lambda(\overline{\boldsymbol{c}})$. (B) Next suppose $\boldsymbol{z}$ features as many houses as $\overline{\boldsymbol{c}}$. Since the first term on the r.h.s. of (8) remains the same, while the second cannot be smaller (than 0), again we have $\Lambda(\boldsymbol{z}) \leqslant \Lambda(\overline{\boldsymbol{c}})$. (C) Finally suppose $\boldsymbol{z}$ features k houses more than $\overline{\boldsymbol{c}}$. Thus $\boldsymbol{z}$ has $\beta - k$ yards only. Because there are β independent edges (Proposition 1 (i)), now only $\beta - k$ of those can be (yard-)covered. Remaining k edges are incident to 2 houses, and hence produce 2 occlusions each. Consider the two vertices (houses) of one such edge. Either one of their *linked neighbors* (Remark 1) is a house; then 2 further occlusions obtain at least. Or none of the linked neighbors is; then two yards are adjacent, are linked by an edge. Another of the graph's edges fail to be (yard-)covered, i.e. produces 2 occlusions. Adding k houses produces at least $4k$ occlusions. Again we have $\Lambda(\boldsymbol{z}) \leqslant \Lambda(\overline{\boldsymbol{c}})$. □

Proposition 2 suggests a straightforward two step procedure for locating maximum daylit windows, $\max_{\boldsymbol{x}} \Lambda(\boldsymbol{x})$. First, the block developer identifies bipartite designs $\overline{\boldsymbol{x}}$ and $\overline{\boldsymbol{y}} = (\boldsymbol{\iota} - \overline{\boldsymbol{x}})$ via eq. (9). And then, second, she chooses from those two designs the one that collects more daylight from outside the block, i.e. $\overline{\boldsymbol{c}}$. If mn is even, she chooses $\overline{\boldsymbol{x}}$ or $\overline{\boldsymbol{y}}$, while if mn is odd, she chooses $\overline{\boldsymbol{x}}$.

We now turn to the block developer's relevant objective, which is maximum block profit. We address necessary conditions first. Substituting total daylit windows on the block (as the expression on the r.h.s. of the second equation of (8)) for Λ in the definition of profit (12), the block developer solves

$$\max_{\boldsymbol{x},N} \ \Pi \ = \ pN \ + \ v\big(4N \ - \ \boldsymbol{x}'\boldsymbol{A}\boldsymbol{x}\big) \quad \text{s.t.} \quad x_{jk} \in \{0,1\} \ \text{ and } \ \boldsymbol{\iota}'\boldsymbol{x} = N. \tag{14}$$

First we replace the N in the objective function by appealing to the equality constraint, then we take the (vector-valued) first derivative of profit with respect to configuration vector $\boldsymbol{x}$. This gives the $3n$ necessary conditions

$$\frac{\partial \Pi(\boldsymbol{x}, N)}{\partial \boldsymbol{x}} \ = \ (p + 4v)\boldsymbol{\iota} - 2\boldsymbol{A}\boldsymbol{x}v \ \leqslant \ 0 \qquad \text{c.sl.} \qquad \boldsymbol{\iota} - \boldsymbol{x} \geqslant 0 \qquad \text{(block f.o.c.)}, \tag{15}$$

where both inequalities and complementary slackness conditions hold component-wise.

The chess block $\boldsymbol{c}$ easily satisfies conditions (15). On the one hand, giving up on any of the chess block's (non-occluded) houses (i.e. $\bar{c}_{jk} = 1$) would reduce profit by $p + 4v \geqslant 0$ (because $2\boldsymbol{e}'_{jk}\boldsymbol{A}\overline{\boldsymbol{c}}$ equals 0). On the other hand, building up on any of the chess block's (fully occluded) yards (i.e. $\bar{c}_{jk} = 0$) would increase profit by $p - 4v \leqslant 0$ (because $2\boldsymbol{e}'_{jk}\boldsymbol{A}\overline{\boldsymbol{c}}$ equals 8). Conditions (15) emphasize the block developer's internalizing occlusions. Each occlusion enters (15) twice (witness the 2), rather than just once. Conditions (15) only test for local changes in design. We next discuss sufficient conditions for obtaining a globally optimum design. Rather than solve problem (14) by brute force, the block developer follows a two-step procedure (Remark 3).

Remark 3 (Daylight Frontier and Maximum Profit)
Maximum block profit $\Pi(v)$ *obtains when solving*

$$\max_{N,\Lambda} \ pN \ + \ v\Lambda \qquad \text{s.t.} \qquad \Lambda = \Lambda(N). \qquad \text{(block developer profit)}, \tag{16}$$

where

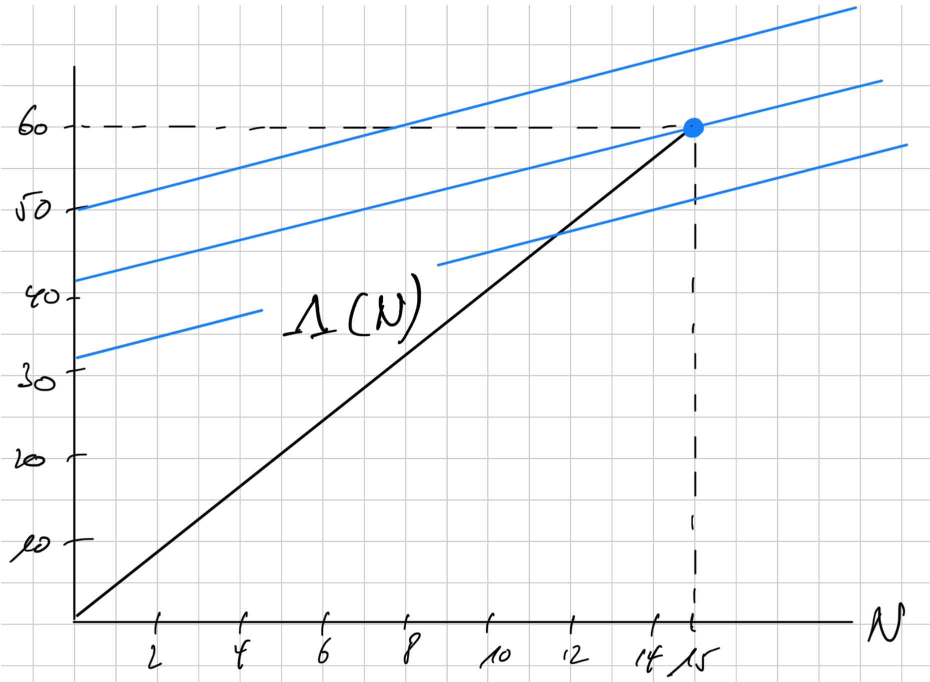
$$\Lambda(N) \ = \ \max_{\boldsymbol{x}} \ 4N \ - \ \boldsymbol{x}'\boldsymbol{A}\boldsymbol{x} \quad \text{s.t.} \quad x_{jk} \in \{0,1\} \ \text{ and } \ \boldsymbol{\iota}'\boldsymbol{x} = N. \tag{17}$$

Proof: Rewrite problem (14) as

$$\max_{N} \ \Big\{ \max_{\boldsymbol{x}} \ pN \ + \ v(4N \ - \ \boldsymbol{x}'\boldsymbol{A}\boldsymbol{x}) \quad \text{s.t.} \quad x_{jk} \in \{0,1\} \ \text{ and } \ \boldsymbol{\iota}'\boldsymbol{x} = N \Big\}$$

After successively shifting pN and v (i.e. constants to the inner maximization problem) outside the curly brackets and substituting $\Lambda(N)$ according to (17), this becomes $\max_{N} \ pN + v\Lambda(N)$ as in (16). $\square$

In a first step, the block developer hence may compute the *daylight frontier*, $\Lambda(N)$. The daylight frontier maps parametric houses N into maximum daylit windows Λ. It just so happens that we have already identified a first point on this frontier; this is point



Notes: It is $\Lambda(N)$ the daylight frontier. Three isoprofit lines are drawn, also. Point $(\overline{N}, 4\overline{N})$ corresponds to the chess block. The slope of any isprofit line is $-p/v$. Changes in p or v have no effect on the choice of optimum.

Figure 4: Chess block as the block developer's optimum solution

$(\overline{N}, 4\overline{N})$, corresponding to the combination of houses and daylit windows provided by the chess block (Proposition 2 (i)). Starting with $\overline{N}$ houses, every house we delete from the chess block must take away 4 daylit windows from the block, until N is 0. Ignoring the integer constraint on N, the block's daylight frontier is

$$\Lambda(N) \;=\; 4N \qquad \text{if} \qquad N \leqslant \overline{N} \qquad\qquad \text{(daylight frontier).} \tag{18}$$

Note the extreme point of the daylight frontier at $(\overline{N}, 4\overline{N})$, illustrated in Fig. (4). In a second step, the block developer determines the best point on that frontier.

Proposition 3 (i) next establishes this section's main result: It is the chess block, $\bar{\boldsymbol{c}}$, that solves (16), and hence solves the original program (14). The solution is illustrated by the profit maximizing point on the frontier $(\overline{N}, 4\overline{N})$ shown in Fig. (4). On the one hand, $-p/v$ the minimum compensation (in terms of daylit windows) the block developer expects for her to voluntarily build a house. On the other hand, 4 is the true compensation (again in terms of daylit windows) the block developer achieves for every house (up until the chess block design is complete). Now, within the city $p + 4v \geqslant 0$ (else not even a fully daylit house ever gets built), or

$$-p/v \;\leqslant\; 4 \qquad \text{if} \qquad N \leqslant \overline{N} \qquad \text{(daylit windows expected vs. received),} \tag{19}$$

equivalently. Thus within the city, building a house yields at least as many daylit windows as the block developer demands to receive, as long as the chess block is incomplete. This is why the chess block maximizes the block's profit, and maximizes it irrespective of the block's location within the city.

In fact, the chess block $\bar{\boldsymbol{c}}$ even is the block developer's *unique* optimum block design (up to the ambiguity allowed for in Definition 1 for when mn is even). It is this result that explains the chess block's popularity among block developers.

Proposition 3 (Chess Block and Maximum Profit)
(i) The chess block $\overline{\boldsymbol{c}}$ *maximizes block profit* Π*, i.e.* $\Pi(\overline{\boldsymbol{c}}) = \max_{\boldsymbol{x}} \Pi(\boldsymbol{x})$.
(ii) Maximum block profit $\Pi(p, v)$ *equals* $\overline{N}(p + 4v)$.

> **Proof**: (i) In (N, Λ)-space, profit contours have slope $-p/v$, while the daylight frontier within the city's remit slopes upward by $4 > -p/v$. Maximum profit obtains where the highest isoprofit line passes through the extreme point of the daylight frontier, at $(\overline{N}, 4\overline{N})$. Thus the chess block is a maximizer. □
>
> (ii) Each of the chess block's $\overline{N}$ houses is daylit. □

Suppose that all cities are built by block developers. Theirs will be an urban system of a highly repetitive design. With no exception, and at any distance from the city center, all blocks are chess blocks. The only possible variation arises if n is even; then possibly either $\overline{\boldsymbol{x}}$ or $\overline{\boldsymbol{y}}$ get built. Of course, various assembly costs preclude those large block developers from emerging across the city. As long as smaller developers prevail, chess blocks not necessarily form. Block configurations will often feature deviations from the chess block, and occluded windows even. Section 4 gives the details. Section 5 then shows how ultimately block developers and their chess block design do come to dominate as households' daylight valuation continues to grow.

4 Sub-Developers

Historically, it is typically two or more developers, called *sub-developers*, who develop subsets of the block independently. The configuration now replaces the plan of the design. Let us partition the block into $S \geqslant 2$ subsets indexed by s, running from 1 to S, called *subdivisions*. Any subdivision must be rectangular, and its dimensions must be at least 2×2. (The limiting case of each sub-developer owning a single lot only is analyzed in Dascher/Haupt (2024).) This latter assumption makes every lot of the subdivision neighbor to at least two other lots of the same subdivision, a property we will repeatedly exploit below. Otherwise we impose no restriction on the subdivision. In particular, we do not require a subdivision's parcels to be connected. A sub-developer's division may be disconnected into subsets not neighbor to each other (because with adjacencies absent, subsets become subdivisions in their own right. Here the sub-developer simply optimizes her subdivision by separately optimizing on the subdivision's subsets.)

To be able to address the lots sub-developer s owns, we introduce the $n \times 1$ ownership dummy $\boldsymbol{\iota}_s$ featuring 1's for all lots belonging to subdivision s and 0 entries everywhere else. Via aggregation, $\sum_{s=1}^{S} \boldsymbol{\iota}_s$ yields $\boldsymbol{\iota}$. To trace *sub-design*, we define sub-design vector $\boldsymbol{x}_s$ by

$$x_{s,jk} = \begin{cases} 0 & \text{if} \quad jk & & \text{not owned by } s \\ 1 & \text{if} \quad jk \text{ has a house} & \text{and} & \text{owned by } s \\ 0 & \text{if} \quad jk \text{ is a yard} & \text{and} & \text{owned by } s \end{cases} \qquad \text{(sub-design } \boldsymbol{x}_s\text{)}, \qquad (20)$$

where $x_{s,jk}$ is the sub-developer's decision on lot jk. This completes our description of sub-design $\boldsymbol{x}_s$. By aggregation, $\sum_{s=1}^{S} \boldsymbol{x}_s$ yields $\boldsymbol{x}$, the configuration of the block. A simple alternative design for the subdivision is $\boldsymbol{y}_s = (\boldsymbol{\iota}_s - \boldsymbol{x}_s)$. This alternative design reverses the roles of houses and yards on all lots of subdivision, and on those lots only.

Finally we define $\boldsymbol{x}_{-s}$ by $\boldsymbol{x}_{-s} = \boldsymbol{x} - \boldsymbol{x}_s$, as well as dummy $\boldsymbol{\iota}_{-s}$ via $\boldsymbol{\iota}_{-s} = \boldsymbol{\iota} - \boldsymbol{\iota}_s$. It is $\boldsymbol{x}_{-s}$ the vector of design decisions taken by all other sub-developers on the block, denoted by subscript $-s$, and with all entries corresponding to lots owned by sub-developer s set to 0. Fig. (5a) illustrates a block partition into two equal-sized subdivisions, while Fig. (7a) illustrates one for which one developer owns all but three of the block's lots. In these figures, lots on one side of the heavy dividing line belong to one sub-developer, lots on the other side to the other sub-developer.

Updating our earlier windows accounting, we have

$$\begin{aligned} \Lambda_{s,i} &= \boldsymbol{x}_s'\boldsymbol{A}(\boldsymbol{\iota}_s - \boldsymbol{x}_s) && \text{(subdivision internally daylit windows)} \quad (21)\\ \Lambda_{s,o} &= \boldsymbol{x}_s'\boldsymbol{A}(\boldsymbol{\iota}_{-s} - \boldsymbol{x}_{-s}) \ + \ \boldsymbol{x}_s'\boldsymbol{f} && \text{(subdivision externally daylit windows),} \quad (22) \end{aligned}$$

so that total daylit windows on the subdivision, Λ_s, can be set out as $\Lambda_{s,i} + \Lambda_{s,o}$. Note how externally daylit windows $\Lambda_{s,o}$ now also include windows daylit by neighboring subdivisions' yards.

From either a compositional or a residual perspective, the subdeveloper's daylit windows, Λ_s, are

$$\Lambda_s = \begin{cases} \boldsymbol{x}_s'\boldsymbol{f} \ + \ \boldsymbol{x}_s'\boldsymbol{A}(\boldsymbol{\iota}_s - \boldsymbol{x}_s) \ + \ \boldsymbol{x}_s'\boldsymbol{A}(\boldsymbol{\iota}_{-s} - \boldsymbol{x}_{-s}) \\ 4\boldsymbol{\iota}'\boldsymbol{x}_s \ - \ \boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_s \ - \ \boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_{-s}. \end{cases} \quad \text{(sub daylit windows),} \qquad (23)$$

where either equation transforms into the other if we make use of the fact that both sides in equation

$$4\boldsymbol{\iota}'\boldsymbol{x}_s \ = \ \boldsymbol{x}_s'(\boldsymbol{f} + \boldsymbol{A}\boldsymbol{\iota}) \qquad \text{(subdivision windows)} \qquad (24)$$

indicate total windows (daylit or not) on the subdivision. The last term on the r.h.s. of the first equation of (23), or $\boldsymbol{x}_s'\boldsymbol{A}(\boldsymbol{\iota}_{-s} - \boldsymbol{x}_{-s})$, captures the daylightings neighboring sub-developers contribute to subdivision s. The last term on the r.h.s. of the second equation of (23), or $\boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_{-s}$, captures the occlusions neighboring sub-developers impose on subdivision s. Either type of externality is specific to the sub-developer context. Neither type of externality arises when developing the entire block.

We next represent the subdivision by its respective graph. That induced graph includes the subset of vertices corresponding to the subdivision's lots joint with the subset of edges that link any pair of those vertices in the original graph, and we label it the subdivision's *sub-graph.* The sub-graph has ν_s vertices and ε_s edges. A remarkable property of the sub-graph is how it inherits bipartiteness from the parent graph (Remark 4). This is important, but it is not surprising. Any (rectangular) subset of the original block retains the block's bipartite character.

Remark 4 (Bipartite Sub-Designs)
(i) The sub-graph is bipartite, and $\overline{\boldsymbol{x}}_s$ *and* $\overline{\boldsymbol{y}}_s$ *are its bipartite sub-designs.*
(ii) Both designs' internally daylit windows, $\Lambda_{s,i}$*, are equal,* $\overline{\boldsymbol{x}}_s'\boldsymbol{A}(\boldsymbol{\iota}_s - \overline{\boldsymbol{x}}_s) = \overline{\boldsymbol{y}}_s'\boldsymbol{A}(\boldsymbol{\iota}_s - \overline{\boldsymbol{y}}_s)$.
(iii) Comparing numbers of houses N_s*,*

$$\boldsymbol{\iota}_s'(\overline{\boldsymbol{x}}_s \ - \ \overline{\boldsymbol{y}}_s) \ = \ \begin{cases} 1 & \text{if } m_s n_s \text{ odd} \\ 0 & \text{if } m_s n_s \text{ even} \end{cases} \qquad (25)$$

Proof: (i) Bondy/Murty (2018).

(ii) Consider any edge in the sub-graph. By definition, that edge also is an edge in the block graph. Because the graph is bipartite, the edge links to both a house and a yard. So the sub-graph is bipartite, too. □

Much as the original graph, the sub-graph has two bipartite designs, denoted $\overline{\boldsymbol{x}}_s$ and $(\boldsymbol{\iota}_s - \overline{\boldsymbol{x}}_s) = \overline{\boldsymbol{y}}_s$, too. These are simply $\overline{\boldsymbol{x}}$ and $\overline{\boldsymbol{y}}$ restricted to the subdivision, respectively. That is,

$$\overline{x}_{s,jk} \;=\; \begin{cases} \overline{x}_{jk} & \text{if} \quad \text{jk owned by } s \\ 0 & \text{if} \quad \text{jk not owned by } s \end{cases}, \tag{26}$$

and hence $\overline{\boldsymbol{y}}_s$ obtains as $(\boldsymbol{\iota}_s - \overline{\boldsymbol{x}}_s)$.

Either bipartite sub-design switches on all ε_s edges on subdivision s. Both bipartite sub-designs generate the same internally daylit windows, $\Lambda_{s,i} = \varepsilon_s$. Much as with the original block, we label that bipartite design the *chess sub-block* that enjoys greater external daylight, and denote it by $\overline{\boldsymbol{c}}_s$ (Definition 2). Only, note that one bipartite sub-design fails to dominate the other. Enjoying greater external daylight not only depends on how houses are positioned vis-a-vis the street, but also how they are positioned vis-a-vis neighboring sub-developers' designs. Figs. (3a) illustrates two subdevelopers' designs on the 6 × 5 block example. These designs are sub-chess blocks, too.

Definition 2 (Chess Sub-Block)

Let $\overline{\boldsymbol{x}}_s$ and $(\boldsymbol{\iota}_s - \overline{\boldsymbol{x}}_s) = \overline{\boldsymbol{y}}_s$ denote the two bipartite sub-designs. Then

$$\overline{\boldsymbol{c}}_s \;=\; \begin{cases} \overline{\boldsymbol{x}}_s \\ \overline{\boldsymbol{y}}_s \end{cases} \quad \text{if} \quad (\overline{\boldsymbol{x}}_s - \overline{\boldsymbol{y}}_s)'\big(\boldsymbol{f} + \boldsymbol{A}(\boldsymbol{\iota}_{-s} - \boldsymbol{x}_{-s})\big) \quad \begin{matrix} \geqslant 0 \\ < 0 \end{matrix}. \tag{27}$$

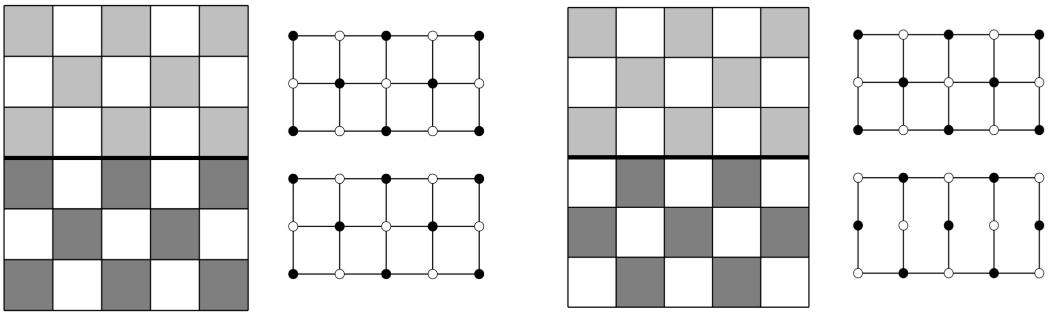
The chess sub-block $\overline{\boldsymbol{c}}_s$ is the sub-design that has more externally daylit windows, $\Lambda_{s,o}$.

Fig. (3a) illustrates sub-developer 1's chess sub-block $\overline{\boldsymbol{c}}_1$ (black lots alternating with white lots, below the heavy dividing line) given sub-developer 2's design choice $\boldsymbol{x}_2$ (grey lots alternating with white ones, above the heavy dividing line). While no daylight is received from the neighboring sub-division (i.e. $\overline{\boldsymbol{c}}_1'\boldsymbol{A}(\boldsymbol{\iota}_2 - \boldsymbol{x}_2) = 0$), 7 daylightings are taken in from the street (i.e. $\overline{\boldsymbol{c}}_1'\boldsymbol{f} = 7$). This gives $\Lambda_1 = 29$ daylit windows altogether on sub 1. Compare this with the alternative bipartite design $(\boldsymbol{\iota}_1 - \overline{\boldsymbol{c}}_1)$, shown in Fig. (5b), which exhausts internal daylightings ($\overline{\boldsymbol{c}}_1'\boldsymbol{A}(\boldsymbol{\iota}_1 - \overline{\boldsymbol{c}}_1) = 22$ again), too, yet fails to haul in as much external daylight from either street (i.e. $\overline{\boldsymbol{c}}_1'\boldsymbol{f} = 4$) or neighboring sub (i.e. $\overline{\boldsymbol{c}}_1'\boldsymbol{A}(\boldsymbol{\iota}_2 - \boldsymbol{x}_2) = 2$). So Fig. (3b)'s design only yields $\Lambda_1 = 28$ daylit windows. This is 1 daylit window less.

As we document next, the chess sub-block maximizes the daylit windows internal to the subdivision, $\Lambda_{s,i}$ (Proposition 4 (ii)). In fact, the chess sub-block even maximizes daylit windows on the subdivision, Λ_s (Proposition 4 (iii)). These results extend our earlier results (in Proposition 2) to the subdivision. Maximum daylit windows on the subdivision obtain for insisting on the chess sub-block. This is true *whatever* neighboring developers' designs are.

Proposition 4 (Chess Sub-Block and Maximum Daylit Windows)

The chess sub-block $\overline{\boldsymbol{c}}$ ...
(i) has ε_s internally daylit windows.
(ii) maximizes internally daylit windows on the subdivision, $\Lambda_{s,i}$.
(iii) maximizes daylit windows on the subdivision, Λ_s.



(a) **Inefficient Nash-Equilibrium** $(\overline{\boldsymbol{c}}_1, \overline{\boldsymbol{c}}_2)$
$0 \leqslant -p/v < 1$

$$\begin{aligned}
\boldsymbol{\iota}'\overline{\boldsymbol{c}}_1 &= 8 = \boldsymbol{\iota}'\overline{\boldsymbol{c}}_2 \\
\overline{\boldsymbol{c}}_1'\boldsymbol{A}(\boldsymbol{\iota}_1 - \overline{\boldsymbol{c}}_1) &= 22 = \overline{\boldsymbol{c}}_2'\boldsymbol{A}(\boldsymbol{\iota}_2 - \overline{\boldsymbol{c}}_2) \\
\overline{\boldsymbol{c}}_1'\boldsymbol{A}\overline{\boldsymbol{c}}_2 &= 3 = \overline{\boldsymbol{c}}_2'\boldsymbol{A}\overline{\boldsymbol{c}}_1 \\
\overline{\boldsymbol{c}}_1'\boldsymbol{A}(\boldsymbol{\iota}_2 - \overline{\boldsymbol{c}}_2) &= 0 = \overline{\boldsymbol{c}}_2'\boldsymbol{A}(\boldsymbol{\iota}_1 - \overline{\boldsymbol{c}}_1) \\
\overline{\boldsymbol{c}}_1'\boldsymbol{f} &= 7 = \overline{\boldsymbol{c}}_2'\boldsymbol{f} \\
\Lambda_1 &= 29 = \Lambda_2 \\
\Lambda &= 58
\end{aligned}$$

(b) **Efficient Nash-Equilibrium** $(\boldsymbol{z}_1, \overline{\boldsymbol{c}}_2)$
$1 < -p/v < 4$

$$\begin{aligned}
&\boldsymbol{\iota}'\boldsymbol{z}_1 = 7,\ \boldsymbol{\iota}'\overline{\boldsymbol{c}}_2 = 8 \\
&\boldsymbol{z}_1'\boldsymbol{A}(\boldsymbol{\iota}_1 - \boldsymbol{z}_1) = 22,\ \overline{\boldsymbol{c}}_2'\boldsymbol{A}(\boldsymbol{\iota}_2 - \overline{\boldsymbol{c}}_2) = 22 \\
&\boldsymbol{z}_1'\boldsymbol{A}\overline{\boldsymbol{c}}_2 = 0,\ \overline{\boldsymbol{c}}_2'\boldsymbol{A}\boldsymbol{z}_1 = 0 \\
&\boldsymbol{z}_1'\boldsymbol{A}(\boldsymbol{\iota}_2 - \overline{\boldsymbol{c}}_2) = 2,\ \overline{\boldsymbol{c}}_2'\boldsymbol{A}(\boldsymbol{\iota}_1 - \boldsymbol{z}_1) = 3 \\
&\boldsymbol{z}_1'\boldsymbol{f} = 4,\ \overline{\boldsymbol{c}}_2'\boldsymbol{f} = 7 \\
&\Lambda_1 = 28,\ \Lambda_2 = 32 \\
&\Lambda = 60
\end{aligned}$$

Notes: (i) Here the block is divided into two equal-sized subdivisions. Subdivision 1 comprises all (black or white) lots below the horizontal line, subdivision 2 all (grey or white) lots above it. (ii) Expressions in the captions successively document subdivisions': total houses ($\boldsymbol{\iota}'\boldsymbol{x}_s$), within-block daylightings ($\boldsymbol{x}_s'\boldsymbol{A}(\boldsymbol{\iota}_s - \boldsymbol{x}_s)$), cross-block occlusions or negative externalities ($\boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_{-s}$), cross-block daylightings or positive externalities ($\boldsymbol{x}_s'\boldsymbol{A}(\boldsymbol{\iota}_{-s} - \boldsymbol{x}_{-s})$), street daylighting ($\boldsymbol{x}_s'\boldsymbol{f}$) and daylit windows Λ_s. (iii) Panel (a) shows two chess sub blocks, $\overline{\boldsymbol{c}}_1$ and $\overline{\boldsymbol{c}}_2$. Panel (b) has a chess sub block for subdivision 2, $\overline{\boldsymbol{c}}_2$, but not for subdivision 1.

Figure 5: Inefficient and efficient Nash-equilibria (symmetric partition of the block)

Proof: (ii) Since the chess sub-block is a bipartition on the subdivision (Remark 4), all of the sub-graph's ε_s edges become daylightings. □

(iii) (1) As with the block developer, adding a house to the chess sub-block adds 2 externally daylit windows at best, while reducing internal daylightings by 2 at least. Adding a house fails to raise daylit windows Λ_s. (2) Every house in the chess sub-block is occluded (by neighboring sub-developers) twice at most. So deleting any house reduces daylightings by 2 at least. (3) Much as with the block developer, and in view of Definition 2, there is no extra internal or external daylighting benefit to be had from rearranging houses. □

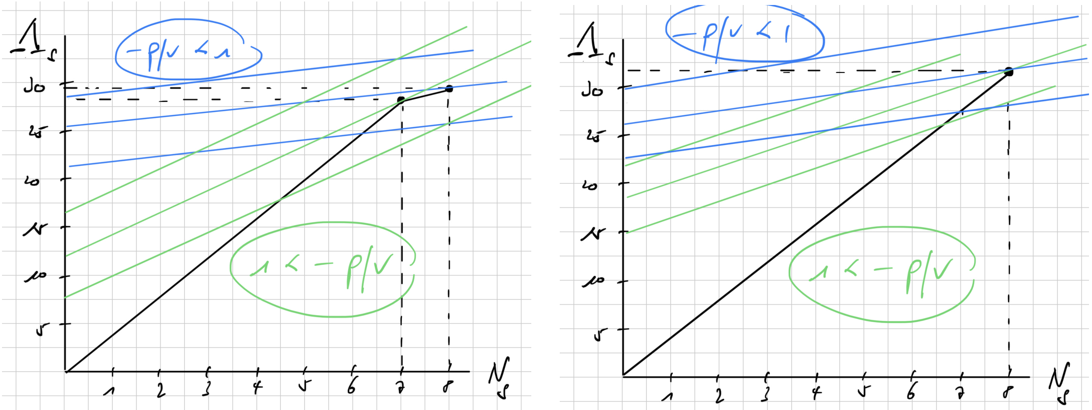
Of course, as with the block developer, sub-developer s is interested in maximum profit, not maximum daylit windows. Sub-developer s solves

$$\begin{aligned}
\max_{\boldsymbol{x}_s, N_s} \quad \Pi_s \quad &= \quad pN_s \ + \ v\Big(4N_s \ - \ \boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_s \ - \ \boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_{-s}\Big) \\
\text{s.t.} \quad &\quad x_{s,jk} \in \{0, 1\} \text{ and } \boldsymbol{\iota}'\boldsymbol{x}_s = N_s. \qquad (28)
\end{aligned}$$

Again we break down the profit maximization objective into two steps. First we identify the subdivision's daylight frontier, then we let the sub-developer choose the point on that frontier that is best (Remark 4).

Remark 4 (Subdivision Daylight Frontier and Maximum Profit)
Maximum subdivision profit $\Pi_s(v)$ *obtains when solving*

$$\max_{N_s, \Lambda_s} \ pN_s \ + \ v\Lambda_s \qquad \text{s.t.} \qquad \Lambda_s = \Lambda_s(N_s) \qquad \text{(sub-developer profit)}, \qquad (29)$$



(a) Subdivision frontiers $\Lambda_s(N_s)$ for Fig. (5a) (b) Subdivision frontier $\Lambda_1(N_1)$ for Fig. (5b)

Notes: (i) The panel on the left shows the daylight frontier for both subdivision 1 and subdivision 2, given each other's respective choice of design $\overline{\boldsymbol{c}}_1$ and $\overline{\boldsymbol{c}}_2$, respectively, from Fig. (5a). (ii) The panel on the right shows subdivision 2's frontier for subdivision 1's choice of design $\overline{\boldsymbol{z}}_1$ in Fig. (5b). (iii) Profit contours are green for when $-p/v < 1$, and blue else.

Figure 6: Subdivisions' daylight frontiers

where $\Lambda_s(N_s)$ *is defined below.*

Proof: Rewrite problem (28) as

$$\max_{N_s} \quad \left\{ \max_{\boldsymbol{x}_s} \; pN_s \; + \; v\Big(4N_s - \boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_s - \boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_{-s}\Big) \;\; \text{s.t.} \;\; x_{s,jk} \in \{0,1\} \text{ and } \boldsymbol{\iota}'\boldsymbol{x}_s = N_s \right\}$$

Shifting pN_s (i.e. a constant to the inner maximization problem) outside the curly brackets gives

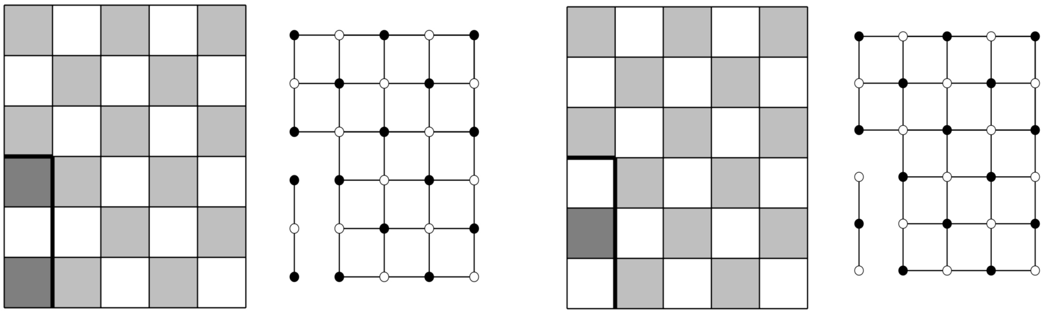
$$\max_{N_s} \quad pN_s \; + \; v\Bigg\{ \underbrace{\max_{\boldsymbol{x}_s} \; \Big(4N_s - \boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_s - \boldsymbol{x}_s'\boldsymbol{A}\boldsymbol{x}_{-s}\Big) \text{ s.t. } x_{s,jk} \in \{0,1\} \text{ and } \boldsymbol{\iota}'\boldsymbol{x}_s = N_s}_{\Lambda_s(N_s)} \Bigg\}$$

If we label the maximum value function within curly brackets $\Lambda_s(N_s)$, i.e. the subdivision's daylight frontier, (29) is equivalent to (28) above. □

Now, maximum daylit windows on the subdivision (Proposition 4 (iii)) identify $(\overline{N}_s, \overline{\Lambda}_s)$ as the first point on the subdivision's daylight frontier, $\Lambda_s(N_s)$. Further points on that frontier obtain as we successively remove houses, and reevaluate the optimal design at each step. The subdivision frontier's shape depends on the specifics of its context, i.e. on the size of the subdivision and the design choices of the neighboring subdevelopers.

Deriving $\Lambda_s(N_s)$ for the general case is not straightforward; one needs to solve a non-linear program. Here we analyze the special case in which (i) the block is partitioned into two equal-sized subdivisions of odd dimensions m_s and n_s and (ii) sub-designs are chess sub-blocks $\overline{\boldsymbol{c}}_1$ and $\overline{\boldsymbol{c}}_2$. The resulting block configuration $(\overline{\boldsymbol{c}}_1, \overline{\boldsymbol{c}}_2)$ is shown in Fig. (5a)), and represents a Nash-equilibrium if the block is close to the center, i.e. if $-p/v$ is less than 1 (Proposition 5). If $-p/v$ exceeds 1, $(\overline{\boldsymbol{c}}_1, \overline{\boldsymbol{c}}_2)$ no longer is a Nash-equilibrium. Then Nash equilibrium is shown by Fig. (5b).

Proposition 5 (Nash-Equilibrium with 2 Sub-Developers)
An inefficient Nash-Equilibrium $(\overline{\boldsymbol{c}}_1, \overline{\boldsymbol{c}}_2)$ *exists if and only if:*



(a) **Inefficient Nash-Equilibrium** $(\overline{\boldsymbol{c}}_1, \overline{\boldsymbol{c}}_2)$
$0 \leqslant -p/v < 1$
$\boldsymbol{\iota}'\overline{\boldsymbol{c}}_1 = 27$, $\boldsymbol{\iota}'\overline{\boldsymbol{c}}_2 = 3$
$\Lambda = 72$

(b) **Efficient Nash-Equilibrium** $(\boldsymbol{z}_1, \overline{\boldsymbol{c}}_2)$
$1 < -p/v < 4$
$\boldsymbol{\iota}'\boldsymbol{x} = 17$
$\Lambda = 68$

Notes: Here the block is divided into two unequally-sized subdivisions. Subdivision 1 comprises all (black or white) lots below the horizontal line, subdivision 2 all (grey or white) lots above it.

Figure 7: Efficient and inefficient Nash-equilibrium (asymmetric partition of block)

(i) $-p/v < 1$*, (ii)* m_s, n_s *are odd for both* s *and (iii)* $m_s, n_s < 7$ *for both* s*.*

Proof: (If) Sub-design $\boldsymbol{c}_1$ from Fig. (5a) maps into extreme point $(\overline{N}_1, \overline{\Lambda}_1)$, or $(8, 29)$, on the daylight frontier $\Lambda_1(N_1)$ in Fig. (6a). Now reduce N_1 by 1, to 7. Maximum daylight for 7 houses obtains with the sub-design for subdivision 1 shown in Fig. (5b), $\boldsymbol{z}_1$, and equals $\Lambda_1(7) = 28$. This identifies the second point on subdivision's daylight frontier, $(7, 28)$. Since with 6 houses, every houses is fully daylit, successive points are $(6, 24)$, $(5, 20)$, etc. Sub-developer 2's daylight frontier, $\Lambda_2(N_2)$, for given $\boldsymbol{c}_1$, obtains similarly. Applying isoprofit lines with slope $-p/v < 1$ to Fig. (6a) yields $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ as sub-developers' choices. In Nash-equilibrium, 6 occluded windows obtain.

(Only if) (i) First suppose that $1 < -p/v$. Since $\boldsymbol{c}_2$ is the same in Fig. (5a) and (5b), $\Lambda_1(N_1)$ is the same as in this Proposition's first part. Moreover, $\Lambda_2(N_2)$ simply equals $4N_2$ for $N_2 \leqslant \overline{N}_2$ (Fig. (6b)). Applying isprofit lines of slope $-p/v$ greater than 1 yields the sub-design choices $\overline{\boldsymbol{z}}_1$ and $\overline{\boldsymbol{c}}_2$ shown in Fig. (5b). These two designs match up to the chess block design for the entire block. (ii) $\square$

Further insights on Nash-equilibrium, above and beyond Proposition 5, are to be had. For example, Fig. (7) indicates a Nash-equilibrium for an asymmetric partition of the block into subdivisions. Here if $-p/v < 1$, Fig. (7a)'s strategy pair $(\overline{\boldsymbol{c}}_1, \overline{\boldsymbol{c}}_2)$ indicates a Nash-equilibrium, also. At the same time, if $1 < -p/v$, then Nash-equilibrium is the pair of strategies shown in Fig. (7b). These observations make use of the same tools as the proof above, i.e. they first construct, and then evaluate, each subdivision's underlying daylight frontier.

5 Daylight Valuation and Industry Consolidation

Let us denote subdivisions' equilibrium daylightings and profits by $\Lambda_s(p, v)$ and $\Pi_s(p, v)$, respectively (where daylit windows and profits both are homogeneous of degree 0 with respect to (p, v)). Revisiting our examples in Fig. (5a) and (7a), total daylit windows in Nash-equilibrium $\Lambda_1(p, v)$ and $\Lambda_2(p, v)$ are strictly less than daylit windows obtained for the chess block design, $\Lambda(p,v)$. Generally, any two sub-developers' union will never do worse than those sub-developers separately. But then we also have the following superadditivity property with respect to daylit windows and profit (Proposition 6):

Proposition 6 (Superadditivity of Daylit Windows and Profit)
(i) Maximum daylit windows are superadditive: $\overline{\Lambda} = \Lambda(p,v) \geqslant \sum_s \Lambda_s(p,v)$.
(ii) Profit is superadditive: $\Pi(p,v) \geqslant \sum_s \Pi_s(p,v)$.

> **Proof**: (i) It is $\Lambda(p,v)$ the block's maximum of daylit windows. This can never be less than the sum of daylit windows maximized separately. Moreover, for the constellation of p/v, m_s and n_s set out in Proposition 5, ... □
>
> (ii) Subdivision developers maximize under (additional) constraints that are irrelevant to the block developer. □

We now address the role of secular changes in working arrangements for urban design. As working hours decline, and as work relocates to the home, *time-at-home* rises. Households value the quality of their home more. A secular increase in valuation v increases developers' incentive to develop the entire block, rather than subdivisions of it (Proposition 7 (i)). We expect the real estate developer and construction industry to consolidate further.

Proposition 7 (Daylight Valuation, Industry Consolidation, House Price)
Let time spent at home rise. A home's daylighting is valued more, so that v rises. Then:
(i) Block profit $\Pi(p,v)$ rises at least as much as, if not strictly more than, $\sum_s \Pi_s(p,v)$.
(ii) The industry consolidates. The block developer replaces subdivision developers.
(iii) Bright rent $p(\delta)+4v$ and land rent $\Pi(\delta,v)$ rise.

> **Proof**: Consider the derivative of the maximum value function $\Pi(p,v)$ to problem (16) with respect to v, $\partial\Pi(p,v)/\partial v$. By the envelope theorem, this is $\overline{\Lambda}$. Similarly, the derivatives of all other subdivision developers' maximum value functions $\Pi_s(p,v)$ to problems (29) with respect to v, $\partial\Pi_s(p,v)/\partial v$, equal $\Lambda_s(p,v)$. But then by Proposition 6 (i), block profit rises by never less than, if not strictly more than, if the block's subdivisions are developed separately:
>
> $$\frac{\partial\Pi(p,v)}{\partial v} = \Lambda(p,v) \geqslant \sum_s \Lambda_s(p,v) = \sum_s \frac{\partial\Pi_s(p,v)}{\partial v}. \quad \square \tag{30}$$
>
> (ii) In text. □

To the extent that (Nash-equilibrium) blocks, of greater density, are shed in favor of the chess block (Proposition 7 (ii)), themselves of lower density, boundary distance $\tilde{\delta}$ must rise. But then the price of housing $p(\delta)+4v$, rises throughout the city, as does the price of land (Proposition 7 (iii)).

6 Extensions

The chess block occludes all of its yards. Yards in the block's interior are occluded four times, yards on the streetfront are occluded twice, and corner yards are occluded twice. Put differently, private space within the block is shaded well, and this provides relief in times of a warming climate. Yet as long as households commute to work and shop, they also spend time on the street. The street aligned by a succession of chess blocks is occluded much less, and discontinuously. Daylit sections of the street alternate with occluded ones. Arguably, this lack of shade in the street, during the commute, or in shared public space may matter more to the household than the chess block's perfection of shade within the block.

Our emphasis has been on the economies of scale to be had from optimizing daylighting at the level of the block, rather than at the level of a subdivision of the block. Besides, any yard can daylight a single house just as well as it can daylight four neighboring houses. But there are many other economies of scale in construction and development that may also drive consolidation in the industry and uniformity of design.

7 Conclusions

Many real-world blocks built today resemble the chess block. We first show that the chess block maximizes daylit houses on the block, daylit windows on the block, and block profit. These properties explain the chess block's appeal to real estate developers. Moreover, these properties have growing appeal to real estate developers as households' appreciation of daylight grows. A secular reduction in working hours, but also work from home, suggest that today households value daylight more than they did in the past. Greater daylight valuation drives consolidation in the real estate industry. Only large developers are able to fully internalize, and hence best exploit, the within-block externalities that characterize urban daylight. Consolidation in turn drives real estate prices up. To be sure, real estate price rise because of chess blocks' lower density (rather than because of a decline in real estate competition).

A companion paper of ours (Dascher/Haupt (2024)) goes beyond the question of optimal *block* design, exploring optimum *urban* design instead. With its discontinuous streetfront, the chess block diminishes the block's attention to the street. There plainly are fewer eyes on the street. Jacobs-style safety in public space suffers if streets' adjoining blocks no longer align with the street. In addition, the chess block offers less retail space aligning with the street. These caveats render the chess block's daylighting properties less impressive. They suggest a trade-off between the quality of its private space and that of its public space. The chess block is not always the optimal urban block.

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