Abstracts

Tinhinane Amina Azzouz, *Spectral theory and ultrametric differential equations*

The spectral theory was the main motivation of Berkovich to introduce his approach to study the analytic manifolds defined over a complete nonarchimedean field k. In this talk, we will introduce the Berkovich spectrum and its main proprieties and use that for the study of the ultrametric differential equations. Precisely, we will show how for a differential equation

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$$

defined over a domain V of $\mathbb{A}_k^{1,an}$ and a non rigid point x of V, we associate a compact subset of $\mathbb{A}_k^{1,an}$, which is the spectrum of an operator. We will see also how this geometrical object behaves when we vary the point x over V . In the end, I will give some concrete computations of such spectrums.

Lev Blechman, *Refined descendant tropical invariants of toric surfaces*

Refined tropical genus zero invariants were introduced by Block–Goettsche and Goettsche–Schroeter, and are Laurent polynomials in one variable y. Those polynomials, under some some appropriate conditions, for $y = 1$ yield Gromov Witten invariants, and for $y = -1$ yield Welschinger invariants of toric del Pezzo surfaces, that count complex, resp. real plane rational tropical curves passing through a generic configuration of points. We will define a refinement of arbitrary rational tropical descendant invariants, which are a generalization of the BG and GS invariants, that count plane rational tropical curves that pass through a more generic configuration of points. Those descendant invariants will be rational functions in y. We will show that those invariants specialize to the refined invariants introduced by H. Marwig and J. Rau as y tends to 1, and, finally, we will show that those invariants are the unique generailzation of both notions in some sense, and discuss some computational aspects. Joint work with Eugenii Shustin.

Roberto Gualdi, *Heights of hypersurfaces in varieties*

In [BPS], the Arakelov geometry of toric varieties has been written in terms of convex geometry. We deduce from the results therein a formula for the height of a hypersurface Y in a toric variety with respect to the choice of toric line bundles equipped with semipositive toric metrics $\overline{L}_0, \dots, \overline{L}_{n-1}$. More precisely, on a base adelic field $(K, (|\cdot|_v, n_v)_{v \in \mathfrak{M}_K})$, such height can be written as

$$h_{\overline{L}_0,\dots,\overline{L}_{n-1}}(Y) = \sum_{v \in \mathfrak{M}_K} n_v \mathrm{MI}_M(\vartheta_{0,v}, \dots, \vartheta_{n-1,v}, \rho_v^\vee),$$

where $\{\vartheta_{i,v}\}$ are concave functions associated to the choice of the metrized line bundles, $\rho_v^\vee$ is the Legendre-Fenchel dual of a suitable v-adic Ronkin function of Y and MI_M denotes the mixed integral of concave functions on a real vector space. A well-known result by V. Maillot for the canonical height of hypersurfaces in smooth projective toric varieties follows easily from our general approach.

[BPS] J. I. Burgos Gil, P. Philippon, and M. Sombra, Arithmetic geometry of toric varieties. Metrics, measures and heights, Astérisque 360, Société Mathématique de France, 2014.

Boulos El Hilany, *Constructing polynomial systems with many positive solutions using tropical geometry*

The number of positive solutions of a system of two polynomials in two variables with a total of five distinct monomials defined in the field of real numbers cannot exceed 15. All previously known examples have at most 5 positive solutions. Tropical geometry is a powerful tool to construct polynomial systems with many positive solutions. The classical combinatorial patchworking method arises when the tropical hypersurfaces intersect transversally. It is known that a system as above constructed using this method has at most 6 positive solutions. In this talk, we show that this bound is sharp. Moreover, using non-transversal intersections of tropical curves, we constructed a system as above having 7 positive solutions.

Paul Alexander Helminck, *Galois covers and explicit algorithms for Berkovich skeleta*

We will discuss the problem of calculating the Berkovich skeleton of a given curve C over a discretely valued field. For all curves of genus less then 7 and residue characteristic greater than 7, we will give an algorithm/technique in terms of Laplacians and Galois coverings. Furthermore, we will discuss its implications for tropicalizing certain moduli spaces.

Marvin Hahn, *A limit linear series moduli problem for Mustafin varieties*

Mustafin varieties are flat degenerations of projective spaces induced by a choice of an n-tuple of lattices in a vector space over a non-archimedean field. These varieties have a rich combinatorial structure as can be seen in pioneering work of Cartwright, Häbich, Sturmfels and Werner. In this talk, we give a new description for Mustafin varieties in terms of images of rational maps, which were studied by Li. We completely classify the irreducible components of Mustafin varieties associated to a tuple of lattices. We relate Mustafin varieties for convex point configurations to Shimura varieties and certain moduli functors which appear in the limit linear series theory developed by Osserman called prelinked Grassmannians. Our methods include tropical convex hull computations and tropical intersection theory. This is joint work with Binglin Li.

Timo Henkel, *An Open Mapping Theorem for analytic rings*

The Open Mapping Theorem for Banach spaces over the complex numbers provides a nice criterion for a continuous surjective map to be open. In this talk we discuss an analogous result for those rings which play a central role in nonarchimedean geometry. As an application we introduce a canonical topology for modules over those rings.

Sevda Kurul, *On the birational geometry of arrangement complements*

The complement Y of a hyperplane arrangement in the projective space can be naturally identified with a subvariety of its intrinsic torus. Tevelev shows that for certain toric varieties one gets nice compactifications, called tropical compactifications, that are constructed by taking the closure of the arrangement complement in these toric varieties. In this talk we will give an overview of this subject and introduce two special cases of tropical compactification: the wonderful compactification and the visible contours compactification. We will then use this theory to prove that every dominant endomorphism of an arrangement complement Y extends to a morphism of its visible contours compactification. We will also show that, for a large class of arrangement complements, every automorphism of Y can be extended to an automorphism of its wonderful compactification.

Sara Lamboglia, *Tropical Fano Schemes*

The classical Fano scheme of a variety X parametrizes linear spaces contained in X. It is well understood for certain class of varieties such as toric varieties or hypersurfaces, but there are still many open questions concerning dimension and irreducibility. In this talk I am going to define the tropical analogue of the Fano scheme and I will show some of its properties. I will then explain how it would be possible to use the tropical Fano scheme to understand its classical counterpart.

Enrica Mazzon, *Essential skeleton of Hilbert scheme of a K3 surface*

Let X be a proper variety over a complete discretely valued field K. The geometry of its Berkovich analytification X^{an} may be captured by the study of some finite simplicial complexes called Berkovich skeleta. They are constructed from models of X over the ring of integers O_K. Following this idea, Mustata and Nicaise recently introduced the notion of essential skeleton of X: it is a finite simplicial complex embedded in X^{an} that enjoys three nice properties. Firstly, it is contained in the Berkovich skeleton of every mildly singular model. Secondly, it is a strong deformation retract of X^{an} by the work of Nicaise and Xu. Finally, it is a birational invariant of X.

In my talk I will focus on a concrete example. I will describe the essential skeleton of the Hilbert scheme of a K3 surface S in terms of the essential skeleton of S, only using techniques from valuation theory and the notion of weight function. Furthermore, the key argument consists in proving that the essential skeleton of the n-th symmetric product of S is isomorphic to the n-th symmetric product of the essential skeleton of S.

Ben Smith, *Tropical Oriented Matroids and Products of Simplices*

Oriented matroids are a combinatorial object that encode data arising from a hyperplane arrangement. We can similarly define the tropical analgoue arising from tropical hyperplane arrangements in the tropical torus. It was conjectured by Ardilla and Develin (later proved by Horn) that tropical oriented matroids are in bijection with polyhedral subdivisions of products of simplices, giving us a concrete combinatorial characterisation of them. We will consider a generalisation of tropical oriented matroids arising from hyperplane arrangements in tropical projective space and investigate whether a similar polyhedral bijection exists.

Alejandro Soto, *Projective models of abelian varieties over arbitrary rank one valuation rings*

Projective regular models of abelian varieties were constructed by Künnemann over a discrete valuation ring. We will discuss this problem over an arbitrary rank one valuation ring. For this, we will discuss Bertin's definition of regularity for nonnoetherian domains and the notion of rigid-noetherianess introduced by Fujiwara-Gabber-Kato.

Yuto Yamamoto, ***Tropical homologies of hypersurfaces in algebraic tori***

Tropical homology is a homology theory for tropical manifolds. It is known that ranks of tropical homologies of tropical projective manifolds coincide with Hodge numbers of generic fibers of corresponding degenerating families of complex projective manifolds. In this talk, we calculate tropical homologies of smooth hypersurfaces in algebraic tori. We also see that they coincide with limit mixed Hodge numbers of corresponding degenerating families.