Function Follows Form

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Abstract: Ever since Victor Gruen opened *Northland* in a Detroit suburb, thousands of shopping centers and office parks have sprung up along city peripheries. While automobile travel is necessary for this transition from the monocentric to the polycentric city, car travel alone, so this paper argues, is not sufficient. Instead, the decentralization of shops and jobs (two important "urban functions") hinges on the initial spatial distribution of the urban electorate, as embodied by the original city's shape ("urban form"). The electorate is more likely to tilt in favor of decentralization the less skewed is original city shape. Hence "function follows form". Given that "form follows function" eventually, too, cities either remain centralized and skewed in the long run, or shed all three: central jobs, central shops, and skew. We turn to a sample of U.S. metropolitan areas to illustrate these ideas.

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1 Introduction

Ever since Victor Gruen opened *Northland*, the first modern mall, in a Detroit suburb, thousands of shopping centers and office parks have sprung up in city peripheries (Lampugnani (1985), Garreau (1991), Hardwick (2010)). Decentralization of jobs and shops could be especially dramatic when highways were built around, or even through, cities (Baum-Snow (2007a, 2007b), Glaeser/Kahn (2010)). At the same time, plans for such radial or tangential highways were often also shelved – witness the success of those grass roots movements against, say, Robert Moses' plans for Lower Manhattan (Jacobs (1961), Caro (1975)), the Spadina Expressway in Toronto, Covent Garden urban renewal in London or Berlin's 1970s highway planning. Surely the city only paves the way for the decentralization of its employment and retail if a majority of its citizens approve.

This paper will add: Support for, and opposition to, such decentralization are imprinted into – and hence may even be predicted by – the city's shape. It is in this sense that function (jobs and shops) follows form (city shape), as in the paper's title. Imagine a monocentric city inhabited by owner-occupiers voting on a proposal to transfer the city's jobs and shops to the ring road encircling it. Surely we can predict these residents' votes. Any resident living near the center (a resident we label centrist) will not want to give up on that center; the opposite is true for any resident living in the periphery (decentrist). A fine line, coincident with the ring halfway from the city's center to its periphery, will divide those who are against decentralization from those who are for it. Even ahead of the vote we are able to assess centrists and decentrists, by counting their respective dwellings.

Ultimately city form drives the extent to which jobs and shops will decentralize. As city form changes over time or varies across cities, decentralization may stall, or accelerate, too. Fogelson (2003) devotes an entire chapter of his book on America's "downtown" to "the specter of decentralization". He writes that one "... phenomenon to which downtown businessmen and property owners attributed decentralization was residential dispersal" (p. 231). More homes at the urban fringe certainly mean more voters in favor of business decentralization. Similarly, Thurston/Yezer (1994) and Brueckner (2000) have discussed how "jobs follow people", as opposed to how "people follow jobs". Our discussion below will cast the interaction between "jobs and people" as one between "function and form".

Of course cities are not solely, and not even predominantly, owner-occupied. Homeownership rates are typically not close to, and often fall well short of, one. Extracting decentralization's urban political economy from urban form here is much less straightforward than for the simple owner-occupied city. Further, note that the bulk of the urban literature – the very literature this paper wants to connect with – departs from a landlord-tenant, rather than an owner-occupier, setup (see the "absentee landlords" in Brueckner (1987) and Helsley (2004) or the "common ownership" framework in Wheaton (1973) or Borck/Brueckner (2018)). For these reasons we will pursue our analysis of centrists' and decentrists' strengths within the more general resident-landlords-cum-tenants framework.

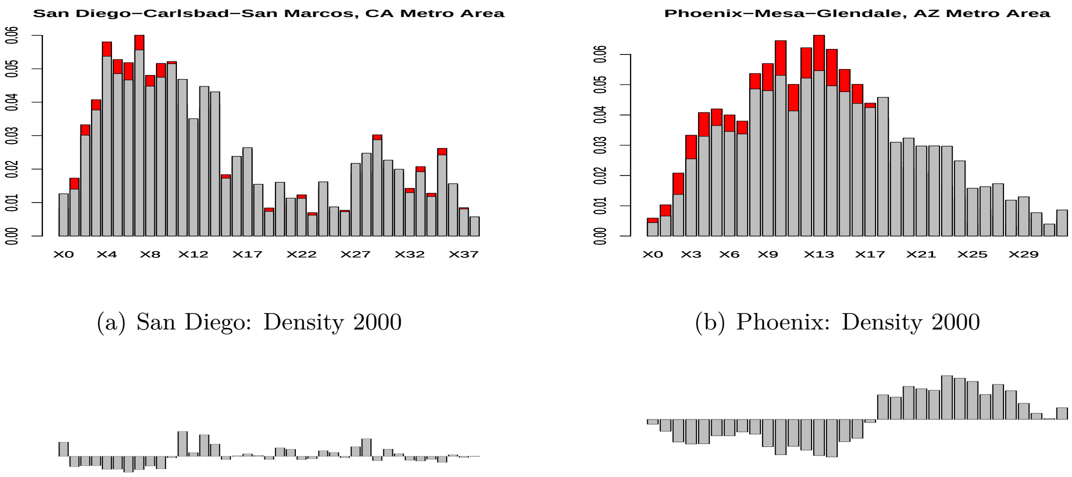
So suppose each landlord owns multiple – two! – properties: one property to live in, the other to rent out. We rarely observe a landlord's full set of properties. We cannot determine that set's average location. We do not know whether a given landlord is a centrist or not. However, we often observe the distribution of population (or housing) across the city's rings, as one representation of urban form. This "city shape" places constraints on the smallest number of centrists that could conceivably nest into it. Consulting suitable subsets of city rings, different lower bounds on centrists will emerge. We may estimate centrists' true (unknown) number by employing the largest of those.

Besides recurring on observable city shape (i.e. aggregate) data only, the largest of our lower bounds (or the "greatest cumulative ring difference", as it is also referred to below) has four remarkable properties. First, it is representable by a formula (and a simple one at that). It may easily be fed shape data from any city presented to it. Second, it will never overestimate centrists' true number. It provides us with conservative estimates of true centrists. Nor will it, third, underestimate centrists' true number by "too much". It provides us with efficient (i.e. not unnecessarily small) estimates. And fourth, it will be bound by city shape's skew. Ultimately we may hope to learn about the city's politics simply by inspecting its shape. E.g., we will see that if that shape "leans more towards" the city center, voters are more inclined to maintain – "lean more towards" – that center.

Urban form is an interesting field in its own right, as forcefully argued in Lynch (1960) and Baranow (1980). This paper adds that urban morphology, as the study of city form, has uses that go beyond the descriptive. Besides, reading restrictions on a city's various political interests off its physical form also complements a prominent view due to Louis Sullivan. According to Sullivan (1896), "...it is the pervading law of all things organic and inorganic, ...that the life is recognizable in its expression". Among architectural theorists, Sullivan's view has become the proverbial *form follows function.* This paper provides a framework of how instead building contours (form) determine buildings' uses (function), or of how ...*function follows form.*

Much as a city's shape previews the minimum share of its centrists does it preview its minimum share of decentrists. We will see that decentrists' minimum share coincides with (minus) the least cumulative ring difference. Important factions of urban political economy now become estimable by inspecting city shape (Proposition 1). We are also interested in understanding the effects of varying that shape. First we will show that more compact cities exhibit more centrists, less decentrists (Proposition 2); and that cities whose shape is skewed more to the right (left) exhibit more centrists (decentrists) (Proposition 3). If skew is positive (negative) enough then a majority holding on to the traditional center (pushing for decentralization) is inevitable. Skew literally reveals urban form's hidden grip on local politics. And, one concept that is graphic predicts another that is political.

We then decompose shape into topography and building height, and discuss the extent to which these "shape shifters" affect urban political economy (Proposition 4). We will suggest that tighter natural constraints on developable land impede decentralization. In that sense, New York is compact because its geography discourages (political support for) the decentralization of its business and shopping. Alternatively, cities able to expand in any direction, such as Houston or Atlanta, have shapes that are skewed less. Here a



(a) San Diego: Density 2000

(b) Phoenix: Density 2000

(c) San Diego: Change in Density 2000-2010

(d) Phoenix: Change in Density 2000-2010

Figure 1: Distribution of Distance to Center (in Miles) – San Diego vs. Phoenix

majority of voters push for decentralization. We will add that tighter limits on building height risk the CBD's demise. Tighter constraints on building height are asymmetric by nature. They tend to weaken centrists and strengthen decentrists.

As time progresses, function and form interact. Political decisions on where to locate jobs and shops not just *reflect* urban form (function follows form), but of course also *drive* it (form follows function). We will address this mutual interaction. Cities that have many centrists and few decentrists initially will not take decisions that weaken the CBD. In contrast, cities that start out with many decentrists and few centrists to begin with are more likely to embrace decentralization. Decentralization subsequently reverses the structure of rents, which has suburban buildings grow in number and height. This in turn reinforces the number of decentrists, and eventually locks city functions into their ultimate ring road position. Two very different equilibria coexist, and either may actually emerge, depending on the city's initial shape (Proposition 5).

U.S. Census Bureau provides detailed data on population as well as population-weighted densities by distance (in miles) from the city center, for all U.S. metropolitan areas. These data have been collected for years 2000 and 2010, with respect to identical geographies (Wilson et al. (2012)). "Population by distance from the center" coincides precisely with this paper's idea of city shape, and so these data are uniquely suited to illustrate aspects of our theory below. For each metropolitan area we avail of the full distribution of the urban population across the city's rings. We briefly preview these shape data on two specific cities, to give a flavor of the path dependence that both (i) is suggested by our theory and (ii) may be consistent with our sample (Fig. (1)).

San Diego, on the one hand, is coastal. From this paper's perspective, this is why its shape not just is noticeably skewed to the right to begin with (diag. (a)). It also barely changes over the decade that follows (diag. (c)). Small relative losses (red on screen in diag. (a)) are balanced by small relative gains (not shown in (a) but shown in (c)) in both, central and peripheral, rings. Phoenix, on the other hand, is continental, and barely land-constrained. And so its shape is much less skewed already back in 2000 (diag. (b)). Our theory has us expect that the mass of its population shifts out over the ten following years, and that is precisely what it does (diag. (d)). Larger relative losses (in red in diag. (b)) impose in, yet are also restricted to, central rings, while larger relative gains (not shown in (b) but shown in (d)) occur in peripheral rings only.

The paper has seven sections. Section 2 sets out city shape as our notion of urban form and reviews the urban political economy of decentralization in an owner-occupier setting. Section 3 shifts to a landlord-tenant setup, and shows how to extract minimum shares of centrists and decentrists from city shape even then. These shares are new to the literature (if we except a companion paper with an independent, linear programming, approach, cf. Dascher (2018)). Section 4 decomposes shape into topography and building height. Section 5 lets shape adjust, too. In the long run, shops and jobs either cluster around the traditional center, with residents staying around, or relocate to the periphery, with residents following suit. Section 6 offers some evidence with respect to the model's key predictions. Section 7 concludes.

2 Shape

We depart from a simple variant of the neoclassical urban model pioneered by Wheaton (1973), Pines/Sadka (1986) and Brueckner (1987). A closed monocentric city extends at most $\widetilde{r}$ miles out from the center – where $\widetilde{r}$, also referred as city size, is determined shortly. Each resident occupies one unit of housing (an "apartment") and initially commutes to the center (CBD) to work and shop. Round trip commuting costs for a resident living at distance r from the center are tr, so that Ricardian rent q becomes $q(r) = t(\widetilde{r} - r)$. City population is s.

Apartments are built by profit maximizing investors. One unit of capital k poured into a building site of unit area yields $h(k)$ units of floor space, where $h' > 0$ and $h'' < 0$ (Brueckner (1987)). If p is the price of capital, investors choose k so as to satisfy the $q(r)h_k(k) = p$ necessary for maximum profit. The optimal capital will clearly depend on rent q and capital price p, and so can be written as $k(t(\widetilde{r} - r), p)$. Let $h(r)$ be shorthand for the corresponding optimal building height $h(k(t(\widetilde{r} - r), p))$.[2] In equilibrium $\widetilde{r}$, the city boundary, has the housing market clear:

$$\int_0^{\widetilde{r}} a(r)h(r)\, dr \;=\; s, \tag{1}$$

where $a(r)$ is available land or "topography" in the unit-width ring r miles away from the CBD. The city size and building heights that the closed city model concludes with are those that this paper's model begins with.

The paper's central policy metaphor is the ring road. The ring road offers an alternative set of locations for jobs and shops, and hence is a potential rival for the traditional CBD.

[2] We ignore the integer constraint arising from apartments needing to fit into buildings.

Locations along it connect just as well, or almost as well, to one another as locations in the traditional core do. Vienna provides an early prominent example of a ring road, so much so that its ring road is actually called the "Ring".[3] Let a costless ring road be proposed, by some interested party (identified shortly). A ring road would shift jobs and shops, two of the city's most important functions, from their inherited position (the CBD) out to the urban boundary $\widetilde{r}$ (the ring road), in a single instant and with t unchanged.[4] Instead of travelling r_i to the center of the city, a resident in ring i now travels $\widetilde{r} - r_i$ to the city's periphery, to work and shop in those office parks and shopping malls strewn along the ring road. The ring road is approved if it captures a majority of the vote.

Urban planners and architects have always debated how rearranging the city's functions – providing room for working, shopping, sleeping, eating, etc. – affects welfare (Dantzig/Saaty (1972)). Economists have long sought to understand the effects of changing density on productivity (see Ahlfeldt/Pietrostefani (2017) for a recent survey of this literature). Shifting business away from the CBD and out to the ring road is unlikely not to affect productivity. Throughout this paper, nonetheless, we will assume away changes in productivity. This we justify by considering our analysis a benchmark. We attempt to understand a shift in jobs and shops if that shift is costless and productivity-neutral, and leave the evaluation of decentralization – necessitating assumptions on technology, construction costs and even externalities (e.g. Brueckner/Helsley (2011)) – to future work.

Arguably there are many different ways to capture a city's form. Our fundamental representation of urban form will be the city's *shape*,

$$f(r) \;=\; a(r)h(r)/s, \tag{2}$$

as defined by Arnott/Stiglitz (1981). City shape $f(r)$ provides us with the share of residents in the one-mile-wide ring r miles from the center. Further aspects of urban form readily derive from f, such as city shape's skew and city shape's compactness. These will be introduced further below. Finally, we may also briefly point to the (city's residential) skyline. This is simply building height h. The skyline indicates the number of residents on a given unit of land at distance r from the center, and so here coincides with the urban literature's concept of population density (e.g. McDonald (1989), Kim (2007)).[5]

To keep our exposition as simple as possible we divide the city into $n = \widetilde{r}$ concentric rings of 1 mile width. Ring i residents travel freely to commuting nodes at $r_i = i - 1/2$, from where they go on to the CBD at cost tr_i. The number of residents in ring i is app. $f(r_i)s = b_i$, and so our city's shape can now be summarized by the vector $(b_1/s, \ldots, b_n/s)$.[6] We also

[3] Victor Gruen appears to have modeled his malls on Vienna's ring (Hardwick (2010)).

[4] While a simultaneous shift of course is unlikely, any shift from the CBD to the city periphery might be helped along by coordination. Rauch (1993) points to the role of business park developers in coordinating industry relocation, while the shopping center industry attests to the importance of retail space developers in coordinating movements in retail. Sometimes it is even the city's government that provides this coordination. For example, Vienna's ring road was where suddenly one could find "the new exchange, the university, a civic and national government section around the new town hall and parliament house, a museums section, the opera house" (Girouard (1989)).

[5] Of course, the latter is only true because we have set housing consumption to 1.

[6] We choose b_i to denote residents in ring i to make our notation consistent with Dascher (2018).

let F denote the corresponding distribution function. It is $F(r_i)$ the share of residents in all rings 1 through i, whereas $\widetilde{F}(r_i)$ denotes that in all rings i through n.[7]

We briefly explore city shape's political implications from an owner-occupier angle. Suppose all residents are owner-occupiers. All those $F(r_{n/2})s$ owner-occupiers who live closer to the center than to the ring road are centrists and will oppose replacing the traditional center with the ring road; while all these $\widetilde{F}(r_{(n/2)+1})s$ owner-occupiers living further from the center are decentrists; they will endorse the ring road. We define owner-occupying centrists' "margin of victory" as the share of centrists minus that of decentrists μ, or

$$\mu \;=\; \sum_{i=1}^{n/2} (b_i - b_{n+1-i}) \Big/ s, \tag{3}$$

or easily computable $F(r_{n/2}) - \widetilde{F}(r_{(n/2)+1})$. If $\mu \geqslant 0$ the city will not decentralize; while if $\mu < 0$ it will.

3 Inspecting Shape

For the reasons outlined earlier, we embrace a landlord-tenant setting for the remainder of the paper. Each landlord owns two apartments, in rings i and j, where i may or may not equal j. One of these apartments she occupies herself, the other she rents out to a tenant. If she lives in i herself she enjoys utility $t(r_n - r_j) - tr_i$ (after discarding invariant wage).[8] Once jobs and shops have migrated out to the ring road (located at $\widetilde{r}$), her utility becomes $-t(\widetilde{r} - r_i) + t(r_j - r_1)$. The resulting change is positive iff

$$(r_i \;+\; r_j)/2 \;\geqslant\; \widetilde{r}/2. \tag{4}$$

Then the landlord is a decentrist. Otherwise we say she is a centrist. Where owner-occupiers differ by how their *single* property compares to "midtown" $\widetilde{r}/2$, landlords differ by how their *average* property does.

Tenants expect the city to extend over the same area always.[9] Equivalently, tenants expect their cost of living, equal to tr_n, to not change.[10] Thus they are indifferent to the ring road, abstaining from the vote on it. It is up to landlords – centrists and decentrists – to decide. As explained, we cannot compute either group's true number. We typically do

[7] In order to address sprawl, for example, Glaeser/Kahn (2004) refer to selected points on the graph of F when providing percentages of population within an inner 3-mile ring, 5-mile ring and 10 mile ring for the 150 largest U.S. metropolitan areas. Likewise, Baum-Snow (2007a) and Kim (2007) compute (changes in) the fraction of metro area population in the central city.

[8] This utility is independent of whether she resides in i or j, which is why further below we may always put the landlord into that of her two properties that suits our exposition best.

[9] Both landlords and tenants are myopic because post-decentralization the city will contract in size as it adjusts to its long run equilibrium (see section 5). Such adjustment surely takes a number of decades. Having residents anticipate these long-run effects of their voting on the ring road could be modeled as a two stage game but should not overturn the essence of our analysis.

[10] This is because wherever a tenant expects to travel longer (less) post-ring-road-construction, he can also expect rent to fall (rise). As long as city size is expected to remain the same, the two changes are expected to just offset each other. This is a familiar property of the simple closed city.

not know which tenant any given landlord is matched up with. Put differently, we do not observe the $n \times n$ matrix of landlord-tenant matchings across all those $i, j = 1, \ldots, n$ city rings. And so we cannot infer landlords' affiliation to either the centrist or the decentrist camp. We can, however, take a guess at the smallest numbers of centrists and decentrists that can possibly nest into the given city shape. These latter estimates later help us predict the outcome of the ring road vote. We start by offering a heuristic estimate of centrists.

Bounding Centrists. Consider all b_1 apartments in the first ring. These must be owned by landlords suffering from (and hence opposing) the ring road ... except for those owned by ring-1-landlords who own their other apartment in ... the last ring, ring n. (For these, we may note, inequality (4) reduces to an equation.) Preparing for the worst, assume that every apartment in that last ring is owned by a ring-1-landlord (rather than by a landlord living in any of the other rings $2, \ldots, n-1$). So of all of ring 1's apartments really only $b_1 - b_n$ apartments can possibly pinpoint centrists. Moreover, those remaining $b_1 - b_n$ apartments might even be occupied by both landlords *and* their tenants, further reducing the number of centrists we can be certain of. We conclude that $(b_1 - b_n)/2 = \underline{l}^c(1)$ is the smallest conceivable number of centrists that housing stocks b_1 and b_n allow for.

This number is our first lower bound on the true number of centrists, l^c. Of course, if the city shape is such that $b_1 < b_n$ then $\underline{l}^c(1) < 0$. Then $\underline{l}^c(1)$ is not a very convincing lower bound. The zero (or "ignorant") lower bound always is a better choice then. Yet this need not bother us. There are many more lower bounds on offer. For example, apartments in the first two rings, $b_1 + b_2$, give another conservative centrist estimate if we allow for (i) all $b_{n-1} + b_n$ tenants to be matched up with some landlord from those first two rings and (ii) remaining apartments $(b_1 + b_2 - (b_{n-1} + b_n))$ to be matched up with one another. Making both adjustments yields $((b_1 + b_2) - (b_{n-1} + b_n))/2 = \underline{l}^c(2)$ as another lower bound on centrists.[11]

Already we have identified two lower bounds on centrists. Let us generalize their underlying common intuition. Including all j first, as well as last, rings, the partial sum $\underline{l}^c(j) = \sum_{i=1}^{j}(b_i - b_{n+1-i})/2$ gives the j-th lower bound on centrists, where $j = 0, \ldots, n/2$.[12] From the set of lower bounds introduced thus, or $\{0, \underline{l}^c(1), \ldots, \underline{l}^c(n/2)\}$, we are naturally inclined to choose the largest, defined as $\underline{l}^c$,

$$\underline{l}^c \;=\; \max_j \sum_{i=1}^{j} \Big(b_i \;-\; b_{n+1-i}\Big)\Big/2 \qquad (j = 0, 1, \ldots, n/2), \tag{5}$$

as the best of our lower bounds on the true number of centrists. Further, if we introduce "ring difference" δ_i as the difference between housing in "leading ring" i and that in its antagonist "lagging ring" $n + 1 - i$, we may also write $\underline{l}^c = \max_j \sum_{i=1}^{j} \delta_i/2$.

[11] It may be helpful to add that the simple (non-cumulative) ring difference $(b_2 - b_{n-1})/2$ cannot be another lower bound. Apartments in the second ring may also house landlords who own their second property in the last, n-th, ring and who hence are strictly better off by adopting the ring road. This disqualifies $(b_2 - b_{n-1})/2$ as lower bound.

[12] We adopt the well-known convention that the sum is zero if j is zero (e.g. Halmos (2012)).

Bounding Decentrists. A similar argument applies towards bounding decentrists. Here consider the last ring first. Note that all of its b_n apartments must be tied up in matches that incite their owners to become decentrists. The only exception are those matched up with an apartment in ring 1. To assess minimum conceivable support for the ring road among owners of ring n apartments suppose then that every resident in ring 1 is linked to another in ring n (rather than in any of the other rings). Then at best $b_n - b_1$ apartments point to landlords who would benefit from the policy proposal. More pessimistically yet, suppose all of these remaining ring n-apartments not just house tenants, but house their respective landlords, too. Then $(b_n - b_1)/2 = \underline{l}^d(1)$ emerges as our first lower bound on the true number of decentrists l^d. Much as above, it may just so happen that $\underline{l}^d(1) < 0$.

But then again, there are many more lower bounds here, too. For example, another lower bound derives from consulting both the two last and first rings, and comes to $((b_n+b_{n-1})-(b_1+b_2))/2 = \underline{l}^d(2)$. Generally, if the last, as well as first, j rings are included, lower bounds on decentrists can be written as $\underline{l}^d(j) = \sum_{i=1}^{j}(b_{n+1-i} - b_i)/2$, where $j = 0, \ldots, n/2$. The greatest from among all these lower bounds $\{0, \underline{l}^d(1), \ldots, \underline{l}^d(n/2)\}$ is

$$\underline{l}^d \;=\; \max_j \sum_{i=1}^{j} \Big(b_{n+1-i} \;-\; b_i\Big)\Big/2 \qquad (j = 0, 1, \ldots, n/2), \tag{6}$$

or $\max_j(-\sum_{i=1}^{j}\delta_i/2) = -\min_j \sum_{i=1}^{j}\delta_i/2$ more briefly. This is the greatest of our lower bounds on decentrists. We add that shortly it will be useful to be able to refer to the maximizer in (5) as j^*, and to that in (6) as j^{**}.

Dividing (5) and (6) by the number of those who actually vote, $s/2$, gives corresponding lower bounds on voter *shares*, $\underline{\lambda}^c$ and $\underline{\lambda}^d$ (Proposition 1). We emphasize that the resulting formulas for $\underline{\lambda}^c$ and $\underline{\lambda}^d$ provide us with *estimators* of centrists and decentrists (rather than just estimates). These formulas provide us with *functions* of city shape. Hence they allow for *any* shape that could possibly be encountered in applications. And by design they are independent of the (unobservable) actual landlord-tenant assignment at hand.

Our heuristic approach does miss one important point, however. As a companion paper points out, minimizing centrists (or minimizing decentrists) with the shape of the city given can be cast as a linear program (Dascher (2018)). Somewhat remarkably, taking the independent approach of solving these linear programs yields minimum centrists just equal to $\underline{\lambda}^c$, and minimum decentrists just equal to $\underline{\lambda}^d$ (Dascher (2018, Proposition 1)). In other words, our shares $\underline{\lambda}^c$ and $\underline{\lambda}^d$ are not just *some* lower bounds to λ^c and λ^d, respectively. They actually are *greatest* lower bounds.[13] At some risk of oversimplification, while greatest lower bounds $\underline{\lambda}^c$ or $\underline{\lambda}^d$ never overestimate λ^c or λ^d (irrespective of which city we analyze), they also never underestimate λ^c or λ^d by more than is "just necessary".

Proposition 1: (Shape Previews Political Economy)

The minimum share of centrists, $\underline{\lambda}^c$, and that of decentrists, $\underline{\lambda}^d$, in the total vote on

[13]Since the minimum exists, it coincides with the greatest lower bound. This is because (i) the minimum is a lower bound and (ii) there is no lower bound strictly greater than the minimum.

decentralization are $\underline{\lambda}^c = \max_j \sum_{i=1}^{j} \delta_i/s$ *and* $\underline{\lambda}^d = -\min_j \sum_{i=1}^{j} \delta_i/s$*, respectively. Cities where* $\underline{\lambda}^c > 0.5$ *vote against, while cities where* $\underline{\lambda}^d > 0.5$ *vote for, decentralization.*

Greatest lower bounds $\underline{\lambda}^c$ and $\underline{\lambda}^d$ uncover the city's political economy from its physical shape. They tie one concept that is physical to another that is political. If $0.5 < \underline{\lambda}^c$ then $0.5 < \lambda^c$, too, and so no decentralization occurs; while if $0.5 < \underline{\lambda}^d$ then $0.5 < \lambda^d$ also, and hence decentralization unfolds. It is in this sense that the city's form predicts its functions' ultimate location: "function follows form". Admittedly, both lower bounds may fail to exceed 0.5. (This will be true of our sample of metro areas in section 6.) But even then either greatest lower bound can still be valuable. Centrists' influence should be strictly increasing in $\underline{\lambda}^c$ and strictly decreasing in $\underline{\lambda}^d$, and a similar property should apply to decentrists' weight in local politics. In the empirical analysis below we will examine these (weaker) statements.

We have extracted minimum centrist share $\underline{\lambda}^c$ and minimum decentrist share $\underline{\lambda}^d$ from city shape $(b_1/s, \ldots, b_n/s)$. Not only does this extraction take little computational effort (as illustrated in section 6). Also, it relies on easily observable, because aggregate, city shape data only. Moreover, note that $\underline{\lambda}^c$ and $\underline{\lambda}^d$ sum to 1 at best, or $\underline{\lambda}^c + \underline{\lambda}^d \leqslant 1$. Equivalently,

$$\sum_{i=1}^{j^*} \delta_i + \Big(- \sum_{i=1}^{j^{**}} \delta_i \Big) \;\leqslant\; s. \tag{7}$$

That this inequality is true is easy to see. If, for example, $j^{**} < j^*$ then the sum on the l.h.s. reduces to $\sum_{i=j^{**}+1}^{j^*} \delta_i$, which surely is smaller than s.[14] Finally, one may suspect that $1 - \underline{\lambda}^d$ is an upper bound on the true centrist share, and that $1 - \underline{\lambda}^c$ is one on true decentrists' share, and this is indeed the case (Dascher (2018, Proposition 3)).

A slightly different perspective is instructive here. Recalling how we have introduced δ_i, F and $\widetilde{F}$, we may also cast minimum centrists in terms of the c.d.f. of city shape, i.e. as $\max_j \Big(\sum_{i=1}^{j} b_i/s - \sum_{i=1}^{j} b_{n+1-i}/2 \Big)$ or

$$\underline{\lambda}^c \;=\; \max_j \Big(F(r_j) \;-\; \widetilde{F}(r_{n+1-j}) \Big), \tag{8}$$

and a similar statement is true for minimum decentrists, to whom

$$\underline{\lambda}^d \;=\; -\min_j \Big(F(r_j) \;-\; \widetilde{F}(r_{n+1-j}) \Big) \tag{9}$$

applies. Our two lower bounds share juxtaposing the shape distribution's two *tails*. But they differ in the (common) length they assign these two tails. While $\underline{\lambda}^c$ chooses a common tail length (of j^*) that renders the surplus of the left tail over the right greatest, $\underline{\lambda}^d$ picks the common tail length (equal to j^{**}) that minimizes that surplus.

Compactness. Let us next establish a causal link from "urban compactness" to city functions, also. Let us say that a city is more *compact* if the distribution of distance to

[14]The case $j^* < j^{**}$ is discussed in a similar fashion. And the case where j^* and j^{**} are equal even has the l.h.s. of (7) drop to zero.

the traditional center, $F(r_i)$, is strictly greater at all r_i except r_n (where F attains 1).[15] Assume this is the case. Then all of the differences in brackets on the r.h.s. of (8), as the choice set, obviously expand. And hence the maximum expands, too. And so $\underline{\lambda}^c$ is strictly increasing in greater compactness. In a similar fashion, $\underline{\lambda}^d$ is strictly decreasing in greater compactness. Anything that makes the city more (less) compact strengthens (weakens) the CBD.

Proposition 2: (Compactness Previews Political Economy)

The minimum share of centrists, $\underline{\lambda}^c$, is strictly increasing in urban compactness, while the minimum share of decentrists, $\underline{\lambda}^d$, is strictly decreasing in it.

We briefly pause to look into how $\underline{\lambda}^c$ and $\underline{\lambda}^d$ not just bound important urban interests (Proposition 1), but may inversely even, in and by themselves, reveal part of the city's shape. To sketch this idea, best we focus on $\underline{\lambda}^c$. Assume that we know $\underline{\lambda}^c$ to just have increased. By (8), then we also know the difference between the city shape distribution's relevant tails to have grown. We may think of this change as a combination of two different scenarios. Either the left tail is fatter than the right one initially, and now relevant tail lengths have grown. Or, both tails retain their initial length yet now the left tail has become thicker while the right tail has thinned out. Either way the shape distribution's tails have become more distinct, to the extent of creating the impression of growing city shape *skewness.*

Skew. Now we are set to bring in our perspective on city shape *skew.* To best uncover skew's relationship with urban political economy, we make use of our own (simple) index.[16] Based on weights $w_i = 1 - r_i/r_{n/2}$, we define skew σ by

$$\sigma \;=\; \sum_{i=1}^{n/2} w_i \left(b_i \;-\; b_{n+1-i}\right)\Big/ s \tag{10}$$

or $\sum_{i=1}^{n/2} w_i \delta_i / s$, a weighted sum of ring differences. Since weights w_i are decreasing in i, early ring differences (i close to 1) receive a larger weight than late ones (i close to $n/2$). Intuitively, this is how it should be. For instance, early positive ring differences (i close to 1) make us perceive shape to be positively skewed even if later ring differences (i close to $n/2$) are negative.[17]

[15] Alternative indicators of urban form are found in the literature. Burchfield et al. (2006), for instance, calculate the percentage of undeveloped land in the neighborhood of each unit of a metro area's developed land, then average this percentage across developed cells, indicating the extent to which urban development is "scattered" or "compact". This index shifts the focus from the compactness of population towards the connectedness of the settlement area. The latter must be important for environmental purposes (as when large rainfall strikes an urban area as in Houston 2017), but may be less relevant when assessing political power in the struggle over whether to relocate urban functions and when analyzing the dynamics of decentralization (given its lack of emphasis on population density).

[16] This index first appears to be introduced in Dascher (2014). There it is shown to also be a multiple of the landlord class's utilitarian welfare, $t\sigma(\tilde{r}/2)s$.

[17] As the diagrams on selected cities in the paper's sample (Section 6, Fig. (2)) below will illustrate, σ does fall as shape becomes visibly less skewed to the right. Also, σ passes the necessary test for symmetry. By definition, if shape is symmetric then $b_i = b_{n+1-i}$ for all i and hence $\sigma = 0$. We also already note that $-1 < \sigma < 1$, as is shown shortly (in the context of Proposition 3).

To focus on centrists first, the following short sequence of inequalities documents how σ bounds $\underline{\lambda}^c$ from below. We have

$$\sigma \;=\; \sum_{i=1}^{n/2} w_i\,\delta_i/s \quad\leqslant\quad \max_j \sum_{i=1}^{j} w_i\,\delta_i/s \tag{11}$$

$$<\quad \max_j \sum_{i=1}^{j} \quad \delta_i/s \;=\; \underline{\lambda}^c. \tag{12}$$

Inequality (11) is certainly true because the sum (of weighted ring differences) is greater if its upper limit can be chosen freely. This leaves us with inequality (12). That it holds is easy to agree with if $\delta_i \geqslant 0$ for all i. Replacing any initial weights smaller than 1 by 1 will certainly not decrease the sum (of weighted ring differences). If, more generally, $\delta_i < 0$ for some or even most i the proof is somewhat subtler. It needs to resort to exploiting properties of the maximum, and hence is delegated to the Appendix.

Skew bounds minimum centrists, and certainly true centrists, from below. When skew is positive it permits the city analyst to assess centrists' minimum strength. Not too surprisingly, there is an analogous statement for decentrists. Much as above,

$$\sigma \;=\; \sum_{i=1}^{n/2} w_i\,\delta_i/s \quad\geqslant\quad \min_j \sum_{i=1}^{j} w_i\,\delta_i/s$$

$$>\quad \min_j \sum_{i=1}^{j} \quad \delta_i/s \;=\; -\,\underline{\lambda}^d$$

The proof parallels that given for the previous set of inequalities. It is obvious if all ring differences are negative. Then replacing weights smaller than 1 with a uniform weight of just 1 will only reduce the overall sum (of weighted ring differences). When some ring differences are positive, then again a subtler argument, akin to the one set forth in the Appendix, is needed.

Skew bounds the negative of minimum decentrists from above. Equivalently, $-\sigma < \underline{\lambda}^d$. We conclude that skew is useful whatever its sign. Joining the inequalities discussed above and recalling that our lower bounds are shares, we also conclude that $-\underline{\lambda}^d < \sigma < \underline{\lambda}^c$. From this we immediately see that skew's range is restricted to the interval $(-1, 1)$. We may even add that minimum centrists and minimum decentrists are monotonic in σ as long as city size $\widetilde{r}$ remains the same. As σ goes up, so must at least one of all those ring differences δ_1 to $\delta_{n/2}$ featuring in it. Since the choice set in the definition of $\underline{\lambda}^c$ rises, too, $\underline{\lambda}^c$ must (weakly) rise. An increase in skew foreshadows greater "centrism" (Proposition 3).

Proposition 3: (Skew Previews Political Economy)

Skew σ bounds shares of minimum centrists and minimum decentrists as in $-\sigma < \underline{\lambda}^d$ and $\sigma < \underline{\lambda}^c$. Moreover, $\underline{\lambda}^c$ is increasing, and $\underline{\lambda}^d$ decreasing, in σ (at given $\widetilde{r}$).

Towards the end of this section we connect $\underline{\lambda}^c$ and $\underline{\lambda}^d$ with margin μ (see (3)), too. At one extreme, if all ring differences are positive then $j^* = n/2$. Then margin μ and minimum share of centrists $\underline{\lambda}^c$ coincide. At the other extreme, if all ring differences are negative then $j^{**} = n/2$. Then margin μ and the negative of minimum decentrists' share, $-\underline{\lambda}^d$, coincide. Turning to the general case, where neither j^* nor j^{**} necessarily attain $n/2$, we have both $\mu \leqslant \underline{\lambda}^c$ and $-\underline{\lambda}^d \leqslant \mu$. Putting both inequalities together gives $-\underline{\lambda}^d \leqslant \mu \leqslant \underline{\lambda}^c$. It is in this sense that the owner-occupied city's politics really are "framed" by (are a special case of) the interests of the resident-landlords-cum-tenants city.

4 Decomposing Shape

This far we have established initial city shape's relevance for the political economy of decentralization. A natural next step is to look into the forces behind initial city shape. To this end we decompose shape f into topography a and skyline h, as in definition (2). At first sight not too much can be said for general functions a and h. But consider two extreme cases. On the one hand, suppose the neoclassical city is height-unconstrained but linear. Then f is decreasing in r, ring differences are strictly positive, and hence $\underline{\lambda}^c = \mu$ while $\underline{\lambda}^d = 0$ (by Proposition 1). On the other hand, suppose a height limit restrains buildings throughout the city not to be taller than 1, while a always equals the full circular land area. Then f is strictly increasing in r, and so ring differences are strictly negative always. Here $\underline{\lambda}^d = -\mu$ and $\underline{\lambda}^c = 0$ (by Proposition 1).

Analyzing the constraints of topography or building height in a more general city is more difficult. How do $\underline{\lambda}^c$ and $\underline{\lambda}^d$ differ from one equilibrium that does not submit to any constraints (indexed 0) to another (indexed 1) that does? Our results, if based on making somewhat stylized assumptions, will be straightforward: Natural barriers in the vicinity of the city center – and not just coastlines or lakeshores – drive $\underline{\lambda}^c$ up, while building height limits, as the quintessential and pervasive aspect of zoning, raise $\underline{\lambda}^d$ (Proposition 4 below).

Topography. Let our city be located close to water bodies like oceans, rivers, wetlands or lakefronts, or to terrain that is difficult to develop such as steep hills and mountains. And specifically let this "first nature" impediment leave land supply a unchanged at distances $r \leqslant r_{n_0/2}$ yet reduce it down to midtown's land area, $a(r_{n_0/2})$, or less at *all* distances $r > r_{n_0/2}$. Given housing market clearance (1), imposing such a constraint drives city size $\tilde{r}$ up from $\tilde{r}_0$ to some higher $\tilde{r}_1$.[18] Then Ricardian rent $t(\tilde{r} - r)$ and building heights h must inevitably go up along with it. Moreover, distribution function F's support expands from $\{r_1, \ldots, r_{n_0}\}$ initially to $\{r_1, \ldots, r_{n_0}, r_{n_0+1}, \ldots, r_{n_1}\}$ then. Let F_0 and F_1 denote corresponding initial and subsequent city shapes, respectively.

We can show that centrists' minimum share must rise, too. By equation (8), $\underline{\lambda}^c$ is the maximum of all cumulative ring differences. Consider the two following (equal-sized) sets of cumulative ring differences

$$\{F_0(r_1) - \widetilde{F}_0(r_{n_0}), \ldots, \quad F_0(r_i) - \widetilde{F}_0(r_{n_0+1-i}) \quad, \ldots, F_0(r_{n_0/2}) - \widetilde{F}_0(r_{(n_0/2)+1})\}, \quad (13)$$

$$\{F_1(r_1) - \widetilde{F}_1(r_{n_1}), \ldots, \quad F_1(r_i) - \widetilde{F}_1(r_{n_1+1-i}) \quad, \ldots, F_1(r_{n_0/2}) - \widetilde{F}_1(r_{n_1+1-n_0/2})\}. \quad (14)$$

[18]Would city size not increase then the same building heights would apply to a smaller land area, to the extent that housing supply fell short of demand. This would contradict housing market equilibrium.

The first set, given in (13), is the full constraint set from which to choose $\underline{\lambda}_0^c$. The second set, shown in (14), is a proper subset of the constraint set for finding $\underline{\lambda}_1^c$. The second set is not the full constraint set in that it lacks cumulative ring differences for $i = n_0/2+1, \ldots, n_1/2$. These latter cumulative ring differences we ignore for now ... only to conclude shortly that we may ignore them for good. Our single key insight is that any (i-th) element in the second set (given in (14)) must exceed any corresponding (i-th) element in the first (see (13)).

First, $F_0(r_i) < F_1(r_i)$ for $i = 1, \ldots, n_0/2$. Consistently taller buildings on an unchanged land area endow any of the constrained city's *first* i rings with a greater share of residents (than the unconstrained city's *first* i rings). Next, $\widetilde{F}_0(r_{n_0+1-i}) > \widetilde{F}_1(r_{n_1+1-i})$ for $i = 1, \ldots, n_0/2$. Identical building heights[19] joint with consistently less land provide the constrained city's *last* i rings with a smaller share of residents (than the unconstrained city's *last* i rings). And so any entry in the second set is *greater* than its corresponding entry in the first. Picking the greatest element from the second set must result in a "greater greatest cumulative ring difference" (than picking from the first). Accounting for those cumulative ring differences not even included yet in (14) can only improve our choice of maximum further. And so $\underline{\lambda}_0^c < \underline{\lambda}_1^c$ (Proposition 4, Part (i)). Land constrained cities are more robust vis-à-vis proposals to decentralize jobs and shops.[20]

Building Height. Building height h depends not just on distance to the center. It also depends on bedrock solidity, zoning, neighborhood characteristics, and so on. There are many reasons why building height may vary across cities and over time. Here we offer a stylized analysis of the effect of imposing building height constraints. Specifically, we impose a binding height limit on all rings up to $r_{n_0/2}$. One immediate consequence of this is to increase city size from $\widetilde{r}_0$ to some higher $\widetilde{r}_1$, to the extent that rents throughout the city rise, too. Another is that cumulative ring differences up to ring $n_0/2$ will obviously fall.

In the height-constrained city, now smaller (because height-constrained) buildings populate the first $n_0/2$ rings (than in the unconstrained city). This is why $F_0(r_i) > F_1(r_i)$ for all $i = 1, \ldots, n_0/2$. At the same time, the height-constrained city features identical building heights[21] on larger lands across the last $n_0/2$ rings (than the unconstrained city does). And so $\widetilde{F}_0(r_{n_0+1-i}) < \widetilde{F}_1(r_{n_1+1-i})$ for all $i = 1, \ldots, n_0/2$. Each (i-th) element of set (14) falls short of its corresponding (i-th) element in set (13). A height limit reduces *every* one of those initial $n_0/2$ cumulative ring differences. Thus the minimum of all these must be even smaller than before and hence the minimum share of decentrists, $\underline{\lambda}^d$, (as the negative of that latter minimum) must be even greater (Proposition 4, Part (ii)).[22] In that sense,

[19] Ring $n_1 + 1 - i$ is as far in from the city border $\widetilde{r}_1$ as ring $n_0 + 1 - i$ is in from the city border $\widetilde{r}_0$. Thus Ricardian rents are the same in both cases. Hence building heights are.

[20] As one referee has pointed out, actual land-constrained cities may be more robust for the simple reason that they lack the peripheral land to be offered to businesses fleeing the CBD. (The model circumvents this issue by having assumed that business demand for land is zero.)

[21] Building heights in these rings are identical following the same argument as in footnote 19.

[22] Skew should also suffer, even as we do not provide a formal proof here. In their policy simulation on Bangalore, Bertaud/Brueckner (2005) illustrate how the introduction of a floor-to-area restriction (FAR) reduces that city's skew.

the height-controlled city is *less resilient* to calls for decentralization than the uncontrolled city is.

Proposition 4: (Topography and Height Limits Preview Political Economy) *(i) Centrists' minimum share $\underline{\lambda}^c$ is greater in cities constrained by their topography than in cities unconstrained. (ii) Decentrists' minimum share $\underline{\lambda}^d$ in cities with binding height limit exceeds that in cities without it.*

5 Evolving Shape

Up to this section city shape has been given. This may be justified by confining the analysis to the short run. Now allow for the long run, assume away any constraints to topography or building height, and let shape adjust to the ring road decision, or, let "form follow function".[23] First, suppose $0.5 < \underline{\lambda}^d$ initially. Then $0.5 < \lambda^d$ a fortiori, and the decentrist majority reverses the roles of center and periphery in an instant. Shops and jobs now locate on the ring road circling the disk-like residential area, and residents commute out to this ring road to both work and shop. Let subscripts "cc" and "rr" denote initial ("central city") and new ("ring road") equilibrium, respectively.

Surely the new maximum distance from shops and jobs (now located at $\widetilde{r}$) to the new marginal resident (now at $\widetilde{r}_{rr}$) may no longer equal $\widetilde{r}$: It must be less. (The disk "has a hole".) This we quickly show by contradiction. Suppose residents still live in the former CBD, post-decentralization. That is, $\widetilde{r}_{rr} = 0$. Then the schedule of rents merely reverses. Investors fill large rings close to the ring road with buildings as large as those they had previously built on those tiny rings close to the historical CBD. Because these large peripheral rings have greater area, this raises aggregate housing supply beyond what it ever was. This in turn contradicts our housing market clearance condition (1) because demand (or population) s never changed. We conclude that $0 < \widetilde{r}_{rr} < \widetilde{r}$. Noone lives near the historical center anymore.

Let us define $\widehat{r}$ as the distance of the ring at which old and new rent schedule intersect. And define $\widehat{j}$ as the index of the ring that contains $\widehat{r}$. For all $j < \widehat{j}$, buildings must now be smaller than what they used to be. There $F(r_j)$ must also be smaller than before. Moreover, for all $j > \widehat{j}$ buildings grow in height. And so in these rings, $\widetilde{F}(r_j)$ must be greater than what it used to be. Equivalently, there $F(r_{j-1})$ must be smaller than what it used to be. To summarize, $F(r_j)$ drops for all j except n. In terms of earlier terminology, decentralization makes the city less compact. In turn this has minimum centrists fall, to $\underline{\lambda}^c_{rr} < \underline{\lambda}^c_{cc}$, and minimum decentrists rise, to $\underline{\lambda}^d_{rr} > \underline{\lambda}^d_{cc}$ (Proposition 2). Note the causality reversal here. Now it is decentralization driving our political shares, rather than political shares driving decentralization (as was still true in the previous section).

[23] In what follows assume that all land beyond the city's original boundary $\widetilde{r}_0$ is not to be developed. This is not a restrictive assumption. If such marginal land could be developed, this would strengthen our results – residents would even settle on *both* sides of the ring road and hence locate even further from the original center.

Any attempt to reinstate the CBD must fail. This is because all those intermittent long run height and shape adjustments only reinforce the ring road's attraction. Shape adjustments strengthen decentrists, and weaken centrists. We have reached a stable ring road ("rr") equilibrium. Office buildings and large residential buildings line the ring road, to be succeeded by ever smaller residential structures as we near the city's former center. Even before we get to that center, at last, ring population has dropped to zero.[24] – So far we have discussed a city with $\underline{\lambda}^d > 0.5$ initially. Alternatively consider a city where $\underline{\lambda}^c > 0.5$ to begin with. This city *cannot help but* turn down the ring road proposal, whatever the actual landlord-tenant matching. Neither shape nor the fact of a centrist majority are subject to change. This is the central city ("cc")-equilibrium. It is stable, too.

Proposition 5: Function and Form Become Locked In

There are two types of long run equilibria. In cc-equilibrium, all shops and jobs locate in the CBD, while in rr-equilibrium all shops and jobs locate on the ring road. We may rank the two equilibria w.r.t.: (i) (Size) $\tilde{r} - \tilde{r}_{rr} < \tilde{r}_{cc}$*, (ii) (Compactness)* $F_{rr}(r) < F_{cc}(r)$ *at all* r *(except* r_n*), (iii) (Centrism)* $\underline{\lambda}^c_{cc} > \underline{\lambda}^c_{rr}$ *and (iv) (Decentrism)* $\underline{\lambda}^d_{cc} < \underline{\lambda}^d_{rr}$*.*

To illustrate, back in Fig. (1) San Diego could be considered settled at "cc", while Phoenix might more properly be thought of as still adjusting towards "rr". Or to take a very different example, true support for revitalizing Detroit's historical center (Owens et al. (2017)) may be unlikely given that a majority of voters have long turned decentrist.

6 Illustrating Shape

U.S. Census Bureau provides data on "population by distance from the center" and "weighted population density by distance from the center" for all 366 metro areas and the two years 2000 and 2010 (for extensive documentation see Wilson et al. (2012)), at constant geographies. These data provide exactly the detailed information on city shapes that is at the heart of the theoretical model.[25] We equate our model's r_n with the distance of the last ring exhibiting population-weighted density greater than 500 from the CBD, thereby excluding "non-urban" metro area parts.[26] We sum population over all rings 1, ..., n, to then simply divide each ring's population b_i by $s = \sum_{i=1}^{n} b_i$. This yields the sample counterpart of the theoretical model's shape $(b_1/s, \ldots, b_{n/2})$.

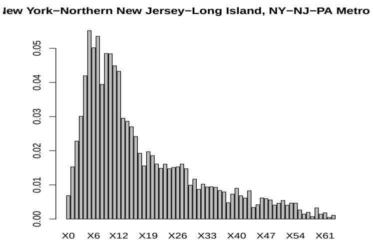
Calculating $\underline{\lambda}^c_{it}$, $\underline{\lambda}^d_{it}$, σ_{it} and μ_{it} for each metropolitan area i and year $t = 2000$ follows formulas (8), (9), (10) and (3), respectively.[27] Figure (2) illustrates shape and skew σ for

[24] This stark description appears reminiscent of Detroit, see Owens III et al. (2017, section 2).

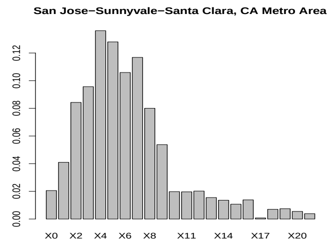
[25] Of course the usual caveats apply: Metro areas are no longer monocentric even at the earliest year in the data (2000) (see Glaeser/Kahn (2004)). And, tenants do not constitute exactly one half of the population, housing ownership is not evenly distributed across landlords, and the jurisdiction deciding on decentralization may not coincide with the metropolitan area.

[26] In some metro areas the first ring fails to meet this criterion. Here we include the first ring nonetheless.

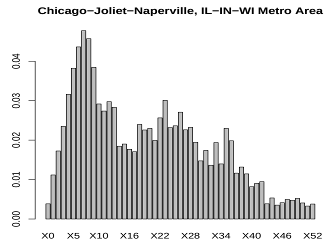
[27] Conforming with section 5's focus on initial city area, we calculate $\underline{\lambda}^c_{2010}$, $\underline{\lambda}^d_{2010}$ and σ_{2010} for the metro area support in 2000. In other words, F_{2010} is truncated to $\{1, \ldots, n_{2000}\}$.



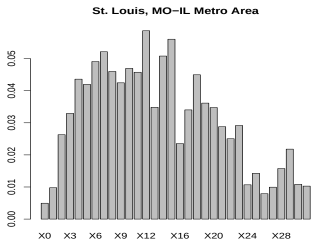
(a) New York: $\sigma = 0.44$



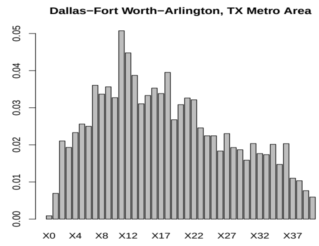
(b) San José: $\sigma = 0.39$



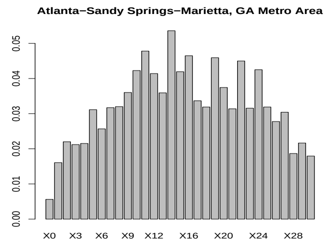
(c) Chicago: $\sigma = 0.25$



(d) St. Louis: $\sigma = 0.13$



(e) Dallas: $\sigma = 0.10$



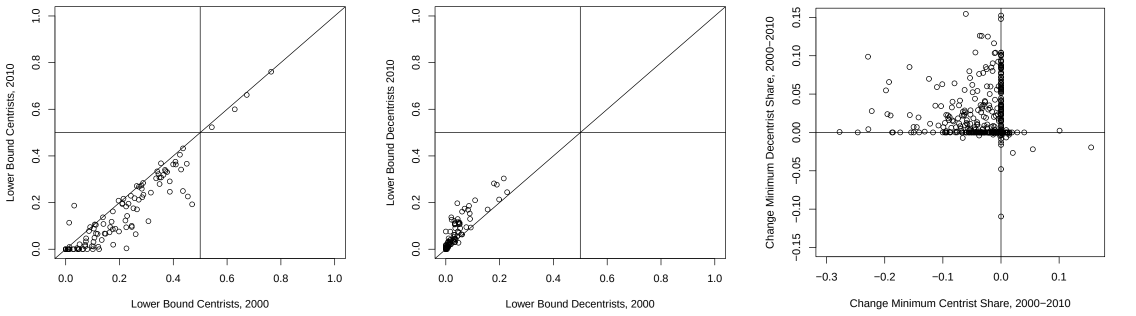
(f) Atlanta: $\sigma = -0.04$

Figure 2: Six US Metro Areas in 2000: Shape and Skew σ (subtitle)

six large US metro areas as of 2000. These metros give a first impression of the variety of shapes present in the sample, reflecting the fact that the interplay of skyline h and land supply a can give rise to virtually any type of shape ah. Among Figure (2)'s cities, New York is most clearly skewed towards the periphery, earning it a σ of 0.44. Chicago appears slightly less skewed. Skew drops further as we gradually shift to almost symmetric Dallas. Atlanta, finally, even has a negative skew, of $\sigma = -0.04$.

Let us now focus on the 100 metro areas that were largest in 2000. Median σ in 2000 was 0.09, a fourth of all observations were skewed by 0.18 or more, and another fourth by 0.02 or less. Next, median $\underline{\lambda}^c$ was 0.20, and one fourth of all metro areas had a minimum share of centrists of 0.34 or more. But really only four cities had minimum centrist shares in excess of one half: Miami ($\underline{\lambda}^c$=0.54), L.A. (0.63), New York (0.67) and San José (0.76). Moreover, median $\underline{\lambda}^d$ was 0.01, and the 75% quartile was 0.03 only. No metro area's $\underline{\lambda}^d$ passed the 0.5 threshold. The four greatest shares of minimum decentrists could be found in Chattanooga ($\underline{\lambda}^d = 0.16$), El Paso (0.18), Augusta (0.19) and Poughkeepsie (0.20).

Fig. (3)'s first two diagrams plot the values for $\underline{\lambda}^c$ and $\underline{\lambda}^d$ in 2000 (a diagram's horizontal axis) against corresponding "constant geography" values for 2010 (vertical axis). In diagram (a), metros exhibiting $\underline{\lambda}^c_{2000}$ in excess of one half show no inclination to see their $\underline{\lambda}^c$ drop. Metros exhibiting $\underline{\lambda}^c_{2000}$ smaller than 0.5, at the same time, often tend to see their $\underline{\lambda}^c$ fall subsequently. While we must be careful not to confuse $\underline{\lambda}^c$ with λ^c, this picture offers some (preliminary) evidence in favor of the function-form-lock-in detailed in Proposition 5. Metro areas with $\underline{\lambda}^c_{2000}$ beyond one half (again, Miami, L.A., New York, and San José) successfully resisted decentralization (Proposition 1). Consequently no shape adjustment occurred over the decade that ensued. This in turn implies that their $\underline{\lambda}^c$ did not change



(a) $\underline{\lambda}^c_{2000}$ vs. $\underline{\lambda}^c_{2010}$ (b) $\underline{\lambda}^d_{2000}$ vs. $\underline{\lambda}^d_{2010}$ (c) $\underline{\lambda}^c_{2010} - \underline{\lambda}^c_{2000}$ vs. $\underline{\lambda}^d_{2010} - \underline{\lambda}^d_{2000}$

Figure 3: U.S. Metro Areas: Changes in $\underline{\lambda}^c$ and $\underline{\lambda}^d$ over Decade 2000-2010

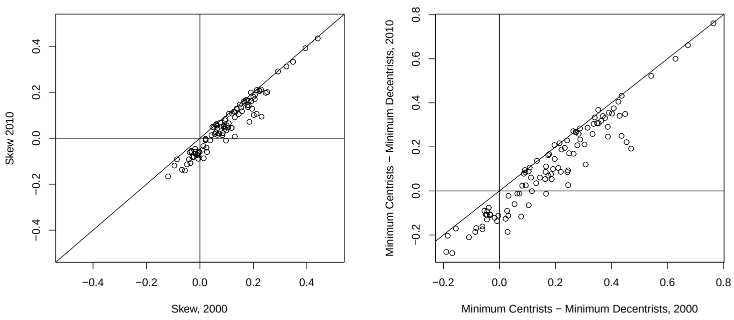
either. In contrast, metros with $\underline{\lambda}^c_{2000}$ short of one half may well have had decentrist majorities. This could be why these metro areas' minimum centrist shares so often receded in the intervening years.

Fig. (3)'s diagram (b) next shows $\underline{\lambda}^d$. Here observations lie either on or above the 45 degree line. No metro area has decentrists that are obviously decisive. However, metros with strictly positive $\underline{\lambda}^d_{2000}$ tend to see their $\underline{\lambda}^d$ rise over time, while those with a zero $\underline{\lambda}^d_{2000}$ go on to have a zero $\underline{\lambda}^d$. Diagram (c) provides a bird's eye perspective on ongoing U.S. suburbanization, plotting $\underline{\lambda}^c_{2010} - \underline{\lambda}^c_{2000}$ against $\underline{\lambda}^d_{2010} - \underline{\lambda}^d_{2000}$. Points coinciding with the origin correspond to metro areas with centrist majorities in 2000 ("cc"-equilibrium); while points in the second quadrant's interior or on one of its axes correspond to metro areas with decentrist majorities in 2000 ("rr"-equilibrium). Note that there are few observations that cannot be rationalized in this way. Neither do we ever see both shares moving into the same direction. Nor do we see a significant number of cities that "recentralize".

City shape's skew σ is bound by our lower bounds, i.e. $-\underline{\lambda}^d < \sigma < \underline{\lambda}^c$. To the extent that one or even both bounds $\underline{\lambda}^c$ and $-\underline{\lambda}^d$ shift downwards over time (Fig. (3)'s diagram (c)), it might be reasonable to expect cities to experience a reduction in skew, too. Fig (4)'s diagram (a) shows how skew σ has changed over time. For many cities we do see a skew drop between 2000 and 2010. At the same time, this loss has been more pronounced for those metros who were not very skewed to begin with.

Unobservable effects – such as federal decisions on the highway network, the extent to which parts of the traditional CBD have shifted out already, etc. – should affect metro areas' lower bounds, too. To the extent that these unobservables (i) are time-constant and (ii) affect both lower bounds in likewise fashion, we may remove part of the bias that these effects have by taking differences. The resulting differential $\underline{\lambda}^c - \underline{\lambda}^d$ we expect to remain the same (drop) over those years 2000 through 2010 if greater (smaller) than zero initially. This is clearly not the case, however (Fig. (4b)). And still, a greater initial differential does appear to protect better against any subsequent loss of differential.

Fitting a straight line to the data shown in Fig. (4)'s diagram (b) gives an estimated



(a) σ_{2000} vs. σ_{2010} (b) $\underline{\lambda}^c_{2000} - \underline{\lambda}^d_{2000}$ vs. $\underline{\lambda}^c_{2010} - \underline{\lambda}^d_{2010}$

Figure 4: U.S. Metro Areas: Changes in $\underline{\lambda}^c$, $\underline{\lambda}^d$ and σ over Decade 2000-2010

equation of

$$\widehat{(\underline{\lambda}^c_{i,2010} - \underline{\lambda}^d_{i,2010})} \;=\; -0.081 + 1.036\,(\underline{\lambda}^c_{i,2000} - \underline{\lambda}^d_{i,2000}) \tag{15}$$

with a standard error of 0.032 for the slope coefficient. While the slope coefficient is greater than one, we also cannot reject its being: just one. This mirrors the mixed results of our descriptive discussion above.

7 Conclusions

While city policy obviously shapes urban form, here urban form also shapes city policy. The more skewed a city's shape, the less conceivable a majority of residents that prefer replacing the traditional center at the CBD by a succession of office parks and shopping malls along a ring road. This theory also relates to architectural theory because it completes the relationship between buildings' function and form. As Frank Lloyd Wright notes (quoted in Saarinen (1954)), "Form follows function – that has been misunderstood. ... Form and function should be one, joined in a spiritual union."

But this theory also relates to mainstream economics. Economics is rightly weary of generalizing an aggregate's properties down towards the aggregate's component members lest it commit a "fallacy of division". Linking the built environment (a society aggregate) to the preferences of at least a majority of its landlords (a decisive subset of society's members) provides an example of where inferring dominant residents' properties does seem justified after all. Whenever the city's shape "leans towards" the city center (a majority of) resident landlords "lean towards" the city center. No fallacy is involved when assessing a city's politics by its shape.

Inspecting cumulative ring differences may not just help understand decentralization and suburbanization. It may also help understand the urban political economy of: limiting building height, taxing carbon and addressing climate change (e.g., Dascher (2018)), constraining urban growth, or fighting central city blight. Centrists and decentrists face off over all sorts of policies that naturally affect them in an uneven fashion.

8 Appendix

Proof of Proposition 3: Suppose not all ring differences are positive. Decomposing $\max_j \sum_{i=1}^{j} w_i\delta_i/s$, and now letting j^* denote that problem's maximizer, gives

$$\sum_{i=1}^{j^*-2} w_i\,\delta_i/s \;+\; \underbrace{w_{j^*-1}\delta_{j^*-1}/s \;+\; \overbrace{w_{j^*}\delta_{j^*}/s}^{+}}_{+}\,. \tag{16}$$

Not only is this three-term-sum positive. By the principle of optimization, also, both the last and the two last of these terms must be (as is indicated) positive, too (else j^* would not be the maximizer). Let us now replace the sum (16) by the sum that follows:

$$\begin{aligned} &\sum_{i=1}^{j^*-2} w_i\,\delta_i/s \;+\; \underbrace{w_{j^*-1}\delta_{j^*-1}/s \;+\; w_{j^*-1}\delta_{j^*}/s}_{+} \\ =\; &\sum_{i=1}^{j^*-2} w_i\,\delta_i/s \qquad\qquad + \; w_{j^*-1}\underbrace{\sum_{k=j^*-1}^{j^*}\delta_k/s}_{+}, \end{aligned} \tag{17}$$

where the only adjustment involved is the change in the weight attached to the last term in (16). This change only makes the overall sum larger, in its attaching greater weight $w_{j^*-1} > w_{j^*}$ to what we know is a positive term.

Given that the entire second term in (17) is positive, so is the sum contained in it (because weights are strictly positive always). This, too, has been indicated. But then we may raise the weight attached to it, from w_{j^*-1} to w_{j^*-2}. This gives even bigger

$$\begin{aligned} &\sum_{i=1}^{j^*-3} w_i\,\delta_i/s \;+ \underbrace{w_{j^*-2}\delta_{j^*-2}/s \;+\; w_{j^*-2}\sum_{k=j^*-1}^{j^*}\delta_k/s}_{+} \\ =\; &\sum_{i=1}^{j^*-3} w_i\delta_i/s \qquad\qquad + \; w_{j^*-2}\underbrace{\sum_{k=j^*-2}^{j^*}\delta_k/s}_{+} \end{aligned} \tag{18}$$

It remains to explain why the sum of the last two terms in (18) is positive (as indicated). Note first, again by the principle of optimization, that the sum of any last $k \leqslant j^*$ weighted ring differences $w_i\delta_i/s$ is always positive (else j^* could not be the maximizer). And second, carefully adjusting the weights on suitable parts of that sum (e.g. as was done when moving from (16) to (17)) will make it only bigger.

Proceeding to replace weights in this fashion ultimately gives $w_1\sum_{i=1}^{j^*}\delta_i/s$. For this latter expression in turn it is true that

$$w_1\sum_{i=1}^{j^*}\delta_i/s \;<\; \sum_{i=1}^{j^*}\delta_i/s \;\leqslant\; \max_j\sum_{i=1}^{j}\delta_i/s, \tag{19}$$

where the last (weak) inequality observes the fact that j^* not necessarily is the maximizer to the last program in (19). Connecting all of the inequalities in this Appendix at last implies

$$\max_j \sum_{i=1}^{j} w_i\delta_i/s \;<\; \max_j\sum_{i=1}^{j}\delta_i/s. \;\Box$$

9 Literature

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