

# Workshop on “Calculus of Variations and Partial Differential Equations”

organised by Helmut Abels, Harald Garcke, and Carolin Kreisbeck  
supported by [University of Regensburg, Faculty of Mathematics](#)  
and [DFG-Research Training Group 2339 – \*IntComSin\*](#)

Regensburg, 25–27 April, 2024

On the occasion of the **60th birthday of Georg Dolzmann** we will enjoy interesting talks on calculus of variations, partial differential equations and their applications, meeting old and new friends, and enjoy some social programme around this one-and-half-day workshop and colloquium.

On Thursday the colloquium talk will take place in [lecture hall H 31](#). The talks of the workshop will take place in the seminar room [M 104](#), while coffee breaks will be in the common room [M 201](#). On Friday we will have lunch together at the university canteen.

In case you want to participate please register with an email to [saskia.lindenberg@ur.de](mailto:saskia.lindenberg@ur.de).

## Preliminary Programme

### Thursday, April 25, 2024

Thursday	
14:45 – 15:30	Sergio Conti
15:30 – 16:15	Dorothee Knees
16:15 – 17:00	<i>Coffee break</i>
17:00 – 18:00	Colloquium talk by Carolin Kreisbeck
19:00	<i>Dinner at <a href="#">Leerer Beutel</a></i>

## Friday, April 26, 2024

We will have 45 minute talks (including discussion) by the following speakers.

Friday	
10:15 – 11:00	Ricardo Nochetto
11:00 – 11:45	Antonio DeSimone
11:45 – 13:30	<i>Lunch break</i>
13:30 – 14:15	Jan Kristensen
14:15 – 15:00	Bernd Kirchheim
15:00 – 15:45	<i>Coffee break</i>
15:45 – 16:30	Alessandra Pluda
16:30 – 17:15	Sören Bartels
17:15 – 17:45	<i>Open discussions</i>
18:30	<i>Workshop dinner at <a href="#">Bischofshof</a></i>

## Saturday, April 27, 2024

For those who wish to stay on Saturday, as a social programme we will enjoy a hike to “Walhalla” near Donaustauf. Afterwards we will have lunch together, such that in the afternoon everyone has enough time to travel home. The departure from Regensburg to Donaustauf is planed for 9:30 and lunch around 12:00–12:30.

## Detailed workshop programme – Thursday

**Sergio Conti** (University of Bonn)

Thursday, 14:45 – 15:30

Title: **A variational phase-field model of cohesive fracture**

**Abstract:** I shall discuss a phase-field model of cohesive fracture, and its convergence to a sharp-interface model within the framework of  $\Gamma$ -convergence. A damage function which depends both on the phase field and the regularization parameter  $\epsilon$  permits to reconstruct many traction-separation laws. In the limit, the deformation belongs to GSBV and the energy contains an (elastic) volume energy, an opening-dependent jump energy concentrated on the fractured surface, and a Cantor part representing diffuse damage and plastic deformation. The talk is based on joint work with Flaviana Iurlano and Matteo Focardi.

**Dorothee Knees** (University of Kassel ) Thursday, 15:30 – 16:15

Title: **A damage model based on microstructure evolution**

**Abstract:** Macroscopic damage and failure phenomena of solids typically are the result of the accumulation of small cracks or defects on a microscopic scale. In engineering literature, various multi-scale or homogenized models are proposed in order to describe time-dependent damage phenomena with microscopic origins like the growth of micro-cracks or micro-voids. In this lecture, we discuss these approaches in the framework of homogenization and evolutionary  $\Gamma$ -convergence, allowing for micro-defects that may grow individually with respect to the time-dependent loadings. The lecture relies on joint work with Hauke Hanke (formerly WIAS Berlin).

## Colloquium talk – Thursday

**Carolin Kreisbeck** (Catholic University of Eichstätt-Ingolstadt)

Lecture hall H31, Thursday, 17:00–18:00

Title: **A variational perspective on auxetic metamaterials**

**Abstract:** Auxetic metamaterials are specifically designed to have the counterintuitive property that they expand perpendicular to applied forces under stretching. In this talk, we will focus on a variational model to describe elastic materials made of (almost) rigid and soft components arranged into a periodic small-scale pattern of checkerboard type. While such structures have been investigated by the mechanics community and tested in experiments and applications over the last few years, establishing a mathematically rigorous foundation of these findings is an interesting task that requires new techniques. We will discuss how combining tailored arguments from calculus of variations, asymptotic analysis, and geometry can help approach this problem. This is based on joint work with Wolf-Patrick Düll (University of Stuttgart) and Dominik Engl (KU Eichstätt-Ingolstadt).

## Detailed workshop programme – Friday

**Ricardo H. Nochetto** (University of Maryland, College Park)

Friday, 10:15 – 11:00

Title: **Liquid Crystal Polymeric Networks: Modeling, Approximation, and Computation**

**Abstract:** We discuss modeling, numerical analysis and computation of liquid crystal networks (LCNs). These materials couple a nematic liquid crystal with a rubbery material. When actuated with heat or light, the interaction of the liquid crystal with the rubber creates complex shapes. Thin bodies of LCNs are natural candidates for soft robotics applications. We start from the classical 3D trace energy formula and derive a reduced 2D membrane energy as the formal asymptotic limit of vanishing thickness and characterize the zero energy deformations. We design a sound numerical method and prove its Gamma convergence despite the strong non-linearity and lack of convexity properties of the membrane energy. We present computations showing the geometric effects that arise from liquid crystal defects as well as computations of nonisometric origami within and beyond the theory. This work is joint with L. Bouck and S. Yang.

**Antonio DeSimone** (SISSA Trieste )

Friday, 11:00 – 11:45

Title: **tba**

**Abstract:** tba

**Jan Kristensen** (University of Oxford)

Friday, 13:30 – 14:15

Title: **The Burkholder Functional on Classes of Planar Quasiregular Maps**

**Abstract:** The area formula of Gronwall and Bieberbach can be viewed as a precise way to express that the Jacobian of a planar Sobolev map is a null Lagrangian. In this talk I discuss a quasiconvexity inequality for the Burkholder functional in the context of planar quasiconformal maps. This inequality can be viewed as an extension of the area formula to an  $L_p$  context. If combined with Stoilow factorization and blow-up arguments it also allows a proof of semicontinuity, and hence to prove existence of minimizers for the Burkholder energy on classes of planar quasiregular maps. The talk is based on joint work with Kari Astala (Helsinki), Daniel Faraco (Madrid), Andre Guerra (ETH), and Aleksis Koski (Aalto).

**Bernd Kirchheim** (University of Leipzig)

Friday, 14:15 – 15:00

Title: **Convexity and Uniqueness in the Calculus of Variations**

**Abstract:** Whereas general existence results for minimizers of (vectorial) variational problems are clearly related to (coercivity) and Morreys quasiconvexity, the situation becomes much more constrained if also uniqueness of the minimizers is required for all linear perturbation of the energy. In this case a rather natural notion of functional convexity arises in a general Banach space context. We will discuss what are the specific implications for energy densities of integral cost functions.

This is joint work with J. Campos Cordero (Mexico), J. Kollar (Prag) and J.Kristensen (Oxford).

**Alessandra Pluda** (University of Pisa)

Friday, 15:45 – 16:30

Title: **tba**

**Abstract:** tba

**Sören Bartels** (University of Freiburg)

Friday, 16:30 – 17:15

Title: **Babuska's paradox in linear and nonlinear bending theories**

**Abstract:** The plate bending or Babuska paradox refers to the failure of convergence when a linear bending problem with simple support boundary conditions is approximated using polygonal domain approximations. We provide an explanation based on a variational viewpoint and identify sufficient conditions that avoid the paradox and which show that boundary conditions have to be suitably modified. We show that the paradox also matters in nonlinear thin-sheet folding problems and devise approximations that correctly converge to the original problem.