Fuzzy categories and type theories

Lawvere observed in the 1970's that metric spaces are a certain kind of enriched category (over the nonnegative reals). This perspective bridges the gap between the 'discrete' world of category theory and the 'continuous' world of analysis.

In this talk, I will report on some attempts to work in this gap. First, in a straight-forward application of the above idea, I will explain some beginnings of a categorical version of harmonic analysis.

In working on applications such as this, it becomes clear (at least to one with a certain background) that the categorical arguments can be made more cleanly in an internal, logical language for such enriched categories. I will also report on this development, which produces a type theoretic version of fuzzy logic, a logic invented by Zadeh in the 1960s while takes truth values not in the booleans but in some more general set of truth values, for instance the unit interval [0,1].