Federal Coordination of Local Housing Demolition in the Presence of Filtering and Migration

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Abstract: Vacant housing and even housing demolition have recently become an issue in a number of countries. Given this renewed interest in demolition, this paper contributes to the literatures on (i) housing demolition and (ii) policy coordination. The paper extends Sweeney (1974a)'s analysis of demolition and filtering, by letting households also choose their location. Then when demolishing part of its housing stock, a city effectively evicts some of its residents not just out of the housing quality it demolishes but out of every other of its qualities, too. The paper shows that demolition's coordination strengthens local governments' incentive to demolish part of their stock, by shutting down inter city migration within qualities. – A case study on Germany's East illustrates the effects of coordinated, simultaneous every-city demolition.

JEL - Classifications: H73, D61, R21
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1 Introduction

Vacant housing has become an issue in a number of countries today. Following the recent real estate bust, a substantial fraction of US, Irish and Spanish housing is vacant today. Similarly, if for a very different reason, more than one tenth of East Germany's housing has long been vacant, following reunification. Unsurprisingly, the proposal of demolishing vacant housing has surfaced in each of these markets. In fact, Germany has even gone as far as levelling some 250,000 flats in her East since 2002, with another 220,000 flats due to follow over the next four years. Yet demolition is also debated in the US (e.g., Glaeser/Gyourko (2008)) and in Ireland (e.g., Irish Independent (2011)).

Given this renewed interest in demolition, this paper contributes to the literatures on housing demolition and policy coordination. On the one hand, the paper extends the analysis of demolition introduced in Sweeney (1974), and subsequently developed in the filtering literature following it, in three ways. First, in this paper demolition proceeds in a context where households choose not just the quality of their housing but that housing's lot size and spatial location, too. Moreover and second, demolition is no longer restricted to a single city here but may instead simultaneously unfold in many cities, or in even every city. Finally, and as a third innovation with respect to the Sweeney type literature, this paper submits the filtering encountered in models of multiple housing qualities to the objectives and policies of landlord run governments.

On the other hand, the paper also adds to the local public finance literature on local governments' policies, by introducing a variety of different housing qualities into the local jurisdiction framework. With residents mobile across space, any given city's attempt to improve its overall rental income creates fiscal externalities. The paper offers a novel analysis of how such externalities, and the benefits from centralizing decision making (Oates (2011)), operate in the presence of multiple housing qualities and demolition. Empirically, interpreting the large scale demolition programs long underway (Germany) or currently discussed (in the US, Ireland, and Spain) may serve as a particularly striking and relevant illustration of real world policy coordination.

Now, why would countries demolish their vacant housing anyway, rather than watching prices and rents fall, say? Certainly in the US, Ireland and Spain often vacant structures are very modern, having been put up only recently. Nor is East Germany's vacant housing of the worst quality. Intriguing and important as this question may be, in this paper we largely bypass it. We simply assume that local governments are governed by landlords who want to raise – or at least sustain – total local rents. A detailed analysis of the politics underlying various demolition policies – with homevoters' role in these politics on center stage – can be found in Dascher (2012).

Instead this paper focuses on how landlord-ruled cities may best sustain, or even raise, their rental incomes if individuals make simultaneous decisions on housing quality, floor space, and residential location. The paper's model permits us to trace filtering across housing qualities, migration across cities, and rental revenues across governments. In permitting simultaneous filtering and migration, this model puts us in the position to analyze the dilemma a single city faces when pondering demolition.

A single city may demolish some of the stock in its lowest quality segment on its own, without other cities going along. Let us suppose that the city targets vacant housing but out of necessity also needs to demolish some occupied housing. (Buildings rarely are completely vacant.) Then it is certainly true that such demolition will raise rents across all of that city's qualities. On the other hand, this attendant increase in rents will drive residents not only out of the segment about to be demolished but out of every better segment, too. Here coordination, by making cities agree to demolish simultaneously, shuts down the avenue of inter city flight.

We append a case study on Germany's East, to illustrate the paper's concepts. Germany's federal government has introduced new legal instruments, facilitating tenants' eviction from housing earmarked for demolition as well as facilitating the rezoning of residential land. But the federal government also directly subsidizes demolition. In response to these incentives, every second resident now lives in a city which has witnessed at least some demolition (BMVBS (2006)).

Moreover, and second, the case study shows that demolition is not simply meant to dispense with vacant, unwanted housing. If demolition were merely about tearing down flats nobody wants then nobody would miss the flats torn down and hence rents could never rise. Yet in the context of a large data micro data set on rental housing across Germany's regions we find that East Germany's rent, far from keeping in line with West Germany's, *rises* during the period of ongoing demolition. We interpret this as preliminary evidence that demolition is indeed meant to sustain rent in the face of excess supply.

We briefly review this paper's relationship with the literature on demolition and filtering. Sweeney (1974a), and then Ohls (1975), Braid (1981), Braid (1984), Kaneko (2006) and Ito (2007) all discuss households' housing quality choice, too, assuming varying combinations of preference and income heterogeneity. However, these papers neither allow for spatial competition between cities, nor do they allow for households' endogenous choice of floor space. Both Arnott/Davidson/Pines (1986) as well as Arnott/Braid/Davidson/Pines (1999) do consider models where households may choose housing quality and quantity, and where residents also pick their location within the urban area. Yet in their papers, too, is analysis restricted to a single city only.

To the best of the author's knowledge, local demolition efforts do not feature in the public finance literature on policy centralization. Starting with Oates (2011), this literature has focused on issues such as the coordination of taxation in a framework of increasing capital mobility (Zodrow/Mieszkowski (1986)), or as the coordination of fiscal policies in a framework of growing international trade. Yet of course demolition in one city generates much the same type of positive externalities in neighboring cities as does, say, an increase in government expenditures. Cooper/John (1988) offer one systematic treatment of policy coordination in the presence of externalities.

The paper's layout is as follows. Section 2 presents the model, section 3 has the case study, section 4 offers conclusions and section 5 gives the appendix for proposition proofs, respectively.

2 Model

The model is laid out in three subsections. Subsection 2.1 gives the basic setup. Subsection 2.2 explains how local policies affect households' simultaneous location and quality choices. And subsection 2.3 analyzes the role of federal coordination of local policies.

2.1 Basic Assumptions

We introduce a filtering model that incorporates both migration across cities and mobility between housing qualities. Let every city supply a range of $i = 1, \ldots, I$ different qualities of housing. Cities are small, open, and indexed by $j = 1, \ldots, J$, where J is a large number. A pair (i, j) indicates a housing segment's quality and location. The number of those living in city j's segment i is n_i^j, with the total number of city j's residents given by $n^j = \sum_{i=1}^{I} n_i^j$. The total number of the urban system's residents is $N = \sum_{j=1}^{J} n^j$, and fixed.

Rent for a square meter of housing of quality i in city j is q_i^j, and the complete list of rents in city j is $q^j = (q_1^j, \ldots, q_I^j)$. Qualities are numbered in ascending order: 1 refers to the lowest, while I refers to the highest, quality. Quality is measurable on a cardinal scale, with s_i denoting the quality of the i-th segment. Frequently we will refer to quality 1 as simply the low quality.

A total of N/J households initially is born into each city. These households, or natives, are heterogenous with respect to a housing quality taste index θ and the moving cost m they incur when settling elsewhere. Specifically, natives to any city are distributed according to the joint c.d.f. $F(\theta, m) : [\theta', \theta''] \times [0, m'] \mapsto [0, 1]$, with $0 < \theta' < \theta''$ and $0 < m'$. F is assumed location-independent, and to exhibit strictly positive partial derivatives. Marginal distributions are denoted $F(\theta)$ and $F(m)$, and respective marginal densities are $f(\theta)$ and $f(m)$. Because we take θ and m to be independent, $F(\theta, m) = F(\theta)\, F(m)$.

To keep the multitude of potential moves tractable, we impose a restriction on inter city mobility popular in regional economics. Cities are arranged on a circle and then numbered consecutively from 1 to J in clockwise fashion. Households may move along this circle, but from their native city j to neighbor $j+1$ only. – All households have identical preferences. Let us focus on a given city's households, thus dropping the city index for now. Household utility in segment i is

$$\theta s_i \, + \, r(h_i, \, s_i) \, + \, x_i \tag{1}$$

Utility depends on quality s_i, on floor space h_i, on the numéraire x_i as well as on the taste index θ. Besides $r_h, r_s > 0$ we assume $r_{hs} > 0 > r_{hh}$.

Income w is exogenous and the same for every household. All residents are renters because all land is owned by absentee landlords. A resident opting for quality i has budget constraint $x_i = w - q_i h_i$. Demand is found by solving $r_h(h_i, s_i) = q_i$ for h_i. Since r_h is strictly decreasing in h_i (and differentiable), its inverse exists and is also strictly decreasing (and differentiable). Inverting for h_i gives demand for floor space $h(q_i, s_i)$, with $h_q(q_i, s_i) \leq 0$. (We restrict attention to utility functions for which $h_q(q_i, s_i) < 0$.) Demand for floor space falls as rent rises. Maximum utility in quality s_i is

$$V(\theta, s_i, q_i) \;=\; \theta s_i + v(s_i, q_i) + w \tag{2}$$

where $v(s_i, q_i) = r(h(q_i, s_i), s_i) - q_i h(q_i, s_i)$.

Finally, existing housing in segment i, S_i, exhibits homogeneous quality but has operating cost c that vary across units. Ordering square meters by c gives a strictly increasing function $c(S_i)$. Inverting it, and accounting for the fact that only housing with operating cost c below rent q_i will actually be offered, gives a strictly increasing housing supply function $S_i(q_i)$. This we assume to be differentiable, to exhibit $\partial S_i / \partial q_i > 0$ throughout its domain, and to be the same in every city. Here rising rent expands supply only by drawing units into the market that were vacant previously. Changes in rent have no effect on construction, which is set to zero.

Suppose for the moment that for a given quality rents are equal across cities. Let these identical rents carry a bar, to distinguish them from rents that do not share their symmetry: $\bar{q}_i^j \;=\; \bar{q}_i$ for all j and i. Then for every quality indirect utilities are the same across cities, too. Moreover, within any given city households indifferent between neighboring housing qualities i and $i+1$, with $i, i+1 \in \{1, \ldots, I\}$, are identified by setting

$$V(\theta, s_{i+1}, \bar{q}_{i+1}) = V(\theta, s_i, \bar{q}_i) \tag{3}$$

and solving for $\theta(\bar{q}_i, \bar{q}_{i+1})$, or shorter, $\bar{\theta}_{i,i+1}$. Letting i run from 1 to $I-1$ identifies all such boundaries, i.e., $\bar{\theta}_{1,2}, \bar{\theta}_{2,3}, \ldots, \bar{\theta}_{I-1,I}$. For notational convenience we define $\bar{\theta}_{0,1} = \theta'$ as well as $\bar{\theta}_{I,I+1} = \theta''$. Now, V can be shown to be strictly decreasing in q and to be strictly increasing in s (Properties (ii) and (v) in Lemma 1 in Appendix A). But then, following (3), better quality must command higher rent. Moreover,

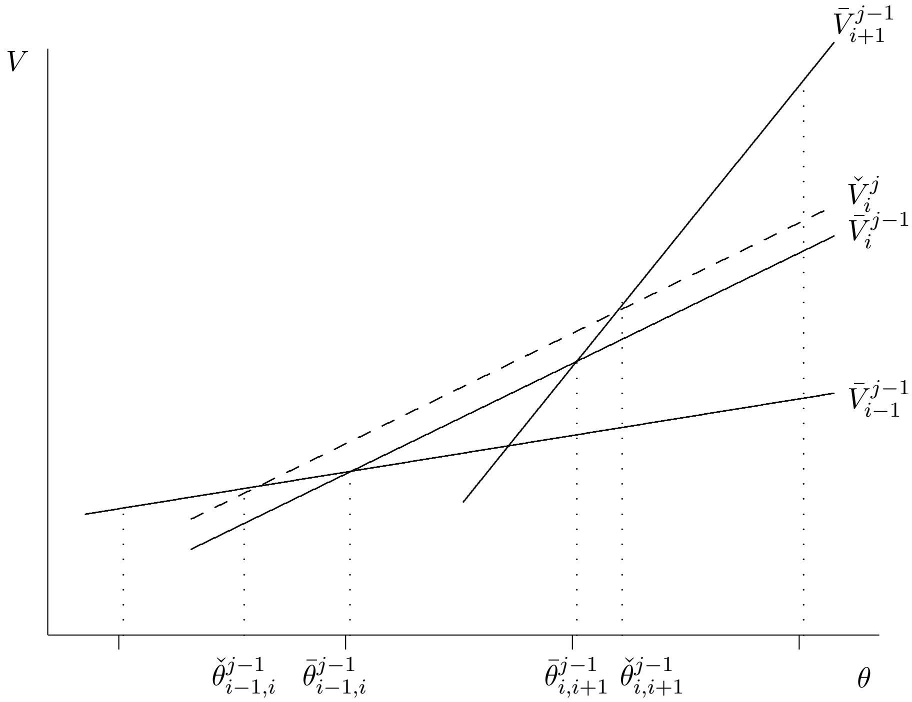

Figure 1: Household Sorting in City $j-1$

Proposition 1 (Resident Sorting By Quality):

Suppose for every quality rent is the same across cities. Then residents within any given city and with taste index θ in the interval $[\bar{\theta}_{i-1,i},\ \bar{\theta}_{i,i+1}]$ sort into housing of quality i, where $i = 1, \ldots, I$.

Figure 3 illustrates such sorting for city $j-1$. (Ignore the Figure's dashed line for the moment, as well as all notation not carrying a bar. These concepts are later employed in the proof of Proposition 2.) Indirect utilities in this city's segments $i-1$, i and $i+1$ are illustrated as straight lines in (θ, V)-space. Corresponding boundaries are shown as $\bar{\theta}_{i-1,i}^{j-1}$ and $\bar{\theta}_{i,i+1}^{j-1}$, respectively. Now, given that cities offer identical menus of utilities no household will want to move from one city to another. The number of residents in segment (i, j) thus simply becomes

$$\bar{n}_i^j = \Big(F(\bar{\theta}_{i,i+1}) - F(\bar{\theta}_{i-1,i})\Big)\ N/J \tag{4}$$

Moreover, with the exception of $\bar{\theta}_{0,1}$ and $\bar{\theta}_{I,I+1}$ every boundary shifts with the two rents governing the segments it separates. Differentiating $\theta(q_i, q_{i+1})$ with respect to q_i and q_{i+1}, at $\bar{q}_i$ and $\bar{q}_{i+1}$, gives

$$\frac{\partial\theta(\bar{q}_i, \bar{q}_{i+1})}{\partial q_i} = \frac{v_{q_i}(s_i, \bar{q}_i)}{s_{i+1} - s_i} < 0 \quad \text{and} \quad \frac{\partial\theta(\bar{q}_i, \bar{q}_{i+1})}{\partial q_{i+1}} = -\frac{v_{q_{i+1}}(s_{i+1}, \bar{q}_{i+1})}{s_{i+1} - s_i} > 0 \tag{5}$$

for $i = 1, \ldots, I-1$. Intuitively, raising segment i's rent reduces, while raising its upper neighbor's rent increases, that segment's market share.

To analyze how small departures of q_i^j from $\bar{q}_i^j$ affect n_i^j we need to distinguish two cases. On the one hand, for rents $\hat{q}_i^j$ above $\bar{q}_i^j$ we have $V(\theta, s_i, \hat{q}_i^j) < V(\theta, s_i, \bar{q}_i^{j+1})$. Hence there must certainly also be migration from segment (i, j) to $(i, j+1)$. On the other hand, for rents $\check{q}_i^j$ below $\bar{q}_i^j$ we have $V(\theta, s_i, \check{q}_i^j) > V(\theta, s_i, \bar{q}_i^{j-1})$. In this case there must certainly also be immigration from segment $(i, j-1)$, and possibly even other city $j-1$ segments, into (i, j). Given this asymmetry of migratory responses, the derivative of the segment population n_i^j with respect to q_i^j at $\bar{q}_i^j$ may depend on from which direction $\bar{q}_i^j$ is approached and, hence, need not exist. Fortunately, Proposition 2 shows that this derivative does exist.

Proposition 2 (Segment Population and Rent): *The partial derivative of the number of residents of segment i in city j, n_i^j, with respect to this segment's rent, q_i^j, at the initial symmetric allocation exists, and is equal to*

$$\begin{aligned}\frac{\partial n_i^j(\bar{q}^j)}{\partial q_i^j} \;&=\; \bar{n}^j\, f(\bar{\theta}^j_{i,i+1})\frac{\partial \theta^j(\bar{q}_i^j, \bar{q}_{i+1}^j)}{\partial q_i^j} \;-\; \bar{n}^j\, f(\bar{\theta}^j_{i-1,i})\frac{\partial \theta^j(\bar{q}_{i-1}^j, \bar{q}_i^j)}{\partial q_i^j} \\ &+\; \bar{n}_i^j\, \left(F(\bar{\theta}^j_{i,i+1}) - F(\bar{\theta}^j_{i-1,i})\right)\, f(\bar{m}_i)\, \frac{\partial V(\bar{q}_i^j)}{\partial q_i^j} \;<\; 0 \qquad (6)\end{aligned}$$

where $\bar{m}_i = 0$.

The derivative's sign follows from the properties set out in (5). Its three terms have an intuitive interpretation. Consider a one Euro rise in q_i^j, for instance. This, first, drives housing consumers away into rivalling segment $(i+1, j)$, with the magnitude of household loss indicated by the first term on the r.h.s. of (6). Second, it drives housing consumers away into rivalling segment $(i-1, j)$, as the second term on the r.h.s. of (6). And third, it makes consumers emigrate to segment $(i, j+1)$, as captured by the third term on the r.h.s. of (6). – Proposition 3 sets out the two other partials of n_i^j, again with signs inferred from consulting (5).

Proposition 3 (Segment Population and Neighboring Segments' Rents): *The partial derivative of the number of residents of segment i in city j, n_i^j, with respect to the next-higher segment's rent, q_{i+1}^j, at the initial symmetric allocation exists, and is equal to*

$$\frac{\partial n_i^j(\bar{q}^j)}{\partial q_{i+1}^j} = \bar{n}^j\, f(\bar{\theta}^j_{i,i+1})\frac{\partial \theta^j(\bar{q}_i^j, \bar{q}_{i+1}^j)}{\partial q_{i+1}^j} \;>\; 0 \qquad (7)$$

Likewise, the partial derivative of n_i^j with respect to the next-lower segment's rent, q_{i-1}^j, at the initial symmetric allocation exists, and equals

$$\frac{\partial n_i^j(\bar{q}^j)}{\partial q_{i-1}^j} = -\,\bar{n}^j\, f(\bar{\theta}^j_{i-1,i})\frac{\partial \theta^j(\bar{q}_{i-1}^j, \bar{q}_i^j)}{\partial q_{i-1}^j} \;>\; 0 \qquad (8)$$

Since city j is small, rent changes induced in neighboring cities by manipulating q_i^j, q_{i-1}^j or q_{i+1}^j will not feed back into j. Nor do they affect rents in any of the many cities elsewhere. Demand spillovers into either of the two neighboring cities $j-1$ and $j+1$ continue to travel along the – large – circle, either in clockwise or in counterclockwise fashion, to fade away eventually. Now, with the derivation of derivatives complete we no longer need to employ the bar notation. Derivatives' evaluation at a symmetric allocation is always implicitly understood.

2.2 Rents and Filtering in a Model of Spatial Mobility

This section analyzes local government's policies. We assume that local government may either set the price of low quality housing q_1^j. Or local government may tear down, and hence evict tenants from, low quality housing. This restricting policies to interfere with low quality housing only may itself seem restrictive. Yet from a modeling perspective, the opposite is true. Local governments are able to manipulate rents in every segment even if their policies are restricted to the low quality segment. It is obvious that local governments are able to manipulate rents in every segment if they may directly reign into that segment.

In the model below demolition is always directed against non-vacant housing. One might argue that in the current discussion demolition typically is about vacant housing. Yet as long as rents have not fully come down (the paper's case study gives one example of this) those units that are currently vacant and about to be demolished might well be units sought after if rents were lower. Put differently, our analysis really also addresses the case where demolition is targeted to units that are vacant yet might be consumed at lower rent.

Now let $-z_1^j > 0$ be the number of tenants evicted from low quality housing about to be demolished. For the sake of simple notation we suggest that local government picks housing units inhabited by immobile (e.g., older) tenants. Then $z_2^j = -z_1^j$ also is the number of those who apply for housing in segment $(j, 2)$. Moreover, $dz_2^j = -dz_1^j$. In any case, initially $z_1^j = z_2^j = 0$.

Aggregation over individual housing demands for quality i in city j yields $H_i^j = (n_i^j\,(q_{i-1}^j, q_i^j, q_{i+1}^j) + z_i^j)\; h(q_i^j, s_i)$, where $z_i^j = 0$ for all j, i initially. Rents $q_2^j, \ldots, q_I^j$

Variable	Magnitude and Sign	Household equivalents of ...
σ_i^j	$-\,n_i^j\left(F(\bar\theta_{i,i+1}^j) - F(\bar\theta_{i-1,i}^j)\right) f(\bar m_i)\frac{\partial V(q_i^j)}{\partial q_i^j} \;>\; 0$	... extra immigration into (i, j)
ε_i^j	$-\;n_i^j\,\frac{\partial h(q_i^j, s_i)}{\partial q_i^j}\,\frac{1}{h(q_i^j, s_i)} \;>\; 0$	... extra demand within (i, j)
η_i^j	$\frac{\partial S(q_i^j)}{\partial q_i^j}\,\frac{1}{h(q_i^j, s_i)} \;>\; 0$	... extra housing supply (i, j)

Table 1: Excess Housing Demand Changes Not Related to Filtering

are determined by perfect competition, satisfying

$$\Big(n_i^j(q_{i-1}^j, q_i^j, q_{i+1}^j) \,+\, z_i^j\Big)\; h(q_i^j, s_i) \;=\; S(q_i^j) \qquad \text{for } i = 2,\,\ldots\,,\, I \tag{9}$$

This system of $I-1$ equations jointly determines the $I-1$ rents observed on competitive housing markets. We assume that an equilibrium exists. Moroover, we also assume that this equilibrium specifies the same rents in every city $j = 1, \ldots, J$.

Next, differentiating (9) with respect to policy parameter $\alpha^j \in \{q_1^j, z_2^j\}$, inserting derivatives (6), (7) and (8), and rearranging gives the following set of $I-1$ equations

$$\begin{aligned} &\left[\Big(\sigma_i^j + \varepsilon_i^j + \eta_i^j\Big) \;-\; n^j f(\theta_{i,i+1}^j)\,\frac{\partial \theta_{i,i+1}^j}{\partial q_i^j} \;+\; n^j f(\theta_{i-1,i}^j)\,\frac{\partial \theta_{i-1,i}^j}{\partial q_i^j}\right] \frac{\partial q_i^j}{\partial \alpha^j} \\ =\;& n^j f(\theta_{i,i+1}^j)\,\frac{\partial \theta_{i,i+1}^j}{\partial q_{i+1}^j}\,\frac{\partial q_{i+1}^j}{\partial \alpha^j} \\ -\;& n^j f(\theta_{i-1,i}^j)\,\frac{\partial \theta_{i-1,i}^j}{\partial q_{i-1}^j}\,\frac{\partial q_{i-1}^j}{\partial \alpha^j} \;+\; \frac{\partial z_i^j}{\partial \alpha^j} \qquad \text{for } i = 2,\,\ldots\,,\, I \end{aligned} \tag{10}$$

with σ_i^j, ε_i^j and η_i^j defined and explained in Table 1. Moreover, $\partial z_i^j/\partial \alpha^j = -1$ if $\alpha^j = z_2^j$ and $i = 1$ and zero else.

Essentially, the first line of (10) collects the effects of the increase in q_i^j induced on market i. First, such an increase pushes residents into segment $(i, j+1)$, of magnitude σ_i^j (i.e., reflecting outmigration). Second, it reduces demand by those remaining in segment (i, j), by ε_i^j (i.e., individual demand response). Third, it raises supply in segment (i, j), by η_i^j units (i.e., aggregate supply response). And fourth, such an increase in q_i^j tends to push residents into neighboring segments $(i+1, j)$ and $(i-1, j)$ (i.e., filtering). In contrast, the effects of the increases induced in q_{i-1}^j or q_{i+1}^j are less varied. Either of these increases simply pushes households towards (i, j), as indicated by the second and third term on the r.h.s. of (10).

Endogenous variables in (10) may be summarized by the $(I-1) \times 1$-vector $y^j = (\partial q_2^j/\partial \alpha^j\,,\,..,\,\partial q_I^j/\partial \alpha^j)$. Let A^j be the $(I-1) \times (I-1)$ matrix of endogenous variables' coefficients. And let b^j be the $(I-1) \times 1$-column vector containing the policy induced shock to the second segment, equal to

$$b_1^j \;=\; \begin{cases} -1 & \text{if} \quad \alpha^j \;= z_2^j \\ \;n^j f(\theta_{1,2})(\partial \theta_{1,2}/\partial q_1^j) & \text{if} \quad \alpha^j \;=\; q_1^j \end{cases} \tag{11}$$

in its first row and zero elsewhere. This shock is strictly negative. Equipped with this notation, (10) can more compactly be written as $A^j\, y^j \;=\; b^j$. Proposition 4 explores the properties of this solution.

Proposition 4 (Dominos)
Rents in segments $i = 2, \ldots, I$ exhibit the following derivatives:

$$(i)\;\frac{\partial q_i^j}{\partial q_1^j} \;>\; 0 \qquad ; \qquad (ii)\;\frac{\partial q_i^j}{\partial z_2^j} \;>\; 0. \tag{12}$$

(i) (Raising Low Quality Rent) Higher segments' rents q_i^j are strictly increasing in low quality rent q_1^j.
(ii) (Demolishing Low Quality Stock) Higher segments' rents q_i^j are strictly increasing in low quality stock's demolition z_2^j.

In essence, Proposition 4 parallels Sweeney (1974a)'s Lemma 7. Sweeney's lemma applies to a more broadly defined set of preferences; while Proposition 4 also admits housing supply's endogenous adjustment, households' endogenous lot size as well as inter city migration. The underlying idea seems very intuitive. Much as in a sequence of dominos, both policies' bolstering of (reducing) demand for the second-lowest quality ultimately pushes up (pulls down) rent in every higher quality. An equally viable metaphor may be that of low quality housing serving as a "pillar" or "corner stone" to housing in qualities towering above it. Should low quality flats' rent give in, so would rents in every other segment.

Segment boundaries are $\theta(q_i^j(\alpha^j), q_{i+1}^j(\alpha^j))$, and hence are composite functions of the policy variables. Let us rewrite these values as $t_{i,i+1}^j(\alpha^j)$. Differentiating $t_{i,i+1}^j(\alpha^j)$ with respect to α^j yields the combined effect of a change in α^j on the segment boundary:

$$\frac{\partial t_{i,i+1}^j}{\partial \alpha^j} = \frac{\partial \theta_{i,i+1}^j}{\partial q_i^j}\frac{\partial q_i^j}{\partial \alpha^j} + \frac{\partial \theta_{i,i+1}^j}{\partial q_{i+1}^j}\frac{\partial q_{i+1}^j}{\partial \alpha^j} \tag{13}$$

for $i = 1, \ldots, I-1$. Total derivatives can thus be calculated by inserting boundaries' partial derivatives, from (5), as well as solutions for rent changes obtained in (12).

Further, to infer boundary changes' signs we use (13) to rewrite (10) as

$$\begin{aligned} n^j f(t_{i-1,i}^j)\frac{\partial t_{i-1,i}^j}{\partial \alpha^j} &= n^j f(t_{i,i+1}^j)\frac{\partial t_{i,i+1}^j}{\partial \alpha^j} \\ &- (\sigma_i^j + \varepsilon_i^j + \eta_i^j)\frac{\partial q_i^j}{\partial \alpha^j} + \frac{\partial z_i^j}{\partial \alpha^j} \end{aligned} \tag{14}$$

for $i = 2, \ldots, I$. This simpler version of (10) explicitly relates the "inflow" of households into segment (i, j) from $(i+1, j)$, as the first term on the r.h.s., to the stream of households "flowing out" of (i, j) into $(i-1, j)$, being the first term on the l.h.s. (Inflows and outflows may either be positive or negative.) The following proposition signs boundary changes, and hence filtering flows.

Proposition 5 (Filtering): *The signs of the derivatives of segment boundaries $t_{i,i+1}^j(\alpha^j)$ with respect to q_1^j and z_2^j are*

$$(i) \quad \frac{\partial t_{i,i+1}^j}{\partial q_1^j} < 0 \quad \textit{for } i = 1, \ldots, I-1 \tag{15}$$

$$(ii) \quad \frac{\partial t_{i,i+1}^j}{\partial z_2^j} < 0 \quad \textit{for } i = 2, \ldots I-1 \quad \textit{and} \quad \frac{\partial t_{1,2}^j}{\partial z_2^j} > 0, \tag{16}$$

respectively.
(i) (Low Quality Rent and Filtering) Raising low quality's rent will make remaining residents filter up consecutive segments $2, \ldots, I$.
(ii) (Low Quality Demolition and Filtering) Demolishing low quality's stock will make remaining residents filter up consecutive segments $3, \ldots, I$. *Moreover, remaining residents in segment 2 partly also filter down to segment 1.*[1]

2.3 Federal Coordination

We compare isolated policies, carried out by a single city only, to coordinated policies, obtained when identical policies are pursued by all cities simultaneously. In order to distinguish between variable changes in these two scenarios, derivatives for coordinated action (isolated action) carry superscript c (superscript nc). For example, $(\partial q_i^4/\partial z_2^4)^{nc}$ is the change in rent in $(i, 4)$ following demolition of low quality housing in city 4 only, while $(\partial q_i^4/\partial z_2)^c$ is the change in rent in segment $(i, 4)$ following uniform demolition of low quality housing in all cities $j = 1, \ldots, J$.

Rent changes under coordinated action result from retracing the previous section's discussion for when the inter city allocation of households remains intact. Coordinated, identical changes in α shut down interurban migration and, hence, wipe σ_i^j off all equations in (10). Let us define A^c as the coefficient matrix that results if σ_i^j is dropped from A^j. Then competitive rents' derivatives under coordinated action are $y^c = ((\partial q_2^j/\partial\alpha)^c, \ldots, (\partial q_I^j/\partial\alpha)^c)' = (A^c)^{-1}b$. The following proposition spells out the attendant changes in detail, and ranks rent increases as well as filtering flows.

Proposition 6 (Coordination): *Should all cities* $j = 1, \ldots, N$ *undertake the same policy* $\alpha^j = \alpha$, *then rent changes within each quality segment* $i = 2, \ldots, I$ *are identical across cities:*

$$(i) \quad \left(\frac{\partial q_i^j}{\partial \alpha}\right)^c = \left(\frac{\partial q_i}{\partial \alpha}\right)^c \tag{17}$$

Moreover, comparing the effect of changing α *under isolated action and under coordinated action on segments* $i = 2, \ldots, I$ *shows that*

$$(ii) \quad \left(\frac{\partial q_i^j}{\partial \alpha}\right)^c > \left(\frac{\partial q_i^j}{\partial \alpha^j}\right)^{nc} \tag{18}$$

$$(iii) \quad \left(\frac{\partial t_{i,i+1}^j}{\partial \alpha^j}\right)^{nc} < \left(\frac{\partial t_{i,i+1}^j}{\partial \alpha}\right)^c < 0 \tag{19}$$

That is,
(i) (Migration) Policy coordination upsets rents equally across cities.

[1] On the one hand, demolition evicts dz_2 households from segment 1 and subsequently induces them to move into segment 2; on the other hand, demolition induces $n^j f(t_{1,2}^j)(\partial t_{1,2}^j/\partial z_2^j)dz_2^j$ households to filter down from segment 2 into segment 1.

(ii) (Rents) Policy coordination reinforces the responses of higher segments' rents.
(iii) (Filtering) Policy coordination dampens filtering flows.

As is familiar from the literature on fiscal federalism, coordination of local policies effectively denies mobile residents the option of leaving the city (Property (i)). Unsurprisingly, any pressure policy may generate in the low quality segment is channeled into higher segments, rather than into other cities. Thus rents increase by more than if the city had only acted on its own (Property (ii)). Intuitively, in the absence of emigration demand for higher quality housing responds less elastically to those rent increases triggered by the local policy. Upper segments' rents must rise by more if these segments are to make room for tenants filtering up. Finally, under coordination the filtering flows into higher segments become less pronounced even as rents increase more strongly (Property (iii)).

At last we turn to local governments' objectives. Let us suppose that local governments are run by landlords, and that landlords may attempt to increase total rental revenues. As Proposition 6 indicates, raising low quality rent or demolishing low quality stock when all other cities also do is more successful in propelling extra rental income than pursuing these policies on one's own. Not just do rents in higher segments increase by more (while low quality rent either rises, too, or remains constant). Also, residents' total demands for housing within each of those higher segments increase by more, too (so that the outflow out of low quality housing is smaller, too). Joint action is unambiguously better than isolated action.

Proposition 7 (Local Government Policies)
Let a city either raise low quality rent or demolish low quality stock. Either policy drives up total rental income in that city by more if accompanied by simultaneous and identical policies in all other cities.

Fundamentally, a city demolishing on its own will drive away residents not just from the segment to be demolished but also from every other of its segments. Coordination remedies this. Simultaneous demolition forestalls migration across cities and within qualities, and thus strongly reinforces a landlord government's incentive to demolish. The following section briefly presents an illustration of this idea.

3 A Case Study

Over 1.2 million flats were vacant in Germany's East in 2001, in a total of 7.6 million units. In response, some 250,000 flats were leveled so far, and another 220,000 flats will follow in the years up to 2016 (see the introduction). Policy makers motivate this large scale demolition by arguing that, first, demolition targets low quality housing noone wants and, that second, demolition "stabilizes housing markets". As one politician argued, a "... clear focus on demolition has brought considerable relief to housing markets ... " (FAZ (2006)).

Of course, these two motivations contradict each other. If it were only unattractive housing that is torn down then overall rents certainly could not be "stabilized", as the filtering model tells us. In any case, both motivations do connect to this paper's model, both in terms of its key assumption as well as in terms of key predictions. First, politicians' self-professed objective that demolition should "stabilize housing markets" connects well with the model assumption that governments strive to increase landlords' rental incomes.

And second, politicians' claim that demolition merely targets unwanted housing has implications within the model that can be tested. If governments merely pull down truly superfluous housing only then East Germany's rents should at best stagnate, if not fall, through the demolition period. In contrast, finding that East Germany's rents in fact increases during this period would point to the opposite interpretation, of demolition in fact taking down housing units that residents would otherwise value, and inhabit. Of course, we need to control for both, individual heterogeneity as well events unfolding during demolition. This we do by (i) employing micro data and (ii) contrasting rent changes in Germany's East with those observed in Germany's West.

We make use of a very large micro data set of East and West German households (the Mikrozensus) available for the years 2002 (the beginning of the demolition program) and 2006 (well into the program). Unfortunately households in 2006 are not the same as those in 2002. Nonetheless, for every respondent we have information on total rent and floor space as well as information contained in dummies on age of the building, number of stories of the building, type of heating, housing tenure, county of residence and degree of agglomeration of both the city and county of residence.

Pooling the observations from 2002 and 2006 gives a data set of well over 230,000 units of rental housing. Let the time dummy $d2$ equal 1 if the year is 2006 and zero otherwise, and let the region dummy dT equal 1 if the region is East Germany and zero else. Following Wooldridge (2010), we specify the following difference-in-difference (DID) model

$$\ln\, q_\ell \;= \gamma_0 \;+\; \beta' \mathbf{x}_\ell \;+\; \delta_0\, d2 \;+\; \gamma_1\, dT \;+\; \delta_1\, (dT \cdot d2) \;+\; u_\ell \tag{20}$$

where ℓ is the household number, $\mathbf{x}_\ell$ is a vector of attributes characterizing the location and quality of that household's flat, β is the vector of these attributes' coefficients and γ_0, γ_1 and δ_0 and δ_1 are the intercept and dummy variable coefficients, respectively.

Roughly, δ_0 gives rent growth for a West German flat while $\delta_0 + \delta_1$ gives rent

	OLS (DID)	**OLS (DID)**	**OLS (FD)**
$\widehat{\gamma}_1$	– 0.143	– 0.184	–
	(0.002)	(0.024)	
$\widehat{\delta}_0$	– 0.025	– 0.068	–
	(0.001)	(0.017)	
$\widehat{\delta}_1$	0.022	0.024	0.026
	(0.003)	(0.015)	(0.008)
Obs.	231,909	610	305
$\bar{R}^2$	0.21	0.61	0.18

Table 2: Rent growth in East and West Germany (Standard errors in parentheses)

growth in Germany's East. Hence δ_1, as the difference between these two differences, has the interpretation of being the excess of the East German rent change over and above the rent change observed for Germany's West. Put differently, δ_1 captures the extent to which rents in East Germany grow by more than do rents in Germany's West, for a flat of identical attributes. It is only this excess that we will attribute to the demolition program taking place just then.

The first column of coefficients in Table 2 documents the estimate of δ_1 as well as estimates of the two dummy variables. (Estimates of control variable coefficients are not shown, but largely are plausible.) All of the three estimates are significantly different from zero. First we see that Eastern rent roughly is 14% smaller than rent of a Western flat of comparable quality and location. This is only a small East-West rent differential, and it suggests that indeed East's demolition activities may target flats that have only ever been offered at fairly expensive rent.

Second we see that Western rents actually have fallen by 2.5% over the four years. So any given Western flat has experienced a decline in rent. In contrast, and third, we see that rent for a similar flat in East has fallen by a mere 0.003%, or not at all. Put differently, rents in East, being the treatment region, have succeeded in rising relative to rent in West, as the control region. To be sure, this is all in spite of an enormous excess supply of housing (not at all present in West) that must in principle have pushed rents the other way. We interpret this finding as refuting the claim that Eastern demolition is innocuous, dispensing with unwanted housing only.

The other two columns of coefficients in Table 2 offer yet two other variations in estimation. If we average the data by county, then we obtain a data set of 305 county averages yearly, or 610 observations. Pooling data from these years and repeating the difference-in-difference procedure set out in (20) gives the coefficient estimates in the second data column. This procedure throws away information, and while estimates still point into the same direction as before it is not surprising that the key coefficient $\widehat{\delta}_1$ no longer is significant here.

However, averaging household data by county becomes meaningful again if we construct a true panel, of average county information. For each county we have average data for 2002 and 2006, and given the county identifier we can match county data for these two years. For each county we then first difference (FD) equation (20), to rule out the possibility that the presence of unobservable effects in each county biases our estimates. Given the resulting estimate $\widehat{\delta}_1$, we again find that Eastern rents grew over and above the Western trend. The estimate, 2.6% , is significant and also reassuringly similar to the estimate we found first.

4 Conclusions

This paper focuses on how landlord-ruled cities may best sustain, or even raise, their rental incomes in the context of multiple quality housing, and given the grown importance of vacant housing. We devise a model in which individuals make simultaneous decisions on housing quality, floor space, and residential location. A single city may wish to demolish some of the stock in its lowest quality segment on its own, without other cities going along. Yet such demolition will drive many of that city's residents out of every of its qualities, thereby diluting the incentive to demolish. Coordinating these local demolition efforts greatly enhances the incentive to demolish.

The paper's focus is on demolition, yet the analysis also addresses issues pertinent to zoning. In the paper, local governments demolish low quality buildings. But much of zoning also is about reducing the supply of low quality housing. For instance, the minimum lot size restrictions that characterize much of Eastern Massachusetts's zoning effectively ration supply (Zabel/Dalton (2011)). If demolition is the reduction of the housing stock, then zoning is that housing stock's prevention. From this perspective we expect institutions facilitating zoning's coordination to also speed up zoning's introduction.

5 Appendix A

Lemma 1 (Indirect Utility Properties):

(i) V exhibits strictly increasing differences with respect to θ and (s, q).

(ii) $V_q = v_q = -h(q_i, s_i) < 0$, by the envelope theorem.

(iii) $V_{\theta s} > 0$, from the definition of V in (2).

(iv) $V_{\theta q} = 0$, from the definition of V in (2).

(v) $V_s = v_s = r_s(h(q_i, s_i), s_i) > 0$, by the envelope theorem.

Proof of Lemma 1 (Indirect Utility Properties):

(i) Consider comparing the maximum utilities that result for two distinct combinations of quality and rent (s', q') and (s'', q''), where $s', s'' \in \{s_1, \ldots, s_I\}$ and $q', q'' \in \mathbb{R}_+$. Suppose $s'' > s'$ and $q'' > q'$. Now surely for all $\theta'' \gtreqless \theta'$ we have $\theta'' s'' - \theta' s'' \gtreqless \theta'' s' - \theta' s'$. But then

$$(\theta'' s'' + v(q'', s'')) - (\theta' s'' + v(q'', s'')) \gtreqless (\theta'' s' + v(q', s')) - (\theta' s' + v(q', s'))$$

also. This proves that

$$\begin{aligned} V(\theta'', s'', q'') - V(\theta', s'', q'') \quad &\gtreqless \quad V(\theta'', s', q') - V(\theta', s', q') \\ &\text{for all} \quad \theta'' \gtreqless \theta' \quad \text{and} \quad (s'', q'') > (s', q') \end{aligned}$$

Put differently, indirect utility V exhibits strictly increasing differences with respect to θ and (s, q). Proofs of (ii) to (v) are indicated in Lemma 1. □

Proof of Proposition 1 (Resident Sorting By Quality):[2]

Given Lemma 1, $V(\theta_{i,i+1}, s_{i+1}, q_{i+1}) - V(\theta, s_{i+1}, q_{i+1}) \gtreqless V(\theta_{i,i+1}, s_i, q_i) - V(\theta, s_i, q_i)$ for all $\theta \lesseqgtr \theta_{i,i+1}$. Inserting indifference condition (3), and also spelling out the resulting pair of inequalities out for segment $i-1$, translates into

$$V(\theta, s_{i+1}, q_{i+1}) \quad \gtreqless \quad V(\theta, s_i, q_i) \qquad \text{for all } \theta \gtreqless \theta_{i,i+1} \tag{21}$$

$$V(\theta, s_i, q_i) \quad \gtreqless \quad V(\theta, s_{i-1}, q_{i-1}) \qquad \text{for all } \theta \gtreqless \theta_{i-1,i} \tag{22}$$

Hence residents with a taste index θ in the interval $[\theta_{i-1,i},\, \theta_{i,i+1}]$ prefer segment i to either of i's neighbors $i-1$ and $i+1$.

In fact, residents in $[\theta_{i-1,i},\, \theta_{i,i+1}]$ prefer i to *every* other quality. Consider, for example, a much better segment s_k, with $k > i+1$. Specifying the pair of inequalities in (22) for $i = k$ gives $V(\theta, s_k, q_k) \gtreqless V(\theta, s_{k-1}, q_{k-1})$ for all $\theta \gtreqless \theta_{k-1,k}$. Since residents in $[\theta_{i-1,i}, \theta_{i,i+1}]$ exhibit $\theta < \theta_{k-1,k}$, they are more strongly attracted to segment $k-1$ than to k. Applying this argument to successively lower segments until segment $i+1$ is reached, shows that $V(\theta, s_k, q_k) < V(\theta, s_{k-1}, q_{k-1}) < \ldots < V(\theta, s_i, q_i)$ for all residents with θ in $[\theta_{i-1,i}, \theta_{i,i+1}]$. A similar argument applies to the case where $k < i-1$. □

[2] The proof merely is an application of the comprehensive treatment in Topkis (1998).

Proof of Proposition 2 (Segment Population and Rent):

We divide the proof into three steps. First we derive the right derivative of n_i^j with respect to q_i^j at $\bar{q}_i^j$, then we derive its left derivative, then we compare the two. (i) Consider a rent $\hat{q}_i^j$ which strictly exceeds $\bar{q}_i^j$. Resulting variable values are denoted $\hat{\theta}_{i,i+1}^j = \theta(\hat{q}_i^j, \bar{q}_{i+1}^j)$, etc. Further, define

$$\hat{m}_i = \bar{V}_i^{j+1} - \hat{V}_i$$

as the pull that segment $(i, j+1)$ exerts on those initially in segment (i, j). Surely only those with taste parameter θ in $[\hat{\theta}_{i-1,i}^j, \hat{\theta}_{i,i+1}^j]$ and mobility cost m beyond $\hat{m}_i$ will remain in segment (i, j). In this case, and given independence between m and θ, $\hat{n}_i^j$ simply equals $\bar{n}^j(F(\hat{\theta}_{i,i+1}^j) - F(\hat{\theta}_{i-1,i}^j))(1 - F(\hat{m}_i))$. In contrast, $\bar{n}_i^j$ equals $\bar{n}^j\,(F(\bar{\theta}_{i,i+1}^j) - F(\bar{\theta}_{i-1,i}^j))$.

Now consider the ratio $(\hat{n}_i^j - \bar{n}_i^j)/(\hat{q}_i^j - \bar{q}_i^j)$. This ratio can be expanded as follows:

$$\begin{aligned} & \bar{n}^j \, \frac{F(\hat{\theta}_{i,i+1}^j) - F(\bar{\theta}_{i,i+1}^j)}{\hat{\theta}_{i,i+1}^j - \bar{\theta}_{i,i+1}^j} \cdot \frac{\hat{\theta}_{i,i+1}^j - \bar{\theta}_{i,i+1}^j}{\hat{q}_i^j - \bar{q}_i^j} \cdot (1 - F(\hat{m}_i)) \\ - \; & \bar{n}^j \, \frac{F(\hat{\theta}_{i-1,i}^j) - F(\bar{\theta}_{i-1,i}^j)}{\hat{\theta}_{i-1,i}^j - \bar{\theta}_{i-1,i}^j} \cdot \frac{\hat{\theta}_{i-1,i}^j - \bar{\theta}_{i-1,i}^j}{\hat{q}_i^j - \bar{q}_i^j} \cdot (1 - F(\hat{m}_i)) \\ - \; & \bar{n}^j \, \frac{F(\hat{m}_i) - F(\bar{m}_i)}{\hat{m}_i - \bar{m}_i} \cdot \frac{\hat{m}_i - \bar{m}_i}{\hat{q}_i^j - \bar{q}_i^j} \cdot (F(\bar{\theta}_{i,i+1}^j) - F(\bar{\theta}_{i-1,i}^j)) \end{aligned} \tag{23}$$

where $\bar{m}_i = 0$.

By the implicit function theorem, $\theta_{i,i+1}$ is a differentiable, and hence continuous, function of $\hat{q}_i^j$ for rents $\hat{q}_i^j$ sufficiently close to $\bar{q}_i^j$. Then as we let $\hat{q}_i^j$ approach $\bar{q}_i^j$, $\hat{\theta}_{i,i+1}$ approaches $\bar{\theta}_{i,i+1}$. Since $F(\theta)$ is differentiable, then the first ratio on the first line of (23) converges to $f(\bar{\theta}_{i,i+1}^j)$.

Taking limits of all other terms in (23) and applying similar arguments eventually gives the right derivative of n_i^j with respect to q_i^j at $\bar{q}_i^j$:

$$\begin{aligned} \lim_{\hat{q}_i^j \to \bar{q}_i^{j+}} \frac{\hat{n}_i^j - \bar{n}_i^j}{\hat{q}_i^j - \bar{q}_i^j} \; &= \; \bar{n}^j \, f(\bar{\theta}_{i,i+1}^j) \frac{\partial \theta^j(\bar{q}_i^j, \bar{q}_{i+1}^j)}{\partial q_i^j} \\ &- \; \bar{n}^j \, f(\bar{\theta}_{i-1,i}^j) \frac{\partial \theta^j(\bar{q}_{i-1}^j, \bar{q}_i^j)}{\partial q_i^j} \\ &+ \; \bar{n}^j \left(F(\bar{\theta}_{i,i+1}^j) - F(\bar{\theta}_{i-1,i}^j) \right) f(\bar{m}_i) \, \frac{\partial V(\bar{q}_i^j)}{\partial q_i^j} \end{aligned} \tag{24}$$

(ii) Consider a rent $\check{q}_i^j$ strictly below $\bar{q}_i^j$. Resulting variable values are $\check{\theta}_{i,i+1}^j = \theta_{i,i+1}(\check{q}_i^j, \bar{q}_{i+1}^j)$, etc. Given $\check{q}_i^j < \bar{q}_i$, segment (i, j) clearly is more attractive than segments of comparable quality elsewhere. We decompose $\check{n}_i^j$ into natives and immigrants. The ratio of the change in natives to the change in rent can be written as

$$\bar{n}^j \, \frac{\left(F(\bar{\theta}_{i,i+1}^j) - F(\bar{\theta}_{i-1,i}^j)\right) - \left(F(\check{\theta}_{i,i+1}^j) - F(\check{\theta}_{i-1,i}^j)\right)}{\bar{q}_i^j - \check{q}_i^j}$$

Letting $\check{q}_i^j$ approach $\bar{q}_i^j$ gives the derivative of natives' numbers with respect to q_i^j at $\bar{q}_i^j$, i.e.,

$$\bar{n}^j\, f(\bar{\theta}^j_{i,i+1})\, \frac{\partial \theta^j(\bar{q}_i^j, \bar{q}_{i+1}^j)}{\partial q_i^j} \;-\; \bar{n}^j\, f(\bar{\theta}^j_{i-1,i})\, \frac{\partial \theta^j(\bar{q}_{i-1}^j, \bar{q}_i^j)}{\partial q_i^j} \tag{25}$$

Next we turn to immigrants to (i, j). Figure 1 (in the text) points to which natives from city $j-1$ may be attracted to (i, j). The Figure shows maximum utility in the three segments $(i-1, j-1)$, $(i, j-1)$ and $(i+1, j-1)$, as well as maximum utility in competing segment (i, j), as functions of the taste index θ. Define

$$\check{m}_i = \check{V}_i^j - \bar{V}_i^{j-1}$$

as the pull that segment (i, j) exerts on those initially in segment $(i, j-1)$. On the one hand, the unknown number of those wanting to emigrate to (i, j), $d\,E$, certainly is smaller than $\bar{n}^{j-1}(F(\check{\theta}^{j-1}_{i,i+1}) - F(\check{\theta}^{j-1}_{i-1,i})\,)F(\check{m}_i)$. On the other hand, this number certainly is greater than $\bar{n}^{j-1}(F(\bar{\theta}^{j-1}_{i,i+1}) - F(\bar{\theta}^{j-1}_{i-1,i}))\,F(\check{m}_i)$. I.e.,

$$\frac{\bar{n}^{j-1}(F(\bar{\theta}^{j-1}_{i,i+1}) - F(\bar{\theta}^{j-1}_{i-1,i}))\,F(\check{m}_i)}{\check{q}_i^j - \bar{q}_i^j} \;\leq\; \frac{d\,E}{\check{q}_i^j - \bar{q}_i^j} \;\leq\; \frac{\bar{n}^{j-1}(F(\check{\theta}^{j-1}_{i,i+1}) - F(\check{\theta}^{j-1}_{i-1,i}))\,F(\check{m}_i)}{\check{q}_i^j - \bar{q}_i^j}$$

The first and the last term in this series of inequalities converge to the same expression as $\check{q}_i^j$ approaches $\bar{q}_i^j$. Hence so does the middle term, by the "squeezing rule". The derivative of the number of migrants from $j-1$ to j thus is the limit of either the first or the last term in the preceding series of inequalities, and hence can be derived as:

$$\bar{n}^{j-1}\left(F(\bar{\theta}^{j-1}_{i,i+1}) - F(\bar{\theta}^{j-1}_{i-1,i})\right) f(\bar{m}_i) \frac{\partial V(\bar{q}_i^j)}{\partial q_i^j} \tag{26}$$

Adding (25) and (26) gives the left derivative of n_i^j with respect to q_i^j at $\bar{q}_i^j$:

$$\begin{aligned}
\lim_{\check{q}_i^j \to \bar{q}_i^{j-}} \frac{\bar{n}_i^j - \check{n}_i^j}{\bar{q}_i^j - \check{q}_i^j} \;&=\; \bar{n}^j\, f(\bar{\theta}^j_{i,i+1})\, \frac{\partial \theta^j(\bar{q}_i^j, \bar{q}_{i+1}^j)}{\partial q_i^j} \\
&-\; \bar{n}^j\, f(\bar{\theta}^j_{i-1,i})\, \frac{\partial \theta^j(\bar{q}_{i-1}^j, \bar{q}_i^j)}{\partial q_i^j} \\
&+\; \bar{n}^{j-1}\left(F(\bar{\theta}^{j-1}_{i,i+1}) - F(\bar{\theta}^{j-1}_{i-1,i})\right) f(\bar{m}_i) \frac{\partial V(\bar{q}_i^j)}{\partial q_i^j}
\end{aligned} \tag{27}$$

(iii) Comparing (27) with (24) shows that the derivative at $\bar{q}_i^j$ exists, if the initial allocation is symmetric. □

Proof of Proposition 3:

The proof is similar to that of Proposition 2. □

Proof of Proposition 4:

(*Existence*) We first document an existence property not explicitly mentioned in the Proposition. Generally, the coefficients of $\partial q_i^j/\partial\alpha^j$, $i = 2, \ldots, I$, are stacked into the $(i-1)$-th column of A^j:

$$\begin{array}{l} \\ \\ \\ \text{row } i-2 \\ \\ \text{row } i-1 \\ \\ \text{row } i \\ \\ \\ \\ \end{array} \left(\begin{array}{c} 0 \\ \vdots \\ 0 \\ n^j f(\theta_{i-1,i}^j)\,\frac{\partial\theta_{i-1,i}^j}{\partial q_i^j} \\ n^j\, f(\theta_{i,i+1}^j)\,\frac{\partial\theta_{i,i+1}^j}{\partial q_i^j} \quad - \quad \left(\sigma_i^j + \varepsilon_i^j + \eta_i^j\right) \quad - \quad n^j f(\theta_{i-1,i}^j)\,\frac{\partial\theta_{i-1,i}^j}{\partial q_i^j} \\ -n^j\ f(\theta_{i,i+1}^j)\,\frac{\partial\theta_{i,i+1}^j}{\partial q_i^j} \\ 0 \\ \vdots \\ 0 \end{array} \right) \tag{28}$$

where, to be sure, the first and the last column of A^j look slightly different.

The effects of a one Euro change in q_i^j on segments $i-1$, i and $i+1$ are found in column $(i-1)$'s three consecutive rows $i-2$, $i-1$, and i, respectively, whereas the column's remaining entries are zero. Now, the element in column $i-1$ and row $i-1$, also shown in (28), is a diagonal element of A^j. In absolute value this (negative) element exceeds the sum of the two (positive) off-diagonal elements found in the column's rows $i-2$ and i. This property applies to any of A^j's columns. Thus A^j is diagonally dominant.

But then A^j also is non-singular (Graybill (1983, Theorem 8.11.2). Hence $(A^j)^{-1}$ exists, and with it a solution to (10). This solution is unique.

(i) (*Raising Low Quality Rent*) Note that $(A^j)^{-1}$ has non-positive entries everywhere (Takayama (1985), Theorem 4.D.3, parts (I") and (III"), or Simon/Blume (1995), Theorem 8.14). Thus every component of the solution vector $(A^j)^{-1}b^j$ is simply the product of a non-positive number, taken from the first column of $(A^j)^{-1}$, and the negative shock (11). Thus every element of the solution y^j is non-negative.

Next we show that the elements of y^j in fact are strictly positive. Consider $\partial q_2^j/\partial q_1^j > 0$ first. Assume, to the contrary, that $\partial q_2^j/\partial q_1^j = 0$. Then if also setting $i = 2$, choosing $\alpha = q_1$, and dropping the city index, equation (10) becomes

$$n f(t_{2,3})\,\frac{\partial t_{2,3}}{\partial q_1} \quad = \quad n\, f(\theta_{1,2})\,\frac{\partial\theta_{1,2}(q_1, q_2)}{\partial q_1} \tag{29}$$

where $\partial t_{2,3}/\partial q_1$ is given in (13) when setting $\alpha = q_1$ and $i = 2$.

On the one hand, the l.h.s. of (29) must be non-negative. After all, $\partial q_3/\partial q_1 \geq 0$, as established above, while $\partial q_2/\partial q_1 = 0$, by assumption. Hence $\partial t_{2,3}/\partial q_1 \geq 0$.

On the other hand, the r.h.s. of (29) is strictly negative (see (5)). We conclude that $\partial q_2/\partial q_1 > 0$. Repeating this argument for successively higher quality segments reveals that $\partial q_i/\partial q_1 > 0$ for all $i = 2, \ldots, I$.

(iii) (*Demolishing low quality stock*) The proof for showing that $\partial q_i/\partial z_2 > 0$ for all $i = 2, \ldots, I$ is similar to (ii). □

Proof of Proposition 5 (Filtering in a Model with Spatial Mobility):

(i) We focus on the case where $\alpha = q_1$ first. The I-th segment version of (14) reads

$$nf(t_{I-1,I})\frac{\partial t_{I-1,I}}{\partial q_1} = -(\sigma_I + \varepsilon_I + \eta_I)\frac{\partial q_I}{\partial q_1} \tag{30}$$

Following Proposition 4, Part (i), the r.h.s. of (30) is strictly negative. Then so must be its l.h.s. Thus $\partial t_{I-1,I}/\partial q_1 < 0$.

Proceeding in this fashion towards successively lower qualities will show that derivatives of all boundaries (except for $t_{0,1}$) with respect to q_1 are strictly negative.

(ii) Now focus on the case where $\alpha = z_2$. Analysis of derivatives of boundaries with respect to z_2 is identical to the analysis in the previous paragraph for $i = 3, \ldots I$. These derivatives are all strictly negative, too.

Treating the case of $\partial t_{1,2}/\partial z_2$ is even simpler. The $i = 2$ version of (13) reduces to:

$$\frac{\partial t^j_{1,2}}{\partial z^j_2} = \frac{\partial \theta^j_{1,2}}{\partial q^j_2}\frac{\partial q^j_2}{\partial z^j_2}$$

which is strictly positive. □

Proof of Proposition 6 (Federal Coordination):

(i) (*Migration*) (We sketch the proof only because the details of the full proof would require introducing additional notation.) To solve for rents in every segment $i = 2, \ldots, I$ in every city $j = 1, \ldots, J$ we need to set up a system of $(I-1)J$ equations. Differentiating this system with respect to $\alpha^j = \alpha$ gives a linear system of $(I-1)J$ equations in the $(I-1)J$ unknowns contained in $y = (y^1, \ldots, y^J)$.

We collect the values found on the right hand sides of these equations in the $(I-1)J \times 1$-vector $b = (b^1, \ldots, b^J)$, where b^j is constant across cities. Note that b exhibits only zeros, except for on the first, and then on every $(I-1)$-th, position.

The system's coefficient matrix, denoted C, essentially is blockdiagonal, with the representative local coefficient matrix $A^j = A$ repeatedly used as block, for a total of J times.

The only extra novelty to be taken care of is that on the two off diagonals $I-1$ entries off the main diagonal we now encounter non-zero elements that capture the effects of changes originating in the two neighboring cities' rents on a given city's housing market. (These latter effects were not present in (28).)

One can show that the resulting matrix C is diagonally dominant. Hence C is non-singular, too, (again Graybill (1983), Theorem 8.11.2) and its inverse C^{-1} exists, and exhibits non-negative entries only.

The solution for y is given by Cb. This product makes use of only the first, I-th, etc. column of C. The coefficients found in these columns repeat themselves every $I-1$-th row. We suggest a solution for y in which rent changes within any given quality segment do not vary across cities.

But when premultiplied by C, this trial solution "works". Since with C being non-singular no other solution can exist, we conclude that for a given quality segment the rent changes found in y indeed are the same in every city.

(ii) (*Rents*) The proof is by contradiction. I.e., there must exist a segment $i \in \{2, \ldots, I\}$ for which both

$$\left(\frac{\partial q_i^j}{\partial \alpha^j}\right)^c \leq \left(\frac{\partial q_i^j}{\partial \alpha^j}\right)^{nc} \qquad \text{and} \qquad \left(\frac{\partial q_{i-1}^j}{\partial \alpha^j}\right)^c \geq \left(\frac{\partial q_{i-1}^j}{\partial \alpha^j}\right)^{nc} \tag{31}$$

hold.

Let us explore i's upper neighbor $i+1$. If $i+1 \leq I$ and if $(\partial q_{i+1}^j/\partial \alpha^j)^c \leq (\partial q_{i+1}^j/\partial \alpha^j)^{nc}$ then we continue by investigating $i+2$. Else we stop. Suppose we stop after l steps, where $l \geq 0$. Then our procedure constructs a "connected sequence" $M = i, i+1, \ldots, i+l$ of adjacent quality segments, of length $l+1$.

Consider M's lower neighbor $i-1$ first. By definition of M, this segment either enjoys a stronger rent rise (if $i > 2$), or the same rent rise (if $i = 2$), under coordinated than under isolated action. Hence $(\partial t_{i-1,i}^j/\partial \alpha^j)^c \leq (\partial t_{i-1,i}^j/\partial \alpha^j)^{nc}$, by inspection of (13).

Likewise, consider M's upper neighbor $i+l+1$ next. If $i+l+1 > I$ then this upper neighbor does not exist. Then $(\partial t_{i+l,i+l+1}^j/\partial \alpha^j)^c = (\partial t_{i+l,i+l+1}^j/\partial \alpha^j)^{nc} = 0$. Alternatively, if $i+l+1 \leq I$ then this upper neighbor exists, and by M's definition experiences a strictly stronger rent rise under coordination. I.e.,

$$\left(\frac{\partial q_{i+l+1}^j}{\partial \alpha^j}\right)^c > \left(\frac{\partial q_{i+1+l}^j}{\partial \alpha^j}\right)^{nc}$$

Hence $(\partial t_{i+l,i+l+1}^j/\partial \alpha^j)^c > (\partial t_{i+l,i+l+1}^j/\partial \alpha^j)^{nc}$. To summarize both cases in one statement, $(\partial t_{i+l,i+l+1}^j/\partial \alpha^j)^c \geq (\partial t_{i+l,i+l+1}^j/\partial \alpha^j)^{nc}$ always.

Recursive substitution in (14) l times gives

$$\begin{aligned} n^j f(t_{i-1,i}^j)\left(\frac{\partial t_{i-1,i}^j}{\partial \alpha^j}\right)^{nc} &= -\sum_{k=0}^{l}\left(\sigma_{i+k}^j + \varepsilon_{i+k}^j + \eta_{i+k}^j\right)\left(\frac{\partial q_{i+k}^j}{\partial \alpha^j}\right)^{nc} \\ &\quad + n^j f(t_{i+l,i+l+1}^j)\left(\frac{\partial t_{i+l,i+l+1}^j}{\partial \alpha^j}\right)^{nc} + \gamma_i \end{aligned} \tag{32}$$

in the uncoordinated case, and

$$\begin{aligned} n^j f(t_{i-1,i}^j)\left(\frac{\partial t_{i-1,i}^j}{\partial \alpha^j}\right)^{c} &= -\sum_{k=0}^{l}\left(\varepsilon_{i+k}^j + \eta_{i+k}^j\right)\left(\frac{\partial q_{i+k}^j}{\partial \alpha^j}\right)^{c} \\ &\quad + n^j f(t_{i+l,i+l+1}^j)\left(\frac{\partial t_{i+l,i+l+1}^j}{\partial \alpha^j}\right)^{c} + \gamma_i \end{aligned} \tag{33}$$

in the coordinated case, with $\gamma_i = (\partial z_2^j/\partial\alpha^j)^{nc} = (\partial z_2/\partial\alpha)^c$ if $i = 2$ and zero otherwise.

Now we make use of the rankings of boundary changes under coordinated and under isolated action derived above. Subtracting the l.h.s. of (33) from the l.h.s. of (32) gives a non-negative number. Subtracting the r.h.s. of (33) from the r.h.s. of (32) gives a strictly negative number, given our assumption (31). Either equation (32) or equation (33) cannot be satisfied. This contradicts the model's assumptions. □

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