Graduate Seminar on New Developments in PDE (S5B3)

Weak Convergence Methods

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Nonlinear partial differential equations arise in many physical models and give rise to challenging mathematical problems. The analysis of such equations often requires to take limits, which however, are usually only weak limits. For example, in order to construct solutions to a nonlinear PDE of the form

$$\mathcal{A}[u] = f,$$

where f is some given function and $\mathcal{A}$ is some nonlinear differential operator, we often approximate the operator $\mathcal{A}$ by a sequence $\mathcal{A}_k$ and seek to pass to the limit $k \to \infty$ for solutions u_k to the approximate PDE

$$\mathcal{A}_k[u_k] = f.$$

Usually, a priori bounds guarantee that (a subsequence of) u_k converges weakly to some function u. In order to pass to the limit in nonlinearities (and verify that u is indeed a solution), this weak convergence is insufficient and one has to better understand the structure of the specific nonlinearity.

In this seminar, we want to study techniques to tackle such problems. The overall philosophy is to use real analysis and in particular measure theory to get a better understanding of the oscillations in the weak convergence. Sometimes, we will want to localize where in physical space the weak convergence fails to be strong. Sometimes, we will want to localize in Fourier space in order to see in which directions the sequence u_k oscillates.

Possible topics for talks reach from fundamental classical results to recent advances in the field. These topics will be introduced in the preliminary meeting and distributed according to the background and interest of participants.

Prerequisites: Partial Differential Equations (in particular Sobolev spaces), Functional Analysis (in particular compactness, weak convergence, duality), Measure Theory (in particular compactness and differentiation of measures)

Venue: Summer term 2021 (online).

Preliminary meeting: February 26, 10:15am–noon via Zoom.
Link
Meeting ID: 930 5983 4504
Passcode: PDE