International Finance

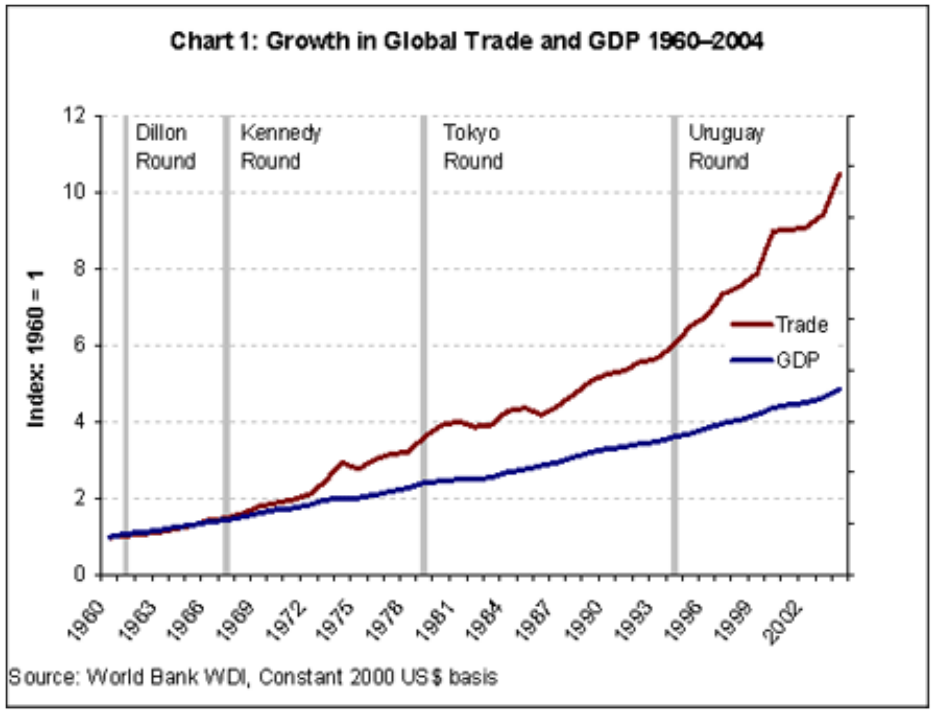
Lutz Arnold
University of Regensburg

Contents

1. International trade and capital flows

Both international trade in goods and services and international capital flows have been growing dramatically over the past decades.

From 1960 through 2004, world trade grew ten-fold (5.4 percent p.a.), while GDP grew five-fold (3.7 percent p.a.). The ratio of trade to GDP rose from 12 percent in 1960 to 26 percent in 2004.



- https://www.researchgate.net/publication/263756369_A_brief_history_of_time_space_and_growth_Waldo_Tobler%27s_first_law_of_geography_revisited_WRSA_presidential_address_2013/figures?lo=1
- http://www.wto.org/english/res_e/reser_e/wtr08_e.htm
- https://www.destatis.de/DE/ZahlenFakten/GesamtwirtschaftUmwelt/VGR/Inlandsprodukt/Tabellen/VerwendungBIP.html

The surge in international trade was caused by several factors:

- increasing specialization in ever more fragmented value chains
- falling transport costs
- trade liberalization by the GATT and the WTO.

- http://www.nytimes.com/2007/12/28/opinion/28krugman.html?_r=0
- https://www.cia.gov/library/publications/the-world-factbook/geos/hk.html
- https://www.cia.gov/library/publications/the-world-factbook/geos/sn.html
- http://www.krannert.purdue.edu/faculty/hummelsd/research/decline/declined.pdf
- www.wto.org/english/thewto_e/whatis_e/who_we_are_e.htm
- http://www.nytimes.com/2007/12/28/opinion/28krugman.html?_r=0

There had been a similar globalization phase in 1870-1913.

Merchandise exports as a percentage of GDP, 1870-1992
(Three year annual average, except for 1950)

	Western developed countries[a]	United States	Western Europe	Japan
1870	...	5.4	13.6	...
1890	11.7	6.7	14.9	5.1
1913	12.9	6.4	18.3	12.5
1929	9.8	5.0	14.5	13.6
1938	6.2	3.7	7.1	13.0
1950	7.8	3.8	13.4	6.8
1970	10.2	4.0	17.4	9.7
1992	14.3	7.5	21.7	8.8

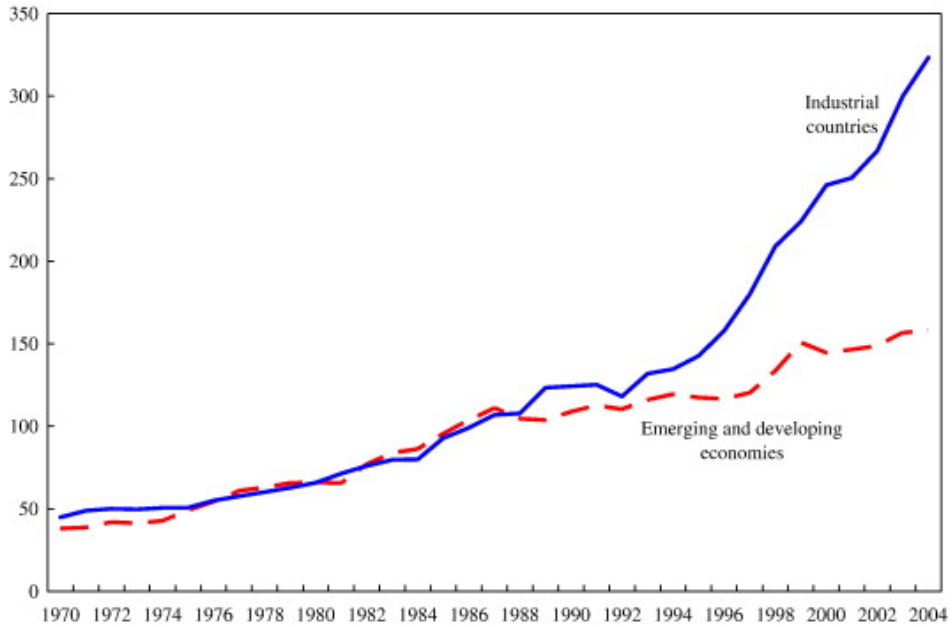
Source: Bairoch (1996a), tables 1 and 4, with additional data for 1870.
a For reasons of statistical consistency, the Western developed countries include the following regions and countries: all Western Europe excluding Yugoslavia, the United States and Canada, Australia, New Zealand and Japan.

Source: Bairoch and Kozul-Wright (1996), “Globalization Myths: Some Historical Reflections on Integration, Industrialization and Growth in the World Economy”, UNCTAD/OSG/DP/113.

- http://earth.columbia.edu/sitefiles/file/about/director/pubs/brookings_q195.pdf
- http://www.unctad.org/en/Docs/dp_113.en.pdf

Capital flows have also been growing dramatically. As in the case of trade in goods and services, this is due

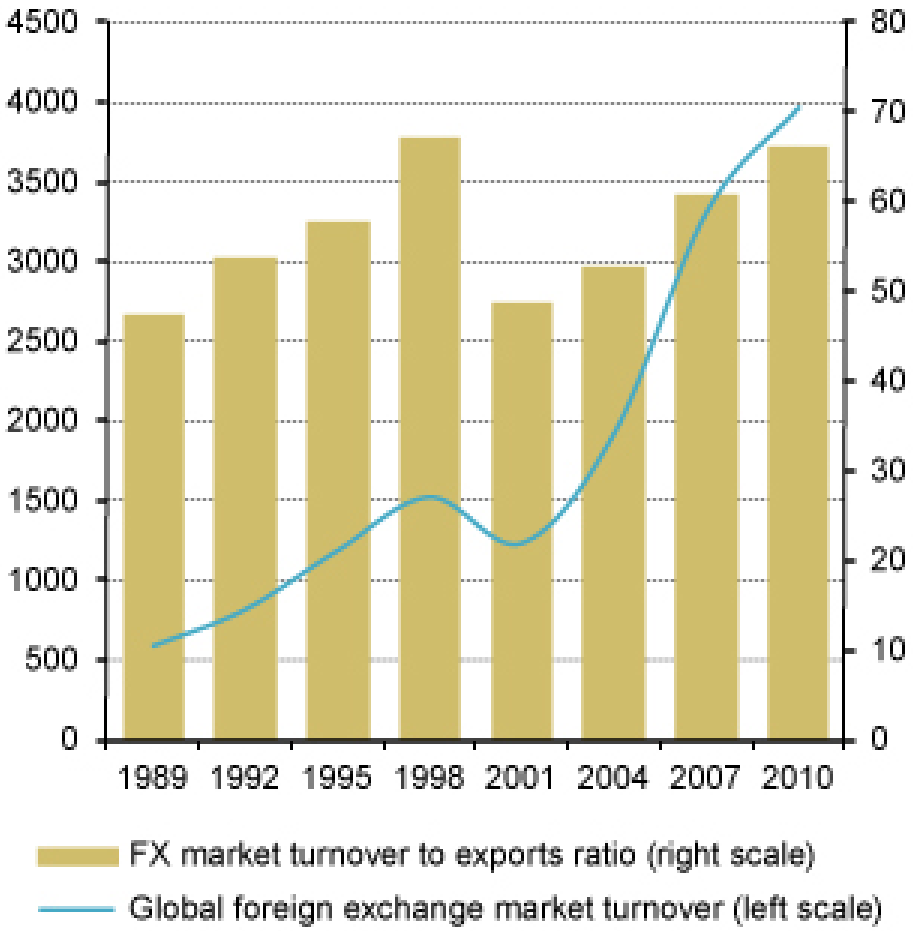
- falling transaction costs and
- liberalization by the OECD.



Source: Lane, Philip R. and Gian Maria Milesi-Ferretti (2007), "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004", *Journal of International Economics* 73, 223-250.

- http://www.oecd.org/daf/inv/investment-policy/codes.htm
- https://www.imf.org/external/pubs/ft/wp/2006/wp0669.pdf
- https://www.bundesbank.de/de/statistiken/aussenwirtschaft/auslandsvermoegen-und-verschuldung/-/tabellen-775718

Turnover on international financial markets exceeds stocks of assets by far.



Source: Bank for International Settlements.

- https://www.bis.org/publ/rpfx13fx.pdf
- https://www.bis.org/statistics/rpfx19.htm

Problems:

1. Let

- Y_t: GDP at date t
- Ex_t: exports at t.

Suppose Y_t and Ex_t grow at constant rates g_Y and g_{Ex}, resp.
(a) Calculate the growth rate of Ex_t/Y_t.
(b) For $g_Y = 3.7\%$ and $g_{Ex} = 5.4\%$, by what factor does Ex_t/Y_t rise over 44 years? Compute Ex_{2004}/Y_{2004} for $Ex_{1960}/Y_{1960} = 12\%$.

2. Consider a product with a five step value chain. At each date the value added is \$10. Each step is performed by a separate firm. Total output is two units of the good. One unit is sold to domestic consumers, the other unit is exported.

(a) Suppose all firms involved are domestic firms. What is the contribution of the good considered to domestic GDP and to exports?

(b) Suppose the first and fourth steps of the value chain are outsourced to foreign firms. What is the contribution of the good to domestic GDP, imports, and exports?

2. Current account and capital account

A country's goods and services trade on the one hand and its international capital flows are not independent of each other:

$$\begin{aligned}\text{current account surplus} &= \text{net capital exports} \\ &\quad + \text{change in reserves.}\end{aligned}$$

Ignore foreign exchange reserves. Then one simple way to see this is to note that

$$\text{net capital exports} = \text{income} - \text{domestic expenditure.}$$

The validity of the formula above follows from the fact that income equals the value of production ("Y") and the value of production is equal to the sum of domestic expenditure ("$C + I + G$") and the current account surplus.

A second proof of the formula uses the condition for equilibrium in the market for foreign exchange:

$$
\begin{aligned}
\text{supply of foreign exchange} &= \text{demand for foreign exchange} \\
\text{exports} + \text{capital imports} &= \text{imports} + \text{capital exports} \\
&\quad + \text{change in reserves} \\
\text{exports} - \text{imports} &= \text{capital exports} - \text{capital imports} \\
&\quad + \text{change in reserves} \\
\text{current account surplus} &= \text{net capital exports} \\
&\quad + \text{change in reserves.}
\end{aligned}
$$

The equation says that a country is either a “surplus country” or a “deficit country” (with regard to both its current account and its capital account).

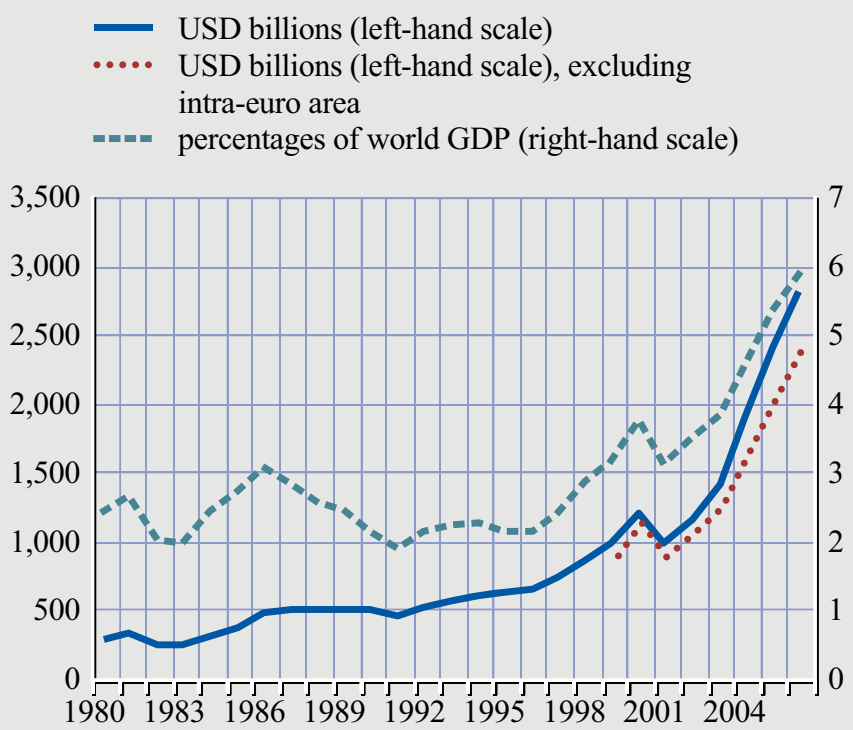
In practice, the identity is somewhat more complicated because of foreign income and international transfer payments.

- https://www.bundesbank.de/de/statistiken/aussenwirtschaft/zahlungsbilanz/zahlungsbilanz-772304
- http://www.economist.com/node/21538100

The “global imbalances” consist of a tendency for individual countries to take a persistent and growing surplus or deficit position.

Chart 2 Sum of current account balances in the world

(USD billions; as a percentage of world GDP)



Source: IMF World Economic Outlook.

http://www.ecb.int/pub/pdf/scpops/ecbocp78.pdf

Problems:
1. Here are the German balance-of-payments data for 2016 (positive capital account figures are net exports):

- Außenbeitrag (trade balance): €216 bn.
- Saldo der Primäreinkommen (net foreign income): €98 bn.
- Saldo der Sekundäreinkommen (net current transfers): €−47 bn.
- Saldo der Vermögensänderungsbilanz (net asset transfers): €1 bn.
- Saldo der Direktinvestitionen (net FDI): €59 bn.
- Saldo der Wertpapieranlagen (net asset purchases): €92 bn.
- Saldo der Finanzderivate und Mitarbeiteraktienoptionen (derivatives and stock options): €22 bn.
- Saldo des übrigen Kapitalverkehrs inkl. Bankkredite (other net capital exports including bank credit): €20 bn.
- Veränderung der Währungsreserven (change in CB reserves): €−1 bn.

Compute the current account balance, the capital account balance, and the statistical discrepancy. Interpret these figures.

3. Efficient allocation of capital

With capital mobility, capital can flow to where it is put to its best use – the allocation of capital becomes more efficient. It's not obvious, however, that this is beneficial to all countries:

- Inhabitants of a country with a low interest rate in autarky get higher returns. But: the interest rises, and domestic business capital formation goes down.
- Firms in a country with a high interest rate in autarky get the financial capital needed to build capacities more cheaply. But: part of the generated income goes abroad, and residents get a lower return.

- http://bookshop.europa.eu/en/capital-market-liberalization-pbC16996005/\textbf{http://www.ecb.int/pub/pdf/scpops/ecbocp78.pdf}

Intertemporal theory of the current account (ITCA):

- Two dates, 1 and 2, no uncertainty.
- One good, whose price is normalized to unity at both dates.
- Representative consumer with linearly homogeneous utility function $U(C_1, C_2)$, where C_t is consumption at $t = 1, 2$ (the somewhat weaker condition of homotheticity would be enough), and exogenous labor supply L. Slope of indifference curves:

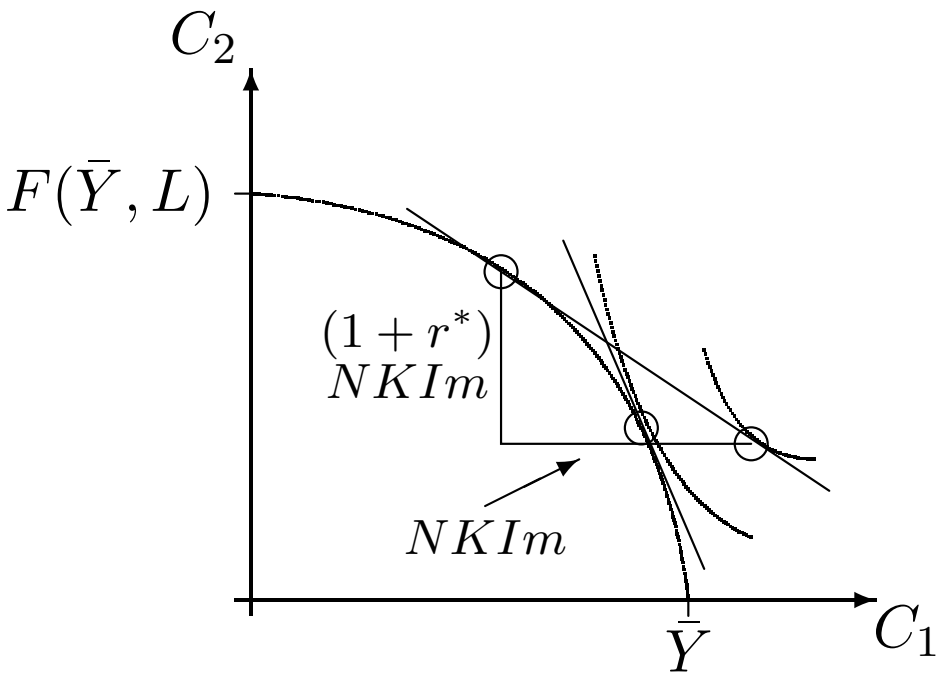
$$-\left.\frac{dC_2}{dC_1}\right|_{dU=0} = \frac{\frac{\partial U(C_1,C_2)}{\partial C_1}}{\frac{\partial U(C_1,C_2)}{\partial C_2}}.$$

- Date-1 GDP $\bar{Y}$ given. Date-2 capital stock: $K = \bar{Y} - C_1$. Full depreciation at date 2.
- Date-2 production function: $F(K, L)$ with the usual properties (positive, diminishing marginal productivity, linearly homogeneous).

Production possibilities frontier (PPF):

$$C_2 = F(\underbrace{\bar{Y} - C_1}_{=K}, L).$$

$$\frac{dC_2}{dC_1} = -\frac{\partial F(\bar{Y} - C_1, L)}{\partial K} < 0, \quad \frac{d^2 C_2}{dC_1^2} = \frac{\partial^2 F(\bar{Y} - C_1, L)}{\partial K^2} < 0.$$



Note the similarity to traditional static international trade models: instead of two different goods at one single date, there is one single good at two different dates.

Analogously as in static international trade theory, the effect of capital mobility is characterized by comparing the small open economy with free capital flows to the autarkic closed economy.

Closed economy

- Factor market clearing: the production point is on the PPF $C_2 = F(\bar{Y} - C_1, L)$.
- Let r denote the interest rate. $1 + r$ is the price per unit of capital. Profit maximization:

$$1 + r = \frac{\partial F(K, L)}{\partial K}.$$

The production point is that point on the PPF where the slope equals $1 + r$.

- Budget line:

$$C_2 = wL + (1 + r)(\bar{Y} - C_1),$$

where wL is date-2 labor income. This is a straight line with slope $1 + r$ through the production point.

- Utility maximization:

$$\max_{C_1} : U\left[C_1, wL + (1 + r)(\bar{Y} - C_1)\right],$$

Necessary optimality condition:

$$-\left.\frac{dC_2}{dC_1}\right|_{dU=0} = \frac{\frac{\partial U(C_1,C_2)}{\partial C_1}}{\frac{\partial U(C_1,C_2)}{\partial C_2}} = 1 + r.$$

That is, the consumption point is that point on the budget line where an indifference curve is tangent.

The autarky equilibrium production and consumption levels (C_1, C_2) are given by that point on the PPF where an indifference curve is tangent. $1 + r$ is equal to the common slope of PPF and indifference curve at that point. Physical capital formation is given by $K = \bar{Y} - C_1$.

Small open economy with perfect capital mobility
The economy can now borrow or lend in the world capital market at an exogenously given interest rate r^*.

- As before, the production point is on the PPF $C_2 = F(\bar{Y} - C_1, L)$.
- Profit maximization implies that it is that point on the PPF where the slope equals $1 + r^*$. The production point is determined independently of preferences.
- As before, the budget line

$$C_2 = wL + (1 + r^*)(\bar{Y} - C_1),$$

is the line tangent to the PPF at the production point.

- Households choose that point on the budget line where an indifference curve is tangent.

The consumption point does not coincide with the production point unless the autarky interest rate coincides with the world market interest rate. If the two interest deviate, then the country borrows (if the autarky interest rate is higher) or lends (if the autarky interest rate is lower) in the world capital market, thereby reaching a higher indifference curve.

Theorem: *If consumers are identical or the government can redistribute income appropriately, international mobility of capital makes the individuals better-off.*

- http://www.tradingeconomics.com/norway/current-account
- http://www.nbim.no/en/the-fund/

The qualification in the theorem is necessary, because the fact that the consumption point is located on a higher indifference curve does not imply that every-one is better-off with capital mobility: if the world interest rate is lower than the autarky interest rate, a consumer whose only source of income is capital is worse-off.

There are gains from capital mobility if one of the following two assumptions is satisfied:

- All consumers are identical (in terms of factor endowments), so each consumer gets the same proportion of aggregate consumption at each date both in autarky and with capital mobility.
- The government can redistribute income (in a lump-sum fashion), so that each consumer gets the same proportion of aggregate income both in autarky and with capital mobility.

The ITCA suggests that the more integrated the international capital market is, the lower the correlation between national saving and national investment:

- Without international capital flows, saving equals investment, so the correlation of the two variables is one.
- With perfect international capital mobility, national savings are not required to finance investment in the home country, so the correlation of the two variables should be close to zero.

Feldstein and Horioka (1980) showed that, surprisingly, the savings-investment correlation was 0.89 for the time period 1960-74. As that is much closer to the no-capital mobility benchmark than to perfect mobility, their finding has been dubbed the “Feldstein-Horioka puzzle”.

- http://www.ecb.int/pub/pdf/scpwps/ecbwp873.pdf

Literature:

Feldstein, Martin, and Charles Horioka (1980), "Domestic saving and international capital flows", *Economic Journal* 90, 314–329.

Obstfeld, Maurice, and Kenneth Rogoff (1996), *Foundations of International Macroeconomics*, Cambridge, MA: MIT Press, Section 1.2.

Appendix: Zero profit and the representative consumer:

A function f of two variables (x_1, x_2) is homogeneous of degree r if for all (x_1, x_2),

$$f(\lambda x_1, \lambda x_2) = \lambda^r f(x_1, x_2)$$

for all $\lambda > 0$. Differentiating with respect to x_i shows that the partial derivatives of a function that is homogeneous of degree 1 are homogeneous of degree zero:

$$\frac{\partial f(\lambda x_1, \lambda x_2)}{\partial(\lambda x_i)} = \frac{\partial f(x_1, x_2)}{\partial x_i}.$$

Setting $r = 1$, differentiating with respect to λ, and setting $\lambda = 1$ yields Euler's law:

$$\frac{\partial f(x_1, x_2)}{\partial x_1} x_1 + \frac{\partial f(x_1, x_2)}{\partial x_2} x_2 = f(x_1, x_2).$$

In the budget constraint $C_2 = wL + (1 + r)(\bar{Y} - C_1)$, we assume $F(K, L) = (1 + r)K + wL$ (so that $C_2 = F(K, L)$). This follows from profit maximization (i.e., equality of marginal productivities and factor prices) and homogeneity of degree one of F:

$$F(K, L) = \frac{\partial F(K, L)}{\partial K} K + \frac{\partial F(K, L)}{\partial L} L = (1 + r)K + wL.$$

Let $\bar{y}^i$ and l^i denote individual i's date-1 endowment and date-2 labor supply, respectively. Let

$$\lambda^i = \frac{wl^i + (1 + r)\bar{y}^i}{wL + (1 + r)\bar{Y}}$$

denote his share in national income.

Assertion: Let all individuals have the same linearly homogeneous utility function U, and suppose C_1 maximizes $U\left[C_1, wL + (1+r)(\bar{Y} - C_1)\right]$. Then $c_1^i = \lambda^i C_1$ and $c_2^i = \lambda^i C_2$ maximize i's utility subject to his budget constraint.

Evidently, these consumption levels satisfy i's budget constraint $c_2^i = wl^i + (1+r)(\bar{y}^i - c_1^i)$. The validity of the necessary optimality condition follows from homogeneity of the utility function:

$$
\begin{aligned}
-\left.\frac{dc_2^i}{dc_1^i}\right|_{dU=0} &= \frac{\frac{\partial U(c_1^i, c_2^i)}{\partial c_1^i}}{\frac{\partial U(c_1^i, c_1^i)}{\partial c_2^i}} = \frac{\frac{\partial U(c_1^i/\lambda^i, c_2^i/\lambda^i)}{\partial (c_1^i/\lambda^i)}}{\frac{\partial U(c_1^i/\lambda^i, c_2^i/\lambda^i)}{\partial (c_2^i/\lambda^i)}} \\
&= \frac{\frac{\partial U(C_1, C_2)}{\partial C_1}}{\frac{\partial U(C_1, C_2)}{\partial C_2}} = 1 + r.
\end{aligned}
$$

Problems:

1. Consider the ICTA with

$$U(C_1, C_2) = C_1^{\frac{1}{2}} C_2^{\frac{1}{2}}, \; F(K, L) = 3K^{\frac{1}{3}} L^{\frac{2}{3}},$$

$L = 1$, and $\bar{Y} = 2.177$.

(a) Derive the equation for the PPF.

(b) Derive the conditions for profit and utility maximization.

(c) Characterize the equilibrium of the autarkic economy (three digits after the decimal point) by solving the equations in parts (a) and (b) for K. Compute r, C_1, C_2, and $U(C_1, C_2)$.

Consider the open economy with world interest rate $r^* = 60\%$.

(d) Compute the profit maximizing capital input K and the production point (i.e., the levels of consumption C_1 and C_2 that would obtain if the country did not trade capital.

(e) State the equation for the consumers' budget constraint (i.e., the straight line through the production point with slope $-(1 + r^*)$.

(f) Compute the utility maximizing consumption levels C_1 and C_2 and indirect utility $U(C_1, C_2)$. Compare to part (c).

2. Consider the ICTA with

$$U(C_1, C_2) = C_1^{\frac{2}{3}} + \frac{1}{2} C_2^{\frac{2}{3}}, \; F(K, L) = 2K^{\frac{1}{2}} L^{\frac{1}{2}},$$

$L = 1$, and $\bar{Y} = 0.75$.
(a) Derive the equation for the PPF.
(b) Derive the conditions for profit and utility maximization.
(c) Characterize the equilibrium of the autarkic economy (three digits after the decimal point) by solving the equations in parts (a) and (b) for K. Compute r, C_1, C_2, and $U(C_1, C_2)$.
Consider the open economy with world interest rate $r^* = 100\%$.
(d) Compute the profit maximizing capital input K and the production point (i.e., the levels of consumption C_1 and C_2 that would obtain if the country did not trade capital.
(e) State the equation for the consumers' budget constraint (i.e., the straight line through the production point with slope $-(1 + r^*)$.
(f) Compute the utility maximizing consumption levels C_1 and C_2 and indirect utility $U(C_1, C_2)$. Compare to part (c).

3. Are $F(K,L) = AK^{\alpha}L^{\beta}$ and $F(K,L) = A(aK^{\alpha} + bL^{\alpha})^{\beta}$ homogeneous? Of what degree r?

4. Prove that $U(C_1, C_2) = C_1^{\gamma} C_2^{\gamma}$ and $U(C_1, C_2) = (C_1^{\gamma} + \beta C_2^{\gamma})^{1/\gamma}$ are linearly homogeneous. Argue why maximizing $U(C_1, C_2) = C_1^{\gamma} + \beta C_2^{\gamma}$ is equivalent to maximizing $U(C_1, C_2) = (C_1^{\gamma} + \beta C_2^{\gamma})^{1/\gamma}$ subject to the budget constraint.

5. Suppose the k-th unit of capital invested in a country contributes $\Delta Y_k = A - Bk$ to GDP. Domestic residents supply K units of capital. Ignore integer problems in what follows. Ignore depreciation, so that the cost of capital is equal to the interest rate.
(a) Compute the autarky interest rate r and illustrate the determination graphically. Compute autarky GDP Y, using

$$\sum_{k=1}^{K} k = \frac{K(K+1)}{2}.$$

(b) Now assume free capital flows. The given world interest rate is r^*. Determine the equilibrium domestic demand for capital K^*. Illustrate graphically. Compute equilibrium GDP Y^*.
(c) Use the results of part (b) to eliminate Y^* and r^* from the gross national income identity $Y^* - r^*(K^* - K)$.
(d) Show that gross national income with capital flows exceeds autarky GDP exactly if

$$K^*(K + 1 + K - K^*) < K(K + 1).$$

Characterize the left-hand side of that inequality as a function of K^*. Show that the inequality holds for $K^* < K$ and for $K^* > K + 1$. Explain, using the figure of part (a), *why* income goes up.

(e) Prove the formula in part (a) via induction.

4. Consumption smoothing

Even if there are no investment opportunities, a country benefits from international capital flows if

- GDP is variable, but
- people have a preference for smooth consumption.

We use a simple finite-horizon one-good endowment economy to show this.

Denote:

- $t = 0, 1, \ldots, T$: dates
- c_t: national consumption at date t
- $u(c_t)$: date-t utility
- $U = \sum_{t=0}^{T} u(c_t)$: total utility
- y_t: date-t GDP.

The period utility function u is strictly concave, so that (given the absence of discounting) the country prefers smooth consumption.

For simplicity, assume that GDP " fluctuates" only at date 0: $y_1 = \cdots = y_T = y$ constant. (One can interpret $t = 0$ as any other date than the initial date. Problem 3 at the end of the section has recurrent fluctuations.)

The homogeneous good is “perishable”. So in order to consume more than the current endowment, the country has to borrow abroad and import the excess of consumption over GDP.

The world interest rate is set equal to zero.

Closed economy

Without international capital flows, the country cannot borrow the money needed to finance imports. So it has to consume its endowment: $c_t = y_t$ for all t. The problem is that the marginal utility at date 0 is then different from the latter marginal utilities if $y_0 \neq y$.

Open economy

Given zero interest, the intertemporal budget constraint is

$$\sum_{t=0}^{T}(y_t - c_t) = 0.$$

Utility maximization implies that the marginal utility of consumption, and hence consumption itself, is constant: $c_0 = c_1 = \cdots = c_T = c$. So

$$\frac{c}{y_0} = \frac{1 + T\frac{y}{y_0}}{1 + T}.$$

When income y_0 is low ($y/y_0 > 1$), consumption exceeds GDP ($c > y_0$), so the country has to borrow abroad, and vice versa.

The fact that the economy maximizes utility by *not* choosing $c_0 = y_0$ implies that the autarky consumption profile is suboptimal.

Empirically, financial liberalization does not seem to have reduced consumption volatility much.

- http://www.imf.org/External/Pubs/FT/staffp/2005/03/kose.htm

Problems:

1. Prove that constant consumption $c_0 = c_1 = \cdots = c_T = c$ is optimal (for any GDP profile $y_0, y_1, \cdots, y_T$) by maximizing the intertemporal utility function subject to the intertemporal budget constraint using Lagrange.

2. Let $T = 1$. Interpret the utility maximization problem using a (c_0, c_1)-diagram.

3. To have more output volatility than only at date 0, let T odd and

$$y_t = \begin{cases} 2y; & t \text{ even} \\ 0; & t \text{ odd} \end{cases}.$$

Compute the value of the country's output and consumption c_t. Describe the country's capital flows at even and odd dates.

4. Allow for a non-zero constant world market interest rate r^* in the budget constraint. Let period utility be logarithmic, and introduce discounting to the intertemporal utility function:

$$U = \sum_{t=0}^{T} \beta^t \ln c_t, \quad 0 < \beta \leq 1.$$

Provide an explicit solution for optimal consumption c_t.

5. Diversification

So far we have considered advantages of capital mobility that accrue in the absence of uncertainty. In the presence of risk, there is another major advantage of international capital mobility: people can spread their financial wealth over a broader set of securities. If foreign assets are not perfectly positively correlated with domestic assets, then this helps reduce their risk exposure.

We present a simple example with one domestic and one foreign asset, both with the same expected return.

We only consider an individual household's portfolio choice, not an equilibrium model.

Denote

- r: return on the domestic asset
- r^*: return on the foreign asset
- $Er = Er^*$: the (identical) expected returns
- σ_r^2, $\sigma_{r^*}^2$: the return variances
- σ_{r,r^*}: the return covariance
- x: the share of wealth invested in the domestic asset
- $\tilde{r}$: the return on the individual's portfolio.

The portfolio return is $\tilde{r} = xr + (1-x)r^*$. The expected return is $E\tilde{r} = Er = Er^*$. Mixing the assets in a portfolio does not increase expected return. The sole question is how to minimize the portfolio variance

$$\begin{aligned}
\sigma^2_{\tilde{r}} &\equiv E(\tilde{r} - E\tilde{r})^2 \\
&\equiv E[x(r - Er) + (1-x)(r^* - Er^*)]^2 \\
&= x^2 E(r - E\tilde{r})^2 + (1-x)^2 E(r^* - E\tilde{r}^*)^2 \\
&\quad +2x(1-x)E[(r - E\tilde{r})(r^* - E\tilde{r}^*)] \\
&\equiv x^2\sigma^2_r + (1-x)^2\sigma^2_{r^*} + 2x(1-x)\sigma_{r,r^*}.
\end{aligned}$$

This is a simplified version of the standard portfolio choice problem for the case of identical expected returns.

We first consider two special cases:

- independent reruns: $\sigma_{r,r^*} = 0$
- perfectly negatively correlated returns: $\rho_{r,r^*} = \sigma_{r,r^*}/(\sigma_r \sigma_{r^*}) = -1$.

With independent returns, we have

$$\frac{d\sigma^2_{\tilde{r}}}{dx} = 2x\sigma^2_r - 2(1-x)\sigma^2_{r^*}.$$

The domestic portfolio share

$$x = \frac{\sigma^2_{r^*}}{\sigma^2_r + \sigma^2_{r^*}}$$

minimizes the portfolio variance. The smaller $\sigma^2_r/\sigma^2_{r^*}$, the larger x. However, $x < 1$ even if $\sigma^2_{r^*}$ is much larger than σ^2_r.

With perfect negative correlation,

$$\begin{aligned}\sigma^2_{\tilde{r}} &= x^2\sigma^2_r + (1-x)^2\sigma^2_{r^*} - 2x(1-x)\sigma_r\sigma_{r^*} \\ &= [x\sigma_r - (1-x)\sigma_{r^*}]^2\end{aligned}$$

or

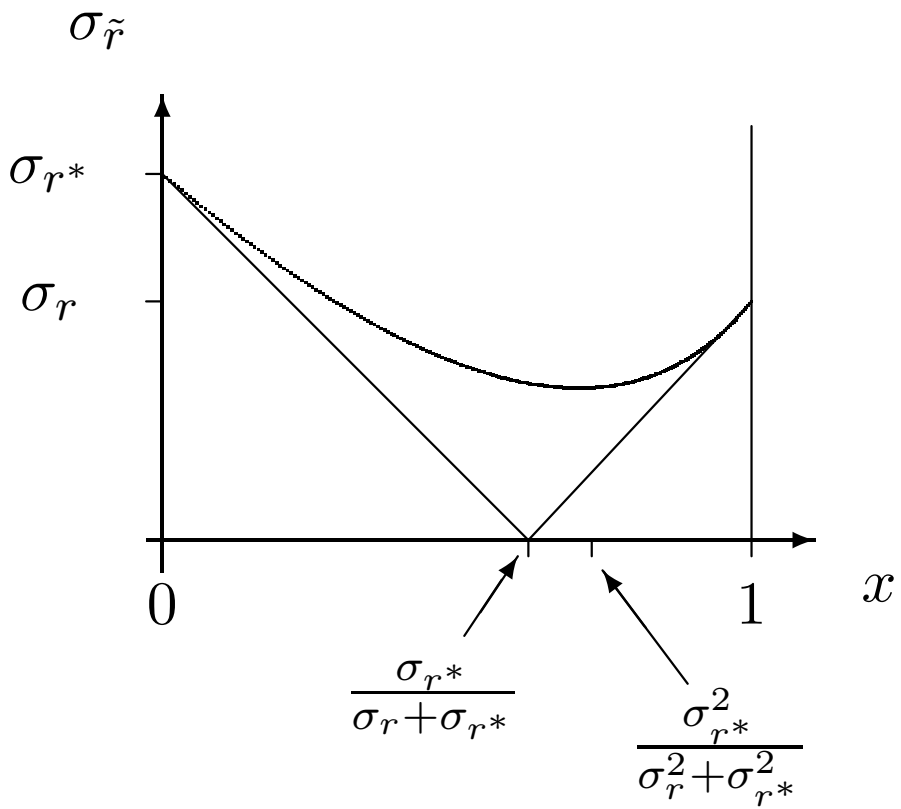
$$\sigma_{\tilde{r}} = |x\sigma_r - (1-x)\sigma_{r^*}|.$$

It is possible to form a riskless portfolio:

$$x = \frac{\sigma_{r^*}}{\sigma_r + \sigma_{r^*}}$$

yields $\sigma^2_{\tilde{r}} = 0$.

Theorem: *International capital mobility allows risk reduction if domestic and foreign returns are independent. Riskless investment is possible if the returns are perfectly negatively correlated.*



From the formula for the portfolio variance,

$$\frac{\partial \sigma^2_{\tilde{r}}}{\partial \sigma_{r,r^*}} = 2x(1-x) > 0.$$

So generally, the lower the correlation of the returns, the greater the scope for risk reduction.

The “puzzle of home bias in equity portfolios” is why investors appear to make so little use of the opportunity to diversify.

Literature:
Danthine, Jean-Pierre, and John B. Donaldson (2005), *Intermediate Financial Theory*, London: Elsevier, Chapter 6 (or any other text on modern portfolio theory).

- http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1025806

Problems:

1. There are three states with r and r^* given by the table below.

	state		
probability	1/6	1/2	1/3
r	2%	6%	8%
r^*	10%	6%	4%
$r - E(r)$			
$r^* - E(r^*)$			

(a) Show: $E(r) = E(r*)$.
(b) Complete the table.
(c) Compute σ_r and σ_{r^*}.
(d) Compute the correlation between r and r^*.
(e) Compute the portfolio share x for the domestic asset which yields a riskless payoff.

2. There are two states with r and r^* given by the table below.

	state	
probability	1/4	3/4
r	5.5%	7.5%
r^*	4%	8%
$r - E(r)$		
$r^* - E(r^*)$		

(a) Complete the table.
(b) Compute σ_r and σ_{r^*}.
(c) Show, using the formula for $\sigma^2_{\tilde{r}}$ that

$$\sigma_{\tilde{r}} = \frac{\sqrt{3}\%}{2}(2 - x).$$

(d) Compute the correlation between r and r^*.
(e) Argue why a decrease in the share of wealth invested at home increases $\sigma_{\tilde{r}}$.

3. (a) From the definition of the variance, derive

$$\sigma^2_{ax+b} = a^2\sigma^2_x$$

and

$$\sigma^2_x = E(x^2) - (Ex)^2.$$

(b) From the definition of the covariance, derive

$$\sigma_{x,y} = E(x\,y) - (Ex)(Ey).$$

4. Suppose an investor has wealth w and is characterized by the quadratic utility function

$$U = c - ac^2,$$

where a is small enough such that $c < 1/(2a)$ and, hence, $dU/dc > 0$. His consumption is $R \equiv (1 + r)w$ in autarky and $\tilde{R} \equiv (1 + \tilde{r})w$ with international diversification.

(a) Express autarky expected utility $E(U)$ as a function of $E(R)$ and σ_R^2, and analogously for the case of diversification.
(b) Let $w = 1$ and $a = 1/16$. Show that diversification raises expected utility by $1/40.000$ for the returns in Problem 1.

5. Suppose the domestic interest rate r is non-random, and $E(r^*) > r$.
(a) Compute $E(\tilde{r})$ and $\sigma_{\tilde{r}}^2$ as functions of x.
(b) Eliminate x from the functions in part (a). Derive the relationship between the portfolio risk premium $E(\tilde{r}) - r$ and the portfolio standard deviation $\sigma_{\tilde{r}}$.

6. Technology transfer

International capital flows are subdivided into

- portfolio investment (acquisition of financial claims) and
- foreign direct investment (FDI) (acquisition of financial claims and control rights).

FDI can realized by buying shares in a host country firm or by setting up or expanding the activity of a foreign subsidiary. Horizontal FDI replaces or duplicates activities at home. Vertical FDI consists of outsourcing stages of the value chain.

The stock of FDI is almost 40 percent of world GDP.

- http://unctad.org/en/Pages/Statistics.aspx
- http://unctad.org/en/pages/PublicationWebflyer.aspx?publicationid=84

- For firms, outward FDI has the benefits of proximity to the foreign market and elimination of exchange rate risk (since costs and revenue are in the same currency).
- Because of the transfer of control rights, FDI is politically sensitive. A major benefit of FDI for the host country is that it provides the home country firm with an incentive to put its best available technologies to use there (technology transfer).
- For workers in high wage countries, FDI in low wage countries (like imports from emerging economies) raises fears of unemployment or growing wage inequality.

- http://www.nber.org/chapters/c9543.pdf
- http://www.nytimes.com/2007/12/28/opinion/28krugman.html?_r=0

Problems:

1. Consider a simple two-period model. At date 1, a firm can sell y units of output abroad at price p (both exogenously given) in a foreign market. It can produce the good at home at unit cost $\bar{c}$, which includes tariffs, and export the good at both dates. Alternatively, it can set up a foreign subsidiary, which produces for the local market at lower unit cost $\underline{c}$, due to lower wages and no tariffs. In that case, the technology becomes common knowledge in the foreign country, so the market is competitive and the firm makes no profits at date 2. Ignore discounting.

Show that the condition for profitability of producing abroad can be expressed in the form "date-2 profit from exporting $<$ cost advantage from relocating production".

2. A domestic monopolist produces a good at unit cost c. The demand curve in a foreign market is $y = a - bp$, where $a > bc$, so that there is positive demand at the competitive price level. Suppose first the monopolist sells his output to a foreign retailer, who is also a monopolist in the foreign market, at price q.

(a) Compute the retailer's monopoly price p, given q. Ignore other costs besides q.

(b) Compute the domestic monopolist's profit maximizing price q, given that he anticipates the impact of his pricing policy on p and y.

(c) Compute the domestic monopolist's profit.

Now assume that the domestic producer sets up a local distribution network in the foreign country (which, as in (a) does not cause costs), so he becomes a standard monopolist.

(d) Compute the monopoly price and monopoly profit. Compare the results to (b) and (c). Explain.

(e) Suppose there is perfect competition between foreign retailers, so that $p = q$. Is there an incentive for vertical integration?

7. Flexible exchange rates

We now turn to monetary international macro models. The present section introduces the monetary model of the exchange rate (MME). This model serves to illustrate the asset approach to exchange rate determination in the present section. It is the basis of the exchange rate overshooting model and first models of currency crises. It helps understand the costs of fixing the exchange rate in general.

Let

- m_t: log money supply at date t
- p_t: log price level
- y_t: log GDP
- i_t: nominal interest rate
- p_t^*: log foreign price level
- s_t: log exchange rate (price of foreign in terms of domestic currency, i.e., "euros per dollar")
- i_t^*: foreign interest rate
- E_t: expectations taken at date t.

Upper-case letters refer to the corresponding non-logged variables.

The models so far did not incorporate money. So we could normalize the price of the consumption good to unity and interpret r as the *real* interest rate. We use the new symbol i_t to denote the *nominal* interest rate.

The MME relies on two crucial "parity conditions":

- Investing one euro at home at t yields $1 + i_t$ euros. Buying $1/S_t$ dollars and investing them abroad yields $(1 + i_t^*)/S_t$ dollars, which can be used to buy $(1 + i_t^*)S_{t+1}/S_t$ euros at $t + 1$. Uncovered interest parity (UIP) states that the expected date-$t + 1$ wealth levels are the same:

$$1 + i_t = (1 + i_t^*)\frac{E_t S_{t+1}}{S_t}.$$

  Let Δ denote the change in a variable. Approximating $(\Delta S_{t+1}/S_t)i_t^* \approx 0$ and $\Delta s_{t+1} \approx \Delta S_{t+1}/S_t$ yields UIP:

$$i_t \approx i_t^* + E_t \Delta s_{t+1}.$$

- The price of domestic goods in terms of euros is P_t. The price of foreign goods in terms of euros is $P_t^* S_t$. Purchasing power parity (PPP) states that the prices, expressed in one currency, are the same: $P_t = P_t^* S_t$ or

$$p_t = p_t^* + s_t.$$

The validity of both parity conditions can be disputed:

- High interest countries do not tend to depreciate.
- PPP is evidently violated in the short run, as exchange rates are much more volatile than prices. For the long run, it is a reasonable assumption.

- https://www.imf.org/external/pubs/ft/staffp/2002/02/flood.htm
- http://www.ssc.wisc.edu/~mchinn/taylor&taylor_PPP_JEP.pdf

The MME nonetheless assumes the two parity conditions plus an LM equation:

$$i_t = i_t^* + E_t \Delta s_{t+1}$$

$$p_t = p_t^* + s_t$$

$$m_t - p_t = \phi y_t - \frac{i_t}{\lambda}.$$

It is a supply-side model: y_t is exogenous.

Solving the model yields an expectational difference equation (EDE) for the exchange rate:

$$s_t = m_t - p_t^* - \phi y_t + \frac{i_t^* + E_t \Delta s_{t+1}}{\lambda}.$$

Finding a solution to the EDE is simple for three special cases.

- Suppose the quantity theory of money holds: $\lambda = \infty$, so $m_t - p_t = \phi y_t$. Then,

$$s_t = m_t - p_t^* - \phi y_t.$$

- The fundamentals m_t, p_t^*, y_t, and i_t^* are constants. Then,

$$s_t = m_t - p_t^* - \phi y_t + \frac{i_t^*}{\lambda}.$$

- The same solution applies if the fundamentals are random walks, since s_t is then a random walk and $E_t \Delta s_{t+1} = 0$.

Let $x_t \equiv \lambda(m_t - p_t^* - \phi y_t) + i_t^*$, so that

$$s_t = \frac{x_t + E_t s_{t+1}}{1+\lambda}.$$

The following result gives the general solution to the EDE.

Theorem:

$$s_t^* = \frac{1}{1+\lambda} \sum_{i=0}^{\infty} \frac{E_t x_{t+i}}{(1+\lambda)^i}$$

is an equilibrium exchange rate (provided that the sum is finite).

Proof:

$$
\begin{aligned}
s^*_{t+1} &= \frac{1}{1+\lambda}\sum_{j=0}^{\infty}\frac{E_{t+1}x_{t+j+1}}{(1+\lambda)^j} \\
E_t s^*_{t+1} &= \frac{1}{1+\lambda}\sum_{j=0}^{\infty}\frac{E_t x_{t+j+1}}{(1+\lambda)^j} \\
&= \sum_{i=1}^{\infty}\frac{E_t x_{t+i}}{(1+\lambda)^i} \\
&= \sum_{i=0}^{\infty}\frac{E_t x_{t+i}}{(1+\lambda)^i} - x_t \\
&= (1+\lambda)s^*_t - x_t \\
s^*_t &= \frac{x_t + E_t s^*_{t+1}}{1+\lambda},
\end{aligned}
$$

where use is made of the law of iterated expectations: $E_t(E_{t+1}x_{t+j+1}) = E_t x_{t+j+1}$.

Note the similarity to standard asset pricing: the price of an asset is the (appropriately discounted and risk-adjusted) sum of the expected cash flow it generates and the expected resale price at the following date. The solution to this EDE gives the asset price as the (appropriately discounted and risk-adjusted) sum of all future expected cash flows the asset generates. Here, the EDE links the current exchange rate to its future expected value. The exchange rate is determined as a weighted sum of all future expected fundamentals.

As shown above, when fundamentals follow a random walk, then so does the exchange rate. When fundamentals are close to a random walk, then so is the exchange rate. The best exchange rate forecast is then the current level. This is one possible explanation for why it is so hard for economic models of the exchange rate to beat the random walk forecast.

- http://www.nber.org/chapters/c6829.pdf
- http://scholar.harvard.edu/files/rogoff/files/monetary_modelsof_dollar_

The result most heavily emphasized by the monetary theory of the exchange rate is that in the simple special cases:

$$\frac{\partial s_t}{\partial m_t} = \frac{\partial S_t / S_t}{\partial M_t / M_t} = 1.$$

That is, a one-percent increase in the stock of money causes a one-percent decrease in the value of the domestic currency (since prices rise one percent and PPP must hold).

Literature:

Mussa, Michael L. (1984), "The Theory of Exchange Rate Determination", in: John F. O. Bilson and Richard C. Marston (eds.), *Exchange Rate Theory and Practice*, Chicago: University of Chicago Press, Chapter 1, pp. 13-78.

- http://www.nber.org/chapters/c6829.pdf

Problems:

1. (a) Plot $x - 1$ and $\ln x$ as functions of x. Argue why $x - 1 \approx \ln x$ for $x \approx 1$.
(b) Check the fit of the approximation for $x = 0.9$, $x = 0.95$, $x = 1.05$, and $x = 1.1$.
(c) Set $x = S_{t+1}/S_t$ in part (a). Show that the percentage change in S_t is approximately equal to the change in s_t.

2. (a) An investor takes a \$100 one-period loan at t and converts the loan to euros. How many euros does he get?
(b) He invests the money in the euro area. What is the resulting wealth at $t + 1$.
(c) How many euros does he need to repay the dollar loan?
(d) Derive the condition that ensures that the investor expects zero profit from the transaction.
(e) Solve the condition in part (d) for i_t.

3. Consider the following MME:

$$m_t - p_t = 0.1 - 9i_t$$

$$s_t + 2 = p_t$$

$$i_t = \frac{10}{9}\% + E_t \Delta s_{t+1}.$$

(a) Suppose the fundamentals are constant. Express s_t as a function of m_t and $E_t \Delta s_{t+1}$.
(b) Compute s_t for a constant money supply $m_t = 4$.
(c) Solve the equation in part (a) for s_t as a function of m_t and $E_t s_{t+1}$.
(d) Consider an unanticipated permanent increase in the supply of money to $m_t = 5$ at $t = 3$. Compute s_t for $t = 0, 1, 2, \ldots$
(e) Now suppose the increase in the money supply is anticipated at $t = 1$ (and $t = 2$). Compute s_t for $t = 0, 1, 2, \ldots$

4. Here is a version of the Keynesian Fleming-Mundell model:

$$i_t = i_t^*$$

$$y_t = \delta(s_t + p_t^* - p_t) - \sigma i_t$$

$$m_t - p_t = \phi y_t - \frac{i_t}{\lambda}.$$

The second equation is the IS equation.
(a) Given that this is a Keynesian model with aggregate demand and price stickiness, which variables are endogenous, which are exogenous? Relate this model to the MME.
(b) Illustrate the equilibrium graphically in a (y_t, i_t)-diagram. Solve analytically for the endogenous variables.
(c) How does an increase in the supply of money m_t affect the exchange rate?

5. Let F_{t+1} denote the forward exchange rate, i.e., the price for the delivery of one dollar at $t+1$ to be paid at t. Covered interest parity (CIP) says that

$$1 + i_t = (1 + i_t^*)\frac{F_{t+1}}{S_t}.$$

(a) Argue that the validity of CIP follows from absence of arbitrage opportunities.
(b) Let

$$FD_{t+1} = \frac{F_{t+1} - S_t}{S_t}$$

be the forward premium of the foreign currency, i.e., the increase in the price of foreign currency predicted by the forward market. Restate the CIP condition in terms of FD_{t+1} instead of F_{t+1}.
(c) Compare UIP and CIP. Which one is the weaker condition? What are the required assumptions on investors' risk attitude?

8. Sticky prices: Overshooting

Exchange rate overshooting (OS) means that the exchange rate overreacts to changes in the macroeconomic environment and then converges to its new equilibrium coming from the wrong direction. This is a standard explanation for high exchange rate volatility.

The response of the exchange rate to a monetary impulse is characterized by OS if goods prices are sticky, i.e., react more slowly to shocks than asset prices:

- A positive money supply shock leads to a depreciation in the long run.
- In the short run it lowers the interest rate.
- UIP implies that the domestic currency must appreciate while it approaches its new long run equilibrium level. So it has to overshoot at first.

- http://scholar.harvard.edu/rogoff/publications/perspectives-exchange-rate-volatility

The model is due to Dornbusch (1976):

$$i_t = i_t^* + E_t \Delta s_{t+1}$$

$$\Delta p_{t+1} = \delta(s_t + p_t^* - p_t) - \sigma i_t$$

$$m_t - p_t = \phi y_t - \frac{i_t}{\lambda}.$$

The model is the MME with one main difference: PPP does not hold. Instead, high demand leads to rising goods prices with a lag. UIP holds. Thus, goods prices change more sluggishly than asset prices.

For simplicity, we set $i_t^* = y_t = p_t^* = 0$ and $m_t = m$ constant.

- http://www.imf.org/external/np/speeches/2001/112901.htm

At a long-run equilibrium, with the exchange rate and the price level constant (i.e., $\Delta s_{t+1} = \Delta p_{t+1} = 0$),

$$i_t = 0,\ p_t = m,\ s_t = m.$$

As in the MME, a one percent monetary expansion leads to a one percent depreciation of the domestic currency.

Now, suppose starting from a long-run equilibrium, there is a once-for-all unanticipated increase in m. What's the reaction of the exchange rate to this shock?

The price moves sluggishly: it does not change at the date when the monetary expansion occurs.

Since no further shocks occur, rational expectations implies $E_t \Delta s_{t+1} = \Delta s_{t+1}$ subsequently. So the dynamics of the exchange rate and the price level is governed by the following system of two ordinary difference equations (ODEs):

$$\Delta s_{t+1} = \lambda(p_t - m)$$

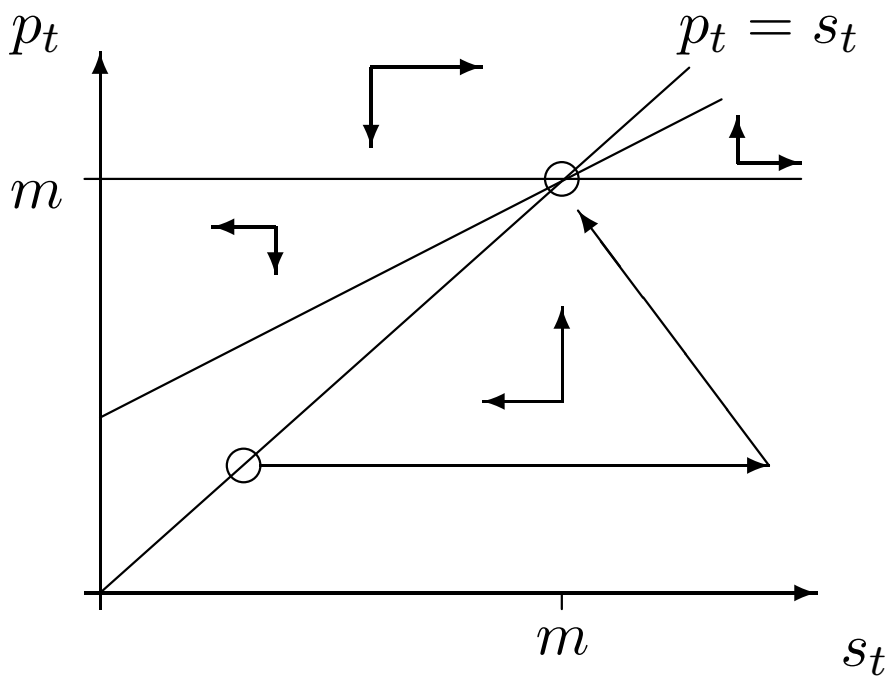
$$\Delta p_{t+1} = (\delta s_t + \sigma\lambda m) - (\delta + \sigma\lambda)p_t.$$

s_t rises for $p_t > m$, and vice versa. p_t rises for

$$p_t < \frac{\delta s_t + \sigma\lambda m}{\delta + \sigma\lambda},$$

and vice versa.

Theorem: *Suppose the exchange rate converges to a new long-run equilibrium after a positive monetary impulse. Then it overshoots at first: the initial depreciation of the domestic currency is stronger than the long-run depreciation; during the convergence it appreciates.*



Let $\tilde{s}_t \equiv s_t - m$ and $\tilde{p}_t \equiv p_t - m$ denote deviations from the long-run equilibrium. From the two ODEs for s_t and p_t, it follows that

$$\tilde{s}_{t+2} - [2 - (\delta + \sigma\lambda)]\tilde{s}_{t+1} + [1 - \delta\lambda - (\delta + \sigma\lambda)]\tilde{s}_t = 0.$$

Consider solutions of type $\tilde{s}_t = Aq^t$ for constants A and q. The eigenvalues q must satisfy

$$q^2 - [2 - (\delta + \sigma\lambda)]q + [1 - \delta\lambda - (\delta + \sigma\lambda)] = 0,$$

that is

$$q_{+/-} = \frac{2 - (\delta + \sigma\lambda)}{2} \pm \sqrt{\left[\frac{2 - (\delta + \sigma\lambda)}{2}\right]^2 - [1 - \delta\lambda - (\delta + \sigma\lambda)]}.$$

The discriminant can be written as

$$\frac{(\delta + \sigma\lambda)^2 + 4\delta\lambda}{4} > 0,$$

so the roots are real. The general solution of the ODE is $\tilde{s}_t = A_+ q_+^t + A_- q_-^t$.

The roots obey $q_+ > 1 > q_-$. So the convergent path satisfies

$$\tilde{s}_t = A_- q_-^t.$$

Convergence requires $q_- > -1$ or

$$\delta + \sigma\lambda + \sqrt{(\delta + \sigma\lambda)^2 + 4\delta\lambda} < 4.$$

(The necessity of this inequality for convergence cannot be inferred from the phase diagram.)

From

$$\begin{aligned} \lambda\tilde{p}_t &= \lambda(p_t - m) = \Delta s_{t+1} = \Delta\tilde{s}_{t+1} \\ &= A_-(q_-^{t+1} - q_-^t) = A_- q_-^t (q_- - 1), \end{aligned}$$

it follows that

$$\tilde{s}_0 = A_- = \frac{\lambda\tilde{p}_0}{q_- - 1}.$$

Since the price level starts below its new equilibrium level ($\tilde{p}_0 < 0$) and $q_- < 1$, the exchange rate jumps beyond the new long-run equilibrium ($\tilde{s}_0 > 0$).

Literature:

Dornbusch, Rudiger (1976), “Expectations and Exchange Rate Dynamics”, *Journal of Political Economy* 84, 1161-76.

Gandolfo, Giancarlo (2001), *International Finance and Open-Economy Macroeconomics*, Berlin: Springer, Subsection 15.3.2.

- https://www.semanticscholar.org/paper/Expectations-and-Exchange-Rate-Dynamics-Dornbusch/36f70eab946e67308909edf4eae4a39f1845f1e5

Problems:

1. Consider an unanticipated increase from $m_t = 1$ to $m_t = 2$ in the following model:

$$i_t = E_t \Delta s_{t+1}$$

$$\Delta p_{t+1} = \frac{1}{2}(s_t - p_t) - \frac{3}{4} i_t$$

$$m_t - p_t = -5 i_t.$$

(a) Illustrate the overshooting result with the phase diagram.
(b) Derive the second-order ODE for the deviation of s_t from its steady state level. Compute the roots of the ODE.
(c) Derive a closed-form solution for $\tilde{s}_t$ from the general solution $\tilde{s}_t = A_- q_-^t$ and the initial condition $p_0 = 1$. State the closed-form solution for s_t. Argue that there is overshooting.

9. Fixed exchange rate

Fixing the exchange rate provides certainty about the future value of cash flows generated abroad in terms of domestic currency. This applies both to sales of goods and services and to returns on investments.

Ever since international trade flourished after the industrialization, major trading economies tried to create a system of fixed exchange rates:

- the classical Gold Standard (1881-1913)
- the the inter-war Gold Standard
- the Bretton-Woods System (1946-1973)
- the European Monetary System (EMS) (1979-1993)
- the European Monetary Union (EMU) (1999-).

- http://research.stlouisfed.org/publications/review/93/03/Gold_Mar_Apr1993.pdf

To fix the exchange rate, it does not suffice to specify the rate. The crucial task is to take care that the equality

$$\begin{aligned}\text{current account surplus} &= \text{net capital exports} \\ &\quad + \text{change in reserves}\end{aligned}$$

is satisfied at the fixed exchange rate. There are two ways of accomplishing this:

- **Financing the imbalance:** The central bank trades

$$\begin{aligned}\text{change in reserves} &= \text{current account surplus} \\ &\quad - \text{net capital exports.}\end{aligned}$$

  It runs down its reserves if the change is negative or buys reserves with fresh money otherwise.
- **Equilibrating the system:** Conduct economic policy such that the equality holds changes in reserves.

One means to equilibrate the system is to conduct monetary policy accordingly. This is the basis of the so-called “open-economy trilemma (OET)”: a country cannot have

- a stable exchange rate,
- international mobility of financial capital, and
- autonomy over monetary policy

at a time.

The standard model used to illustrate this is the Fleming-Mundell model. Here we use the MME to illustrate the OET.

- http://www.youtube.com/watch?v=oLbfAfCVG_4

In the MME, suppose the home country fixes the value of the foreign currency s. Then, from the EDE that governs the evolution of s, it follows immediately that:

Theorem: *Equilibrium requires that the quantity of money obeys*

$$m_t = s + p_t^* + \phi y_t - \frac{i_t^*}{\lambda}.$$

Systems of fixed exchange rates break down when member countries are no longer willing to accept the non-autonomy of monetary policy.

- http://www.nber.org/chapters/c6876.pdf
- https://www.ubs.com/global/en/about_ubs/follow_ubs/ubs-economic-insights/andreas_hoefert/2012/8/31/the-20th-anniversary-of-the-ems-crisis.html

When should a country fix its exchange rate?

The direct effects of fixing the exchange rate are:

- no uncertainty about the value of foreign currency for firms and investors (if it works)
- the exchange rate is of no help in restoring competitiveness.

The indirect effects of giving up autonomy over monetary policy are:

- reduced scope for stabilizing business fluctuations
- the exchange rate may serve as a “nominal anchor” for anti-inflation policies.

So a country should fix its exchange rate if

- it intensively trades goods and capital
- wages and prices react flexibly to changes in competitiveness
- the level of public debt is sufficiently low so that there is scope for anticyclical fiscal policy
- fixing the exchange rate helps to establish credibility of anti-inflationary policies.

The EMU as a system of fixed exchange rates:
The EMU (or, more generally, a monetary union) can be regarded as an attempt to "irrevocably" fix the member states' currencies (in response to the breakdown of the EMS).

The principal difference to tying the currencies one-to-one to each other is that there is one common central bank, the ECB.

- There is not a national central bank that finances the imbalance when
  current account surplus $\neq$ net capital exports.
- There is no national monetary policy that can equilibrate the system.

So equilibrating the system must work via policy measures other than monetary policy. Meanwhile, how does financing the imbalance work?

deficit country	rest of EMU
current account $<$ net capital exports	current account $>$ net capital exports
private capital flows do not enable agents to pay their bills	agents are unwilling to finance the deficit country's imports of domestic goods
private banks borrow the required money from the national central bank	private banks deposit the surplus with the national central bank
national central bank borrows the required money from the ECB (TARGET2 deficit)	national central banks acquire claims to the ECB (TARGET2 surplus)

- https://www.bundesbank.de/de/aufgaben/unbarer-zahlungsverkehr/target2/target2-saldo/target2-saldo-603478
- https://www.degruyter.com/view/j/pwp.2018.19.issue-4/pwp-2019-0002/pwp-2019-0002.xml
- https://www.degruyter.com/view/j/pwp.2019.20.issue-3/pwp-2019-0038/pwp-2019-0038.xml?format=INT

Problems:

1. (a) In the MME from the MME section problems, how does the central bank has to set m_t in order to fix the exchange rate at $s_t = 1$?
(b) In that model, suppose the central bank announces at $t = 1$ that it will fix the exchange rate at $s_t = 1$ starting at $t = 2$, while $m_1 = 4$. Compute s_1. Compare s_1 to the exchange rate that would prevail if the money supply stayed constant at its current level $m_1 = 4$ and to the fixed exchange rate $s_2 = 1$.

2. Recall the Fleming-Mundell model:

$$i_t = i_t^*$$

$$y_t = \delta(s_t + p_t^* - p_t) - \sigma i_t$$

$$m_t - p_t = \phi y_t - \frac{i_t}{\lambda}.$$

Suppose the exchange rate s_t is being fixed.
(a) Solve for y_t.
(b) Which variables are endogenous? Illustrate equilibrium in an (y_t, i_t)-diagram.

3. (a) What are the effects of financing the imbalance on the domestic supply of money, depending on whether current account surplus − net capital exports is positive or negative?
(b) Why might the CB not like the monetary effects of its FX interventions?
(c) How does the CB have to make use of other monetary operations in order to undo the impact of its FX interventions on the money supply ("sterilization policy")?

10. Currency crisis: 1st generation models

1st generation currency crisis models were developed to explain Latin American currency crises in the 1970s, which appear to be caused by a combination of a dollar peg, capital account liberalization, and reluctance to give up autonomy over monetary policy (i.e., accept the OET).

What they add to the OET is a dynamic notion of the crisis: reserves fall slowly at first, then investors buy the remaining stock of reserves when it is still “large” in a speculative attack on the currency.

- http://www.kellogg.northwestern.edu/faculty/rebelo/htm/currency%20crisis%20models%20Ed.pdf

Model:

$$i_t = i^* + \frac{\Delta S_{t+1}}{S_t}$$

$$P_t = S_t P^*$$

$$\frac{M_t}{P_t} = \phi Y - \frac{i_t}{\lambda}$$

$$M_t = R_t + D_t$$

$$\Delta D_t = \mu$$

The first three assumptions (UIP, PPP, and LM) are (a linear, not log-linear, and non-stochastic version of) the MME. The expectations operator does not appear in the UIP condition, because there is no uncertainty. GDP Y is exogenous and constant.

R_t denotes foreign exchange reserves. D_t denotes the sum of

- money creation via central bank credit to the banking system
- monetary financing of government expenditure,

where the latter is nowadays, of course, forbidden in developed countries with independent central banks. The fourth equation says that total money in circulation is the sum of its components. The fourth equation says that there is an autonomous element of monetary policy: a constant exogenous amount of money is continually created via banks or government funding. R_0 and D_0 and, hence, M_0 are given. R_0 is the amount of reserves the country is willing to use in order to defend the exchange rate.

Eliminating the price level and the interest rate from the first three assumptions yields the following ODE for the exchange rate:

$$S_t = \frac{M_t}{\beta} + \frac{\alpha \Delta S_{t+1}}{\beta},$$

where

$$\beta \equiv P^* \left(\phi Y - \frac{i^*}{\lambda} \right)$$

and

$$\alpha \equiv \frac{P^*}{\lambda}.$$

This is the analogue of the EDE for the exchange rate in the MME. The expectations operator is obsolete here because of no uncertainty.

Fixed exchange rate: Suppose the government fixes the exchange rate at date 0 at the level $\bar{S}$ which is consistent with equilibrium:

$$\bar{S} = \frac{M_0}{\beta}.$$

For PPP, the price level has to be stabilized at the level

$$P_t = \bar{S}P^*$$

subsequently. This requires that money supply is held constant at the level

$$M_t = \bar{S}P^* \left(\phi Y - \frac{i^*}{\lambda} \right).$$

As D_t grows, this requires that reserves R_t fall:

$$R_t = R_0 - \mu t.$$

The fixed exchange rate system can survive no more than

$$T' = \frac{R_0}{\mu}$$

periods. In fact, it breaks down earlier.

Flexible exchange rate:
Suppose the currency floats freely after the fixed exchange rate is abandoned. Since there are no reserves left then ($R_t = 0$), we have $M_t = D_t$ or (using $\Delta D_t = \mu$) $M_t = D_0 + \mu t$. So

$$S_t = \frac{D_0 + \mu t}{\beta} + \frac{\alpha \Delta S_{t+1}}{\beta}.$$

This is a non-autonomous ODE for S_t, as time t enters explicitly (not only as a subscript on the exchange rate). We solve the ODE by trying a solution of the form

$$S_t = a_0 + a_1 t,$$

where a_0 and a_1 are constants to be determined. Substituting for S_t and $\Delta S_{t+1} = a_1$ into the ODE yields:

$$(\beta a_0 - \alpha a_1 - D_0) + (\beta a_1 - \mu)t = 0.$$

Validity for all t under a flexible exchange rate implies

$$a_1 = \frac{\mu}{\beta}$$

and

$$a_0 = \frac{\alpha\mu}{\beta^2} + \frac{D_0}{\beta}.$$

So

$$S_t = \frac{\alpha\mu}{\beta^2} + \frac{D_0}{\beta} + \frac{\mu}{\beta}t.$$

Suppose the fixed exchange rate regime is given up at T'. Then

$$\begin{aligned} S_{T'} &= \frac{\alpha\mu}{\beta^2} + \frac{D_0}{\beta} + \frac{\mu}{\beta} T' \\ &= \frac{\alpha\mu}{\beta^2} + \frac{D_0}{\beta} + \frac{R_0}{\beta} \\ &= \frac{\alpha\mu}{\beta^2} + \bar{S}, \end{aligned}$$

hence

$$S_{T'} - S_{T'-1} = \frac{\alpha\mu}{\beta^2} > 0.$$

However, from the fact that M_t/P_t is constant up to $t = T' - 1$ and the condition for equilibrium in the money market, it follows that

$$i_{T'-1} = i^*.$$

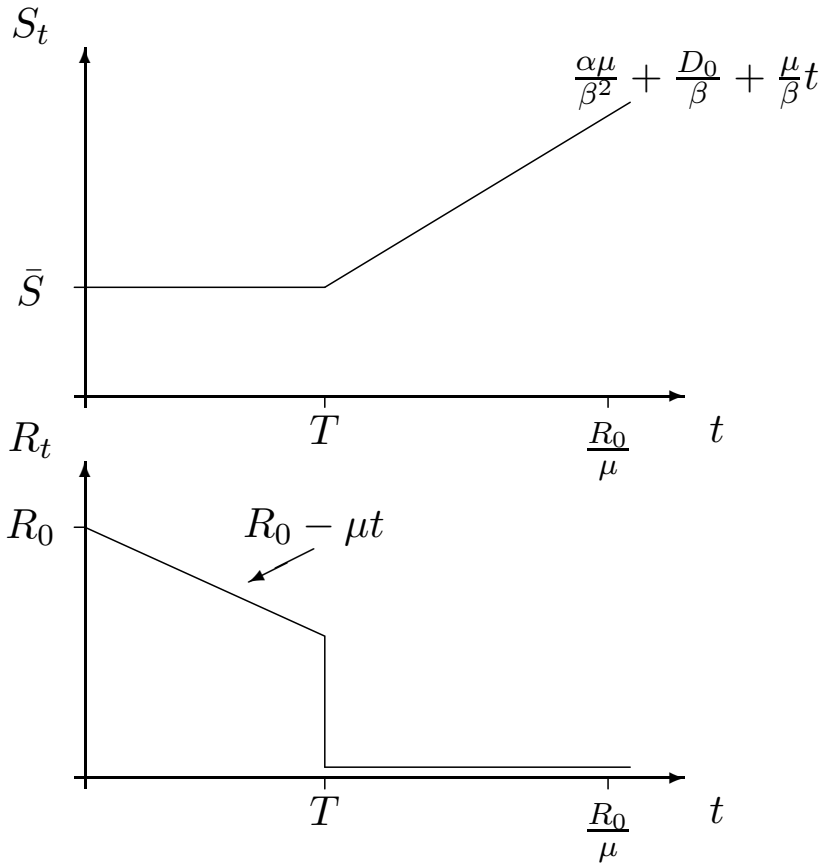
Therefore,

$$i_{T'-1} < i^* + \frac{S_{T'} - S_{T'-1}}{S_{T'-1}}.$$

Since investors anticipate that the fixed exchange rate will be abolished and the foreign currency will appreciate at date T', UIP is violated at date $T' - 1$. This is inconsistent with equilibrium. So the currency peg must break down before date T'.

So *when* does the currency collapse? At that date T when abandoning the peg does not cause a jump:

$$\begin{aligned} \bar{S} &= S_T \\ \frac{M_0}{\beta} &= \frac{\alpha\mu}{\beta^2} + \frac{D_0}{\beta} + \frac{\mu}{\beta} T \\ T &= \frac{R_0}{\mu} - \frac{\alpha}{\beta}. \end{aligned}$$



Theorem: *A positive stock of reserves is depleted at once and the exchange rate starts to float at some $T < T'$.*

Literature:

Flood, Robert P., and Peter M. Garber (1984), "Collapsing Exchange-Rate Regimes: Some Linear Examples", *Journal of International Economics* 17, 1-13.

Gandolfo, Giancarlo (2001), *International Finance and Open-Economy Macroeconomics*, Berlin: Springer, Subsection 16.3.1.

- http://www.macroeconomics.tu-berlin.de/fileadmin/fg124/financial_crises/literature/floodgarber.pdf

Problems:

1. Let

$$\begin{aligned} M_t &= R_t + D_t \\ \Delta D_t &= 0.01 \\ \frac{M_t}{P_t} &= 1 - 10 i_t \\ i_t &= 0.05 + \frac{\Delta S_{t+1}}{S_t} \\ P_t &= S_t, \end{aligned}$$

$R_0 = 0.4$ and $D_0 = 0.6$.

(a) Derive the equation that relates S_t to M_t and ΔS_{t+1}.

(b) Which fixed exchange rate $\bar{S}$ is consistent with equilibrium at $t = 0$?

(c) How long would it take until the country's reserves are exhausted if they fell continually by ΔD_t per period?

(d) Show that, once the exchange rate floats freely, it satisfies $S_t = 1.6 + 0.02t$.

(e) Argue that the return on foreign assets acquired at $T' - 1$ is 25% if the exchange rate started to float at T'.

(f) Compute the date T at which the speculative attack occurs.
(g) Compute the stock of reserves R_T lost in the attack.

11. Currency crisis: 2nd generation models

The major criticism of 1st generation currency crises models is that policy is passive: everyone knows there is a fixed date at which the currency collapses, but all the government does is sell the amount of reserves needed to keep the peg until the crash happens. 2nd generation models move policy *decisions* to the center of the analysis.

Obstfeld (1996) presents a 2nd generation "toy model" (before also presenting a "serious" model). There are two traders and a central bank that tries to defend a currency peg. Speculation consists of taking debt in the domestic currency and investing the money abroad. This speculation pays off if the country's currency depreciates (recall the derivation of interest parity in the MME section). Taking the respective short and long positions comes at a cost c, which is a shorthand for the interest differential and transaction costs.

- http://www.kellogg.northwestern.edu/faculty/rebelo/htm/currency%20crisis%20models%20Ed.pdf

Taking debt in the domestic country and investing the money abroad contributes to the demand for foreign exchange, thereby forcing the central bank to sell further reserves. The currency peg has to be given up if the resulting excess demand for foreign exchange exceeds the central bank's reserves.

Let

- R: central bank's stock of reserves
- ΔS: appreciation of the foreign currency if the peg breaks down
- K: per capita speculative capital
- c: cost of speculation against the currency.

It is assumed that speculation is profitable when a trader takes a position half of the size of the central bank's reserves and the depreciation occurs:

$$\frac{R}{2}\Delta S > c.$$

Case 1: large reserves ($2K < R$)

Reserves are sufficient to cover the excess demand for foreign currency that occurs when both traders speculate against the domestic currency.

		trader 2	
		"attack"	"don't"
trader 1	"attack"	$-c$ / $-c$	$-c$ / 0
	"don't"	0 / $-c$	0 / 0

Dominant strategy equilibrium: ("don't", "don't"). An attack does not occur.

Case 2: low reserves ($R < K$)

A single trader's capital is enough to wipe out the central bank's reserves. It is assumed that if both traders try to borrow K, each one gets $R/2$.

		trader 2	
		"attack"	"don't"
trader 1	"attack"	$\boxed{\frac{R}{2}\Delta S - c \,/\, \frac{R}{2}\Delta S - c}$	$R\Delta S - c \,/\, 0$
	"don't"	$0 \,/\, R\Delta S - c$	$0 \,/\, 0$

Dominant strategy equilibrium: ("attack", "attack"). An attack occurs.

Case 3: intermediate reserves ($K < R < 2K$)

		trader 2	
		"attack"	"don't"
trader 1	"attack"	$\boxed{\frac{R}{2}\Delta S - c \,/\, \frac{R}{2}\Delta S - c}$	$-c \,/\, 0$
	"don't"	$0 \,/\, -c$	$\boxed{0 \,/\, 0}$

There are now two Nash equilibria: ("attack", "attack") and ("don't", "don't"). The fixed exchange rate system breaks down if, and only if, a coordinated attack happens. Expectations are self-fulfilling. This multiplicity of equilibria is due to the strategic interaction of traders and the central bank.

Obstfeld (1996) and Sachs, Tornell, and Velasco (1996) (STV) present a more elaborate model of self-fulfilling currency crises.

Consider a country with a fixed exchange rate. Let:

- L: government's "loss" from giving up the peg
- u: unemployment rate
- $\bar{u}$: "natural" rate of unemployment.

Version 1: no fixed cost of devaluation

To endogenize government behavior, the model introduces government preferences: its loss function is

$$L = \alpha(\Delta s)^2 + u^2.$$

There are two possible reasons why it dislikes a devaluation:

- a reputational loss due to having to give up the currency peg
- imported inflation.

The tradeoff faced by the government between the two arguments of the loss function is

$$u = \bar{u} - \theta(\Delta s - E\Delta s).$$

There are two reasons why unemployment is a decreasing function of unexpected devaluation:

- Export prices are lower than expected (in foreign currency), so domestic firms are more competitive than expected in export markets.
- Imported inflation reduces the real wage rate.

Interpreting s as the price level, this version of the model is simply Barro and Gordon's (1983) model of the time inconsistency of optimal monetary policy.

Substituting the tradeoff into the government's objective function yields the following optimization problem:

$$\min_{\Delta s} : L = \alpha(\Delta s)^2 + [\bar{u} - \theta(\Delta s - E\Delta s)]^2.$$

Necessary optimality condition:

$$\alpha\Delta s + \theta^2(\Delta s - E\Delta s) = \theta\bar{u}.$$

Rational expectations ($E\Delta s = \Delta s$):

$$\Delta s = \frac{\theta}{\alpha}\bar{u} > 0.$$

A devaluation occurs. The larger the impact of the exchange rate on unemployment and the weaker the government's dislike of exchange rate changes, the larger the devaluation.

Rational expectations implies that the devaluation is correctly anticipated, so $u = \bar{u}$, and the value of the loss function is

$$L = \left(1 + \frac{\theta^2}{\alpha}\right) \bar{u}^2.$$

This is the standard time inconsistency problem of discretionary policy: the country would be better-off if the government kept the exchange rate fixed (so that $L = \bar{u}^2$), but that is time inconsistent, since the best response to zero expected devaluation is $\Delta s = [\theta/(\alpha + \theta^2)]\bar{u}$.

- http://gregmankiw.blogspot.de/2006/04/time-inconsistency.html

Version 2: fixed cost of devaluation

Now assume that the reputational cost of giving up the fixed exchange rate contains a fixed cost c:

$$L = \alpha(\Delta s)^2 + u^2 + \mathbf{1}_d c,$$

where

$$\mathbf{1}_d = \begin{cases} 0; & \Delta s = 0 \\ 1; & \Delta s \neq 0 \end{cases}.$$

If the government does not devalue ($\Delta s = \mathbf{1}_d = 0$), unemployment is $u = \bar{u} + \theta E\Delta s$, and the loss is

$$L = (\bar{u} + \theta E\Delta s)^2 \equiv L^f.$$

If it decides to devalue, then the optimum devaluation is given by the same necessary optimality condition as before. Solving for Δs gives

$$\Delta s = \frac{\theta}{\alpha}\lambda(\bar{u} + \theta E\Delta s),$$

where

$$\lambda \equiv \frac{1}{1 + \frac{\theta^2}{\alpha}} < 1.$$

Substitution into the exchange rate-unemployment tradeoff and into the loss function yields

$$u = \lambda(\bar{u} + \theta E\Delta s)$$

and

$$L = \lambda(\bar{u} + \theta E\Delta s)^2 + c \equiv L^d.$$

The higher expected devaluation, the more likely it is that the loss with devaluation is lower than the loss with a fixed exchange rate.

The government chooses to devalue the domestic currency exactly if ($L^d < L^f$, i.e.)

$$\bar{u} + \theta E\Delta s > \sqrt{\frac{c}{1-\lambda}} \equiv k.$$

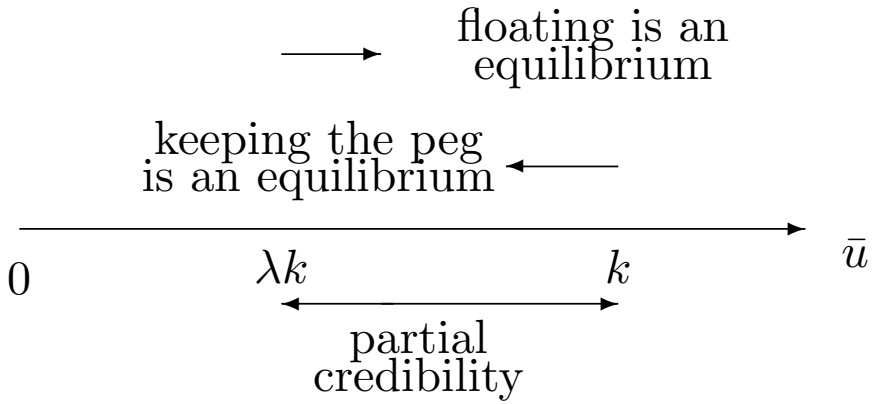
$1/(1-\lambda)$ is large if the marginal impact of devaluation expectations is not much lower with than without devaluation. So k is a measure of the equilibrium cost of devaluation.

Case 1: low cost of devaluation ($k < \bar{u}$):
The condition for devaluation is satisfied for all $E\Delta s \geq 0$. So the government devalues. Market participants correctly anticipate the optimum amount of devaluation $\Delta s = \theta\bar{u}/\alpha$.

Case 2: high cost of devaluation ($k \geq \bar{u}$):
The condition for devaluation is violated for $E\Delta s = 0$. So keeping the exchange rate fixed is an equilibrium.

Is there also an equilibrium with positive devaluation? If Δs is positive, then it is given by the optimum value $\Delta s = \theta\bar{u}/\alpha$. Inserting this and the rational expectations condition $E\Delta s = \Delta s$ into the condition for devaluation yields

$$\lambda k < \bar{u}.$$



Theorem: *For $\lambda k < \bar{u} \leq k$, there are multiple expectational equilibria: the government devalues if, and only if, market participants expect it to do so.*

Literature:

Barro, Robert J., and David B. Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Model", *Journal of Political Economy* 91, 589-610.

Obstfeld, Maurice (1996), "Models of Currency Crises With Self-Fulfilling Features", *European Economic Review* 40, 1037-47.

- http://www.wm.tu-berlin.de/fileadmin/fg124/geldtheorie/literatur/barro-gordon-a.pdf
- http://people.ucsc.edu/~hutch/241B/Ec%20241b%20SYLLABUS%20Winter%202010_files/obstfeld_modelsofcurrencycrises_selfulfilling_eer96.pdf

Sachs, Jeffrey, Aaron Tornell, and Andres Velasco (1996), "The Mexican Peso Crisis: Sudden Death or Death Foretold?", *Journal of International Economics* 41, 265-83.

Problems:

1. Consider a country whose currency (the peso, say) is pegged one-for-one to the dollar. The currency is under pressure to devaluate. One quarter into the future, its price will be \$0.4545 per peso if it devalues. A U.S. trader takes a 100 peso loan with three months maturity in the country at 10% interest and invests the money in the U.S. at 2% interest.
(a) What is the investor's payoff (in dollars) if the currency actually devalues?
(b) What is his dollar loss if the dollar peg is still intact after three months?
(c) Compute the probability of devaluation p above which the expected profit is positive.

2. A country's currency is pegged to the dollar and under pressure to devaluate. If a devaluation occurs, then $\Delta S = 0.5$. Two traders can take a short position in the country at transaction cost $c = 3$. The central bank has reserves $R = 20$.
(a) Prove the validity of the condition that both traders gain if the currency collapses.

(b) Construct the games for the cases of speculator capital $K = 7$, $K = 25$, and $K = 16$. Characterize the equilibria of these games.

3. Let the the central bank's loss function and the devaluation-unemployment tradeoff be given by

$$L = 0.2(\Delta s)^2 + u^2 + \mathbf{1}_d c$$

and

$$u = 6\% - 0.4(\Delta s - E\Delta s),$$

respectively.
(a) Compute the devaluation that occurs for $c = 0$.
(b) Leaving c unspecified, compute L^f, depending on $E\Delta s$.
(c) Compute the best positive amount of devaluation $\Delta s > 0$ and the associated loss L^d.
(d) Characterize the equilibria of the model for all $c > 0$.

4. (a) Construct the payoff matrix that describes the following situation.
A couple plans for the evening either to watch soccer (Borussia Dortmund finishes the Bundesliga season as uncontested champion) or go the cinema. The husband's utility is maximal ($h > 0$, say) when he watches soccer with his wife, intermediate ($l > 0$, say) when they go to the cinema together (unrealistically), and lowest (zero, say) if they spend the evening separately. The wife's payoffs are h if they go to the cinema, l if they watch soccer, and zero if they don't meet.
(b) Find the Nash equilibria of the game.
(c) Suppose the husband moves first. Describe the game in extensive form and find the subgame-perfect Nash equilibrium.

12. Currency crisis: global games

Discontent with the view that unlucky countries are struck by a crisis due to shifts in speculators' expectations led to a new type of model, pioneered by Morris and Shin (1998), in which crises are due to bad fundamentals.

The baseline model is a modified version of Obstfeld's 2nd generation "toy model".

- http://www.kellogg.northwestern.edu/faculty/rebelo/htm/currency%20crisis%20models%20Ed.pdf

A currency is under pressure to devalue. There is a continuum $[0, 1]$ of traders. That is, the real numbers are used to "number" the traders, and aggregation makes use of integration (rather than summation). The traders decide whether to take a short position in the country or not. Let

- 1: profit from taking a speculative position if the currency depreciates
- c: cost of speculation against the currency
- l: "number" (mass) of traders who attack the currency
- θ: indicator of the strength of the economy's fundamentals, such as its reserves.

The country has to give up the fixed exchange rate if the number of traders who speculate on devaluation is large relative to the indicator of the economy's fundamentals, i.e.,

$$I \geq \theta.$$

- If fundamentals are strong ($\theta > 1$), the currency certainly does not devalue.
- If fundamentals are weak ($\theta \leq 0$), the currency certainly devalues.
- The interesting case is that of intermediate fundamentals ($0 < \theta \leq 1$).

Note the similarity to the Obstfeld model of the previous section.

Traders are uncertain about fundamentals θ. Each trader i observes

$$x_i = \theta + \sigma\varepsilon_i, \quad \sigma > 0,$$

where ε_i is a random variable which distorts i's signal. The larger σ, the less precise the signal. The ε_i's satisfy:

- The ε_i's are independent.
- The distribution function $F(\varepsilon_i)$ is identical for all traders i.
- $E\varepsilon_i = 0$.
- The density is symmetric:

$$F(-\varepsilon_i) = 1 - F(\varepsilon_i)$$

for all ε_i.

We confine attention to equilibria which satisfy two conditions:

- There is a "trigger value" x^* such that each trader i attacks the currency exactly if

$$x_i = \theta + \sigma\varepsilon_i \leq x^*.$$

- There is a critical value of the fundamentals θ^* such that devaluation occurs (i.e., $l \geq \theta$) exactly if $\theta \leq \theta^*$.

The second condition implies that the strength of the economy's fundamentals (and not extraneous shifts in expectations) determine whether or not the currency depreciates.

Theorem: *There is a unique equilibrium satisfying the above two conditions.*

Proof: i knows that the attack is successful exactly if $\theta = x_i - \sigma\varepsilon_i \leq \theta^*$ or

$$\varepsilon_i \geq \frac{x_i - \theta^*}{\sigma},$$

i.e., if ε_i is not so small that a low value of x_i is mistakenly interpreted as weak fundamentals. So the probability that the currency devaluates is

$$1 - F\left(\frac{x_i - \theta^*}{\sigma}\right)$$

or, given symmetry of the distribution,

$$F\left(\frac{\theta^* - x_i}{\sigma}\right).$$

Since the profit in case of a successful attack is 1, this is also the expected payoff from participating in the attack.

For $x_i = x^*$ such that

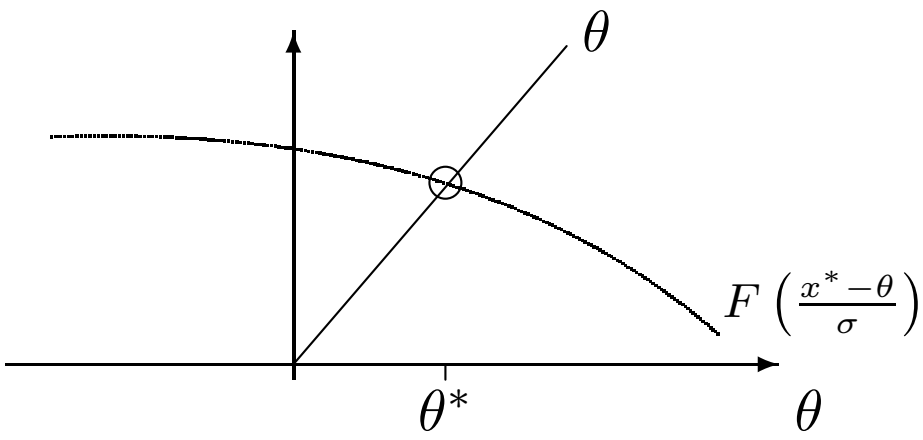
$$F\left(\frac{\theta^* - x^*}{\sigma}\right) = c,$$

trader i is indifferent between participating in the attack or not. For $x_i < x^*$, he prefers to participate, and vice versa. So all i with $x_i = \theta + \sigma\varepsilon_i \leq x^*$, i.e.,

$$\varepsilon_i \leq \frac{x^* - \theta}{\sigma},$$

attack the currency. Given independence of the error terms, the mass of traders for which this holds true is equal to the probability that the condition holds:

$$I = F\left(\frac{x^* - \theta}{\sigma}\right).$$



The attack is successful exactly if $I \geq \theta$, i.e.,

$$F\left(\frac{x^* - \theta}{\sigma}\right) \geq \theta$$

or $\theta \leq \theta^*$, where

$$F\left(\frac{x^* - \theta^*}{\sigma}\right) = \theta^*.$$

Using symmetry, we can solve for the equilibrium critical fundamentals and traders' trigger:

$$\theta^* = F\left(\frac{x^* - \theta^*}{\sigma}\right) = 1 - F\left(\frac{\theta^* - x^*}{\sigma}\right) = 1 - c$$

and

$$x^* = \sigma F^{-1}(\theta^*) + \theta^* = F^{-1}(1 - c) + 1 - c.$$

It can be shown that the two conditions imposed on equilibrium are immaterial. There does not exist an equilibrium that does not satisfy both conditions; equilibrium is unique.

Literature:

Corsetti, Giancarlo, Amil Dagupta, Stephen Morris, and Hyun Song Shin (2004), "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders", *Review of Economic Studies* 71, 87-113.

Morris, Stephen, and Hyun Song Shin (1998), "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks", *American Economic Review* 88, 587-597.

- http://www.macroeconomics.tu-berlin.de/fileadmin/fg124/financial_crises/literature/RES_71__2004__Corsetti_et_al.pdf
- https://economics.mit.edu/files/17461

Problems:

1. Let $c = 1/2$,

$$x_i = \theta + \frac{\varepsilon_i}{2},$$

and

$$f(\varepsilon_i) = \begin{cases} \frac{1}{2}; & -1 \leq \varepsilon_i \leq 1 \\ 0; & \text{otherwise} \end{cases}.$$

(a) State the probability that an attack is successful depending on θ^* and x_i.
(b) State the trigger value x^* depending on θ^*.
(c) State the mass of traders l who participate in the attack and the equation that relates the critical value θ^* to the trigger value x^*.
(d) Solve the equations in parts (b) and (c) for x^* and θ^*.

2. The usual definition of a symmetry around zero is

$$f(\varepsilon) = f(-\varepsilon).$$

(a) Prove that this implies the version used here, i.e., $F(-\varepsilon_i) = 1 - F(\varepsilon_i)$. (Hint: Start with

$$F(-\varepsilon) = \int_{-\infty}^{-\varepsilon} f(y)dy$$

and perform the change of variables $x = -y$.)
(b) Prove that the distribution in Problem 1 is symmetric. Prove that the uniform distribution with mean zero is generally symmetric.
(c) Provide other examples of symmetric distributions.

13. Debt crises

A debt crisis consists of repayment problems either

- in the private sector or
- in the public sector.

Private sector debt and capital inflows:
Repayment problems in the private sector arise when repayment obligations exceed debtors' repayment capacity. This can happen without international capital flows, after a credit boom refinanced by domestic savings (as in Japan in the 1980s or in the U.S. housing market in the 1990s and early 2000s). But rapid capital inflows may also play a role.

- http://dx.doi.org/10.1787/5kgc9kpkslvk-en

Sustainability of external debt:
There are frequent concerns that net debtor countries have to be sufficiently competitive so as to service their debt. There are good reasons for this concern: a net debtor cannot run a trade balance deficit forever.

Let

- D_t: net foreign debt
- r: (constant) interest rate
- TB_t: trade surplus.

The clue to the result is the no-Ponzi game condition:

$$\lim_{t\to\infty} \frac{D_t}{(1+r)^t} = 0.$$

- http://www.cesifo-group.de/DocDL/dicereport113-db2.pdf

The condition says that the absolute level of debt must grow at a rate lower than the interest rate.

- Suppose $\lim_{t\to\infty} D_t/(1+r)^t > 0$, i.e., foreign debt is positive and grows at a rate no less than the interest rate. That is, the country borrows more than enough to pay the interest on its existing debt. This is essentially a Ponzi game. Lenders will not be willing to let the country run this scheme.
- Conversely, $\lim_{t\to\infty} D_t/(1+r)^t < 0$, the home country is a net creditor and continues to give foreigners more than enough money to roll over its debt, a situation the home country will not be willing to sustain.

- http://www.biography.com/people/charles-ponzi-20650909

Theorem: *If $D_0 > 0$, then the present value of the future trade surpluses has to be positive:*

$$D_0 = \sum_{i=1}^{\infty} (1+r)^{-i} TB_i.$$

Proof: Debt evolves according to

$$D_t = (1+r)D_{t-1} - TB_t.$$

The proof is by induction. Suppose

$$D_t = (1+r)^t D_0 - \sum_{i=1}^{t} (1+r)^{t-i} TB_i$$

holds for t.

Then it also holds for $t+1$:

$$\begin{aligned} D_{t+1} &= (1+r)\left[(1+r)^t D_0 - \sum_{i=1}^{t}(1+r)^{t-i} TB_i\right] - TB_{t+1} \\ &= (1+r)^{t+1} D_0 - \sum_{i=1}^{t}(1+r)^{(t+1)-i} TB_i \\ &\quad -(1+r)^{(t+1)-(t+1)} TB_{t+1} \\ &= (1+r)^{t+1} D_0 - \sum_{i=1}^{t+1}(1+r)^{(t+1)-i} TB_i. \end{aligned}$$

The assertion follows from taking the limit and using the no-Ponzi game constraint:

$$0 = \lim_{t\to\infty} \frac{D_t}{(1+r)^t} = D_0 - \sum_{i=1}^{\infty}(1+r)^{-i} TB_i.$$

Public sector external debt and enforceability:
Public sector foreign debt is special in an important respect: though it is signed under prevailing law, who enforces repayment when the sovereign itself defaults?

One common argument is that sovereigns service a self-enforcing implicit contract: they repay in order to build reputation of creditworthiness. But that is not a valid argument. Suppose a country services constant public foreign debt D at interest r. The common argument says that it continually repays $(1 + r)D$ in order to get fresh money D. But that implies a constant net cash flow rD to foreign creditors. The country can avoid this constant negative cash flow by defaulting on its foreign debt.

- http://economichistory.ucla.edu/wp-content/uploads/sites/69/2017/07/tomz_powerandmoney.pdf

This argument generalizes to a setup with non-constant debt and interest (Bulow and Rogoff, 1989). Suppose there is a sequence of external debt levels such that the government agrees to service the debt, even though it is not enforceable. We produce a contradiction. The government defaults on its debt when the debt burden is maximal, and it is not dependent on foreign capital afterwards.

- D_t: foreign debt at t
- $t = -1$: date with maximum D_t (assumed to exist)
- r_t: interest rate
- A_t: foreign wealth after default at date 0.

Claim: if the country defaults at date 0 and invests the money saved by not repaying its external debt abroad, then it accumulates external wealth

$$A_t = \left[\prod_{i=0}^{t}(1 + r_{i-1})\right] D_{-1} - D_t > 0$$

for all $t = 0, 1, 2, \ldots$

If the country repays its debt, it makes a net payment

$$(1 + r_{t-1})D_{t-1} - D_t$$

to foreign lenders at t. The validity of the claim for $t = 0$ is evident:

$$A_0 = (1 + r_{-1})D_{-1} - D_0 > 0.$$

Subsequently, we have

$$A_{t+1} = (1 + r_t)A_t + (1 + r_t)D_t - D_{t+1} > 0.$$

The validity of the claim follows by induction:

$$\begin{aligned} A_{t+1} &= (1 + r_t)\left\{\left[\prod_{i=0}^{t}(1 + r_{i-1})\right] D_{-1} - D_t\right\} \\ &\quad +(1 + r_t)D_t - D_{t+1} \\ &= \left[\prod_{i=0}^{t+1}(1 + r_{i-1})\right] D_{-1} - D_{t+1}. \end{aligned}$$

Theorem: *Reputational concerns cannot provide a sovereign with incentives to service its debt.*

It must be other than reputational costs which make states repay their external debt (De Paoli et al., 2006):

- The cost of debt rises for the private sector. A sovereign default may put severe stress on the domestic financial sector and cause a severe disruption of corporate financing.
- If the government defaults on all of its debt, held at home and abroad, then it also hurts its own residents. If local banks hold substantial amounts of government debt, then the default causes a banking crisis, disintermediation, and recession.

- If the default precipitates capital flight from the country, it puts the currency under pressure, thus causing twin crises (see below). At the same time, the vulnerability to future crises rises if foreign lenders are unwilling to make domestic currency loans after the default.
- The default causes a general loss of credibility of the state. It raises questions about its role as a lender of last resort, adding to financial distress in the banking sector.

Twin crises:

Once a country suffers from a debt crisis, a currency crises often follows, as hot money (does not flow from some institutions to others within the country, but) leaves the country altogether: “In no time, capital flight wipes out reserves and precipitates a currency collapse” (Dornbusch, 2001).

- http://www.macroeconomics.tu-berlin.de/fileadmin/fg124/financial_crises/literature/Kaminsky___Reinhart__1999_.pdf
- http://www.imf.org/external/pubs/ft/wp/2012/wp12163.pdf
- http://www.kellogg.northwestern.edu/faculty/rebelo/htm/currency%20crisis%20models%20Ed.pdf
- http://elsa.berkeley.edu/~eichengr/research/ospainaug21-03.pdf

The problem with such “twin crises” is that the standard measures against a currency crisis aggravate the debt crisis, from which it originates:

- Raising the interest rate to keep money from leaving the country makes debt rollover more expensive.
- Devaluing the currency increases the cost of servicing that part of the debt which is denominated in foreign currency (the “original sin”).

Literature:

Bulow, Jeremy, and Kenneth Rogoff (1989), "Sovereign Debt: Is to Forgive to Forget?", *American Economic Review* 79, 43-50.

De Paoli, Bianca, Glenn Hoggarth, and Victoria Saporta (2006), "Costs of sovereign default", *Financial Stability Paper no.1*, Bank of England.

Dornbusch, Rudiger (2001), "A primer on emerging market crises", in: Sebastian Edwards and Jeffrey A. Frankel (eds.), *Preventing Currency Crises in Emerging Markets*, Chicago: University of Chicago Press.

- http://www.dklevine.com/archive/refs4209.pdf
- https://www.bankofengland.co.uk/financial-stability-paper/2006/costs-of-sovereign-default
- http://www.nber.org/chapters/c10648.pdf

Schmitt-Grohé, Stpehanie, and Martín Uribe (2014), *International Macroeconomics*, Columbia University, Chapter 2.

- http://www.columbia.edu/~mu2166/UIM/notes.pdf

Problems:

1. Consider the following Ponzi game. Each participant (player) in the game has to pay a $10 fee. The Ponzi scheme starts with 10 players. The players are encouraged to draw new players into the game (a pyramid or snowball scheme) and the manager of the scheme himself also tries to attract new participants. He promises each player a 100% return on the fee. He "finances" the payments as follows: whenever for each player there are five new players *and* he anticipates that it will be possible to keep the scheme going for another stage, he takes $0.80 of the latest generation's players' fee and pays the money to the players' pre-predecessor two stages earlier in the game. When the manager perceives that the scheme cannot grow further, he takes the latest generation's fees and runs.

(a) Argue that a player who enters the Ponzi game earlier than at the latest three stages before it breaks down gets the promised 100% return on his fee.

(b) Suppose the manager terminates the scheme after stage 5. Complete the following table.

stage	number players	manager's payoff per player	manager's payoff total	players' payoffs per player	players' payoffs total
1					
2					
3					
4					
5					
total					

(c) Interpret the numbers in the table. Compute the players' realized rates of return depending on the stage at which they entered the scheme.

2. Suppose a country with a positive amount of debt D_0 runs a trade surplus which is proportional to its interest obligations:

$$TB_t = \alpha r D_{t-1}, \quad 0 < \alpha < 1.$$

(a) Derive the solution for D_t from the equation of motion $D_t = (1 + r)D_{t-1} - TB_t$. Will D_t ever converge to zero?
(b) Prove that the no-Ponzi game constraint is satisfied.
(c) Consider the current account $CA_t = TB_t - rD_{t-1}$. Show that, despite its trade balance surplus, the country runs a current account deficit forever.

3. Repeat the proof of the theorem that reputational concerns cannot provide a sovereign with incentives to service its debt for the special case of a constant interest rate r.

4. Let r constant and

$$D_t = \begin{cases} h; & t \text{ odd} \\ l; & t \text{ even} \end{cases},$$

where $0 < l < h$. How does A_t evolve over time after default at $t = 0$?

5. Let r constant and

$$D_t = D_0(1 + g_D)^t$$

(so that the assumption that D_t has a maximum is violated). Under what condition is default at $t = 0$ with non-negative foreign wealth subsequently possible?