ARAKELOV GEOMETRY - ARCHIMEDEAN AND NON-ARCHIMEDEAN ASPECTS

Regensburg, September 5th to 9th 2016

Organizers: José Ignacio Burgos Gil, Walter Gubler, Guido Kings, and Klaus Künnemann

Monday		
9h00 - 9h30		Registration and Welcome
9h30 - 10h30	J.-B. Bost	Constructing vector bundles on arithmetic solids
11h00 - 12h00	H. Chen	Relative Brunn-Minkowski inequality in arithmetic geometry
14h00 - 15h00	R. de Jong	Arakelov invariants of degenerating curves
15h15 - 16h15	R. Wilms	New formulas for Faltings' delta invariant
16h30 - 17h30	P. Jell	Real – valued de Rham cohomology of Berkovich spaces
Tuesday		
9h30 - 10h30	D. Rössler	A new approach to the Grothendieck-Riemann-Roch theorem
11h00 - 12h00	G. Freixas	Flat line bundles and arithmetic intersection theory
14h00 - 15h00	M. Sombra	An arithmetic Bernstein-Kushnirenko inequality
15h15 - 16h15	C. Gasbarri	Liouville's inequality for transcendental points on projective varieties
16h30 - 17h30	A. Navarro	Revisiting higher arithmetic Riemann-Roch theorem
Wednesday		
9h30 - 10h30	M. Maculan	Maximality of hyperspecial compact subgroups avoiding Bruhat-Tits theory
11h00 - 12h00	S. Kawaguchi	Effective faithful tropicalizations associated to linear systems
13h30 - 18h30		Excursion to Weltenburg Abbey
Thursday		
9h30 - 10h30	U. Kühn	$\overline{\omega}^2$ on Fermat curves and some super elliptic curves
11h00 - 12h00	J. Kramer	Mumford's theorem on mixed Shimura varieties: the case of a line bundle on the universal elliptic curve
14h00 - 15h00	A. von Pippich	An analytic class number type formula for $PSL_2(\mathbb{Z})$
15h15 - 16h15	Y. Liu	Algebraic and tropical de Rham theories of non-Archimedean spaces
16h30 - 17h30	J. Scholbach	Modules over the de Rham cohomology spectrum
19h00		Conference Dinner *Restaurant Bischofshof*
Friday		
9h30 - 10h30	F. Pazuki	Bad reduction of curves with CM jacobians
11h00 - 12h00	F. Martin	Differentiability of non-archimedean volumes
14h00 - 15h00	E. Viada	Rational points on curves
15h15 - 16h15	C. Liu	Counting multiplicities in a hypersurface
16h30 - 17h30	D. Scarponi	The realization of the degree zero part of the motivic polylogarithm

All lectures take place in lecture room H51 in the biology department!
During the breaks there is coffee in the lobby of the lecture room.

This conference is supported by the *SFB 1085 Higher Invariants - Interactions between Arithmetic Geometry and Global Analysis* and constitutes the eighth session of the Intercity Seminar on Arakelov Theory organized by José Ignacio Burgos Gil, Vincent Maillot and Atsushi Moriwaki with previous sessions in Barcelona, Kyoto, Paris, and Rome.

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Abstracts

Jean-Benoît Bost: Constructing vector bundles on arithmetic solids

This talk will describe some constructions of vector bundles on surfaces over number fields, starting from analytic and formal data. These constructions rely on the properties of suitable 'infinite rank hermitian vector bundles' and of their invariants defined in terms of theta-functions.

Huayi Chen: Relative Brunn-Minkowski inequality in arithmetic geometry

The classical Brunn-Minkowski inequality asserts that volume function of line bundles on a projective variety is log-concave. In the setting of arithmetic geometry over a function field or a number field, the relative structure permits to obtain finer inequalities. In this talk, I will explain some generalizations of the Brunn-Minkowski inequalities in the relative setting.

Gerard Freixas i Montplet: Flat line bundles and arithmetic intersection theory

The Néron-Tate pairing is defined on the rational points of the Jacobian of a curve, and its construction makes use of the existence and (almost) uniqueness of a flat hermitian metric on a line bundle of degree 0 on a compact Riemann surface (i.e. at the archimedean places). This datum is equivalent to a unitary holomorphic connection. The Jacobian is covered by the so-called universal vectorial extension, that can be seen as the moduli space of line bundles with (algebraic) connections on the curve. It is thus tempting to try to extend the Néron-Tate pairing to the universal vectorial extension, by using the intrinsic connection data of its rational points. This leads to consider the following problem: extend arithmetic intersection theory to vector bundles endowed with flat holomorphic connections at the archimedean places. In this talk I will report on joint work with Richard A. Wentworth (University of Maryland), where we address this question on arithmetic surfaces and in rank 1. We will define the 'admissible' objects in the theory and define an arithmetic intersection pairing. We show that it naturally appears in an arithmetic Riemann-Roch theorem, where the determinant of cohomology no longer comes equipped with the Quillen metric, but with a rather new complex valued version of it, introduced by Cappell-Miller. This way we answer the questions of these authors, whether their torsion object enjoys of analogous properties as the holomorphic analytic torsion in the work of Bismut, Gillet and Soulé.

Carlo Gasbarri: Liouville's inequality for transcendental points on projective varieties

Liouville inequality is a lower bound of the norm of an integral section of a line bundle on an algebraic point of a variety. It is an important tool in may proofs in diophantine geometry and in transcendence. On transcendental points an inequality as good as Liouville inequality cannot hold. We will describe similar inequalities which hold for manytranscendental points.

Philipp Jell: Real-valued de Rham cohomology of Berkovich spaces

Chambert-Loir and Ducros defined real-valued differential forms on Berkovich analytic spaces. These forms should be thought of as analogues of (p,q)-differential forms on complex manifolds. They are defined by formally pulling back superforms on tropical varieties along tropicalization maps. We establish the basic cohomological results for these differential forms. This includes a Poincaré lemma as well as finite dimensionality of the cohomology in certain bidegrees. If time permits we will explain a new approach to this theory, which uses tropicalizations of affine space respectively toric varieties.

Robin de Jong: Arakelov invariants of degenerating curves

In 1984 G. Faltings introduced the delta-invariant of a compact and connected Riemann surface, and asked about its asymptotic behavior near the boundary of the moduli space of curves. Generalizing previous work in this direction from around 1990 by J. Jorgenson and R. Wentworth we present in this talk an explicit asymptotic result for the delta-invariant for arbitrary 1-parameter semi-stable degenerations. Our result shows in particular that the delta-invariant does NOT behave like a good metric on a rational combination of the boundary divisors. Along the way we also present the asymptotics of the Arakelov metric and the Arakelov-Green's function. In each case, the leading terms of the asymptotics are controlled by S. Zhang's admissible pairing on the generic fiber of the degenerating family.

Shu Kawaguchi: Effective faithful tropicalizations associated to linear systems

For a smooth projective curve X of genus $g \geq 2$, global sections of any line bundle L with $\deg(L) \geq 2g+1$ give an embedding of the curve into projective space. We consider an analogous statement in non-Archimedean geometry, in which we study when global sections of a line bundle give a homeomorphim of a Berkovich skeleton (that is allowed to have ends) into tropical projective space preserving the piecewise affine Z-structures (called a faithful tropicalization of a Berkovich skeleton). Let K be an algebraically closed field which is complete with respect to a non-trivial non-Archimedean value. Suppose that X is defined over K, and that Γis a Berkovich skeleton (with possible ends). We show that, if $deg(L) \geq 5g-4$, then global sections of L give a faithful tropicalization of Γ into tropical projective space. If time permits, we would also like to talk on a version in higher dimension. This is a joint work with Kazuhiko Yamaki.

Ulf Kühn: $\bar{\omega}^2$ on Fermat curves and some super elliptic curves

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Jürg Kramer: Mumford's theorem on mixed Shimura varieties: The case of a line bundle on the universal elliptic curve

In the case of a Shimura variety X of non compact type, by a theorem of Mumford, automorphic vector bundles equipped with the natural invariant metric have a unique extension to vector bundles over a toroidal compactification of X equipped with a logarithmically singular hermitian metric. This result is crucial to define arithmetic Chern classes for these vector bundles. It is natural to ask whether Mumford's result remains valid for 'automorphic' vector bundles on mixed Shimura varieties.
In our talk we will examine the simplest case, namely the Jacobi line bundle on the universal elliptic curve, whose sections are Jacobi forms. We will show that Mumford's result cannot be extended directly to this case since a new type of metric singularity appears. By using the theory of b-divisors, we show however that an analogue of Mumford's extension theorem can be obtained in this setting.

Chunhui Liu: Counting multiplicities in a hypersurface

In this talk, I will give an explicit upper bound of a counting function of multiplicity of rational points in a projective hypersurface over a finite field, and I will explain why I choose this counting function. This work is motivated by a problem of counting rational points of a arithmetic variety by the approach of Arakelov geometry. If time permits, I will explain the relation of these two counting problems.

Yifeng Liu: Algebraic and tropical de Rham theories of non-Archimedean spaces

We show that de Rham cohomology sheaves of a smooth non-Archimedean space have a canonical functorial decomposition through local weights. In particular, this answers a question raised by Berkovich for 1-forms. We also reveal a connection between de Rham cohomology sheaves and the complex of real forms defined by Chambert-Loir and Ducros. As an application, we show that integrations of closed real forms on an algebraic cycle vanish if the cycle is cohomologically trivial (in algebraic de Rham cohomology).

Marco Maculan: Maximality of hyperspecial compact subgroups avoiding Bruhat-Tits theory

Metrics on line bundles over non-archimedean fields were introduced in Arakelov geometry to give a uniform treatment of finite and complex places. In this talk we will use them to prove the following result: given a reductive group G over a discrete valuation ring A with fraction field K, the group $G(A)$ is a maximal bounded subgroup of $G(K)$. The advantage of this geometric point of view is that it frees the argument of the combinatorics of Tits systems, which are involved in the construction of the Bruhat-Tits building.

Florent Martin: Differentiability of non-Archimedean volumes

If L is a line bundle on a projective variety Y of dimension n, the volume of L is defined as

$$\mathrm{vol}_Y(L) := \lim_{m \in N} \frac{h^0(L^m)}{m^n/n!}.$$

This can be continuously extended to the real Néron-Severi group of Y (which is a finite dimensional real vector space) and Boucksom, Favre and Jonsson proved that $\mathrm{vol}_Y(\cdot)$ is even C^1. If Y is additionally a complex analytic variety, and L is equipped with a metric $\|\cdot\|$, Boucksom and Berman have proved similar properties for the asymptotic growth of the set of global sections s of L^m with $\|s\| \leq 1$ everywhere.
I will explain analogous results when Y is an analytic variety over a discretely valued non-Archimedean field, thus proving a formula conjectured by Kontsevich and Tschinkel. The main motivation is to extend some results of Boucksom, Favre and Jonsson about non-Archimedean Monge-Ampère equations. This is a joint work with J. Burgos, W. Gubler, P. Jell and K. Künnemann.

Alberto Navarro: Revisiting higher Arithmetic Riemann-Roch theorem

Holmstrom and Scholbach constructed an Arakelovian version of motivic cohomology for arithmetic varieties and proved a higher arithmetic Riemann-Roch theorem. In this talk we will revisit this result and set up a framework so that its proof will allow us to obtain new Riemann-Roch theorems in the classic geometric setting.

Fabien Mehdi Pazuki: Bad reduction of curves with CM jacobians

An abelian variety defined over a number field and having complex multiplication (CM) has potentially good reduction everywhere. If a curve of positive genus which is defined over a number field has good reduction at a given finite place, then so does its jacobian variety. However, the converse statement is false already in the genus 2 case, as can be seen in the entry $[I_0 - I_0 - m]$ in Namikawa and Ueno's classification table of fibres in pencils of curves of genus 2. In this joint work with Philipp Habegger, our main result states that this phenomenon prevails for certain families of curves.
We prove the following result: Let F be a real quadratic number field. Up to isomorphisms there are only finitely many curves C of genus 2 defined over $\overline{\mathbb{Q}}$ with good reduction everywhere and such that the jacobian Jac(C) has CM by the maximal order of a quartic, cyclic, totally imaginary number field containing F. Hence such a curve will almost always have stable bad reduction at some prime whereas its jacobian has good reduction everywhere. A remark is that one can exhibit an infinite family of genus 2 curves with CM jacobian such that the endomorphism ring is the ring of algebraic integers in a cyclic extension of $\mathbb{Q}$ of degree 4 that contains $\mathbb{Q}(\sqrt{5})$.

Anna von Pippich: An analytic class number type formula for $\mathbf{PSL}_2(Z)$

For any Fuchsian subgroup $\Gamma \subseteq \mathrm{PSL}_2(R)$ of the first kind, Selberg introduced the Selberg zeta function in analogy to the Riemann zeta function using the lengths of simple closed geodesics on $\Gamma \backslash H$ instead of prime numbers. In this talk, we report on a generalized Riemann-Roch isometry for the trivial sheaf on $\Gamma \backslash H$, equipped with the Poincaré metric. As arithmetic application, we determine the special value at $s = 1$ of the derivative of the Selberg zeta function in the case $\Gamma = \mathrm{PSL}_2(R)$. This is joint work with Gerard Freixas.

Damian Rössler: A new approach to the Grothendieck-Riemann-Roch theorem

We shall present a new proof of the Grothendieck-Riemann-Roch theorem, which is intrinsic, in the sense that it does not depend on factorisations of morphisms through projective space. It is also possible to put hats on this proof.

Danny Scarponi: The realization of the degree zero part of the motivic polylogarithm on abelian schemes in Deligne-Beilinson cohomology

We use Burgos theory of arithmetic Chow groups to exhibit a realization of the degree zero part of the polylogarithm on abelian schemes in Deligne-Beilinson cohomology.

Jakob Scholbach: Modules over the de Rham cohomology spectrum

We show that the bounded derived category of regular holonomic D-modules on a smooth variety is equivalent to the category of constructible modules over the de Rham cohomology spectrum. This equivalence preserves the 6 functor formalism and therefore extends the classical 6 functors on regular holonomic D-modules to not necessarily constructible objects. Another application is a motivic t-structure on modules over the de Rham spectrum. This is a joint project with Dmitri Pavlov.

Martín Sombra: An arithmetic Bernstein-Kushnirenko inequality

I will present an upper bound for the height of the isolated zeros in the torus of a system of Laurent polynomials over an adelic field satisfying the product formula. This upper bound is expressed in terms of the mixed integrals of the local roof functions associated to the chosen height function and to the system of Laurent polynomials. This result can be seen as an arithmetic analogue of the classical Bernstein-Kushnirenko theorem. Its proof is based on arithmetic intersection theory on toric varieties. This is joint work with César Martínez (Caen).

Evelina Viada: Rational points on curves

We prove a sharp explicit height bound for the points of rank one on curves embedded in a product of elliptic curves. We then show how this can be used to find all the rational points on curves of genus at least 2 embedded in E^N with $E(Q)$ of rank one; we finally present a variety of non-trivial explicit curves for which we list all the rational points.

Robert Wilms: New formulas for Faltings' delta invariant

Faltings' delta invariant of compact Riemann surfaces plays a crucial role in Arakelov geometry of arithmetic surfaces. For example, it appears in the arithmetic Noether formula. We will give new formulas of Faltings' delta invariant in terms of integrals of theta functions. This has several applications: We obtain a lower bound of delta only in terms of the genus and we give a canonical extension of delta to the moduli space of abelian varieties. As an application to Arakelov theory, we can bound the heights of Weierstraß points in terms of the Faltings height."