

ARAKELOV GEOMETRY

Regensburg, September 6th to 10th 2021

Organizers: José Ignacio Burgos Gil, Walter Gubler, and Klaus Künnemann

Monday

9h00 - 9h50	R. de Jong ²	Asymptotics of Arakelov invariants, and applications
10h30 - 11h20	R. Wilms ¹	A uniform Bogomolov type result for tautological cycles in Jacobians of curves in one-parameter families
11h30 - 12h20	J.S. Müller ¹	p -adic Arakelov theory on abelian varieties and quadratic Chabauty
14h00 - 14h50	A. Javanpeykar ²	Recent progress on conjectures of Campana, Lang and Vojta
15h30 - 16h20	D. Eriksson ²	Genus one mirror symmetry and the arithmetic Riemann–Roch theorem
16h30 - 17h20	G. Faltings ²	Estimates related to Arakelov

Tuesday

9h00 - 9h50	F. Pazuki ¹	Northcott property for special values of L-functions
10h30 - 11h20	E. Sertöz ¹	Arithmetic self-intersections and canonical limit mixed Hodge structures
11h30 - 12h20	F. Ballaÿ ¹	Arithmetic Okounkov bodies and positivity in Arakelov geometry
14h00 - 14h50	J. Bruinier ²	Arithmetic volumes of unitary Shimura varieties (with Ben Howard)
15h30 - 16h20	S. Zhang ²	Adelic line bundles over quasi-projective varieties
16h30 - 17h20	A. Carney ²	Variation of canonical heights via adelic line bundles

Wednesday

9h00 - 9h50	X. Yuan ²	Bigness of the admissible canonical bundle of a family of curves
10h30 - 11h20	Y. Liu ²	A Gross-Zagier type formula for higher dimensional Shimura varieties
11h30 - 12h20	H. Chen ²	Arithmetic intersection theory over adelic curves
14h00 - 14h50	G. Freixas ²	Flat vector bundles and Arakelov geometry
15h30 - 16h20	A. Borisov ²	Bi-Euclidean spaces and coherent sheaves on Arakelov curves
16h30 - 17h20	G. Pearlstein ¹	Archimedean height pairings for higher cycles

Thursday

9h00 - 9h50	J. Kramer ¹	A note on Kudla’s modularity conjecture
10h30 - 11h20	M. Sombra ¹	The mean height of a subvariety of a torus
11h30 - 12h20	A. Botero ¹	Chern-Weil theory for positive singular metrics with an application
14h00 - 14h50	Z. Gao ²	A proof of the Uniform Mordell—Lang Conjecture
15h30 - 16h20	F. Shokrieh ²	Jumps in the height of the Ceresa cycle
16h30 - 17h20	A. Navarro ²	A proof of the Riemann-Roch theorem via Spanier-Whitehead duality

Friday

9h00 - 9h50	U. Kühn ¹	Multiple q -zeta values and Hilbert schemes of points on a surface
10h30 - 11h20	X. Ma ¹	Quotient of Bergman kernels on punctured Riemann surfaces
11h30 - 12h20	K. Köhler ²	Asymptotics of equivariant torsion on homogeneous spaces
14h00 - 14h50	R. Pengo ¹	Mahler measure of successively exact polynomials
15h30 - 16h20	F. Bambozzi ¹	The sheafiness problem for Banach rings
16h30 - 17h00	Y. Li ²	Categorification of Harder-Narasimhan Theory
17h15 - 17h45	E. Marmolejo ²	A cube structure in Arakelov Theory

All times are in Central European Time. Speakers talk *onsite*¹ or *online*². Version September 3, 2021

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General Information

Conference website: Please check the conference website at

www-app.uni-regensburg.de/Fakultaeten/MAT/sfb-higher-invariants/index.php/Arakelov2020

for more information and updates of this program.

Tea: Every day there will be *online tea breaks* at 15h00 - 15h30 which take place in *Gathertown*. All participants online and onsite are invited to join for tea in *Gathertown*. There will be also regular tea breaks from 10h00 - 10h30 in the morning.

Place: All lectures take place in lecture hall H2 (see last page).

Access data: Talks will be given either *onsite*¹ or *online*². All talks will be accessible on Zoom.

We will send the access data for the lectures on Zoom and the tea in Gathertown during the week before the conference.

Please contact one of the organizers if you have not received the Zoom and Gathertown access data the week before the conference.

Support: This conference is supported by the DFG via *SFB 1085 Higher Invariants - Interactions between Arithmetic Geometry and Global Analysis*.

Slides and blackboard talks

Onsite: Onsite speakers can give their talks either by using slides or by giving a classical blackboard talk which is then broadcasted over Zoom. Please observe that only one large blackboard is available for the speaker. In the first case please send a pdf with your slides the day before your talk to

gast.kuennemann@ur.de

In the second case you should write an email before the conference to

walter.gubler@mathematik.uni-regensburg.de

that you plan a black board talk.

Online: Online speakers give their talks over Zoom as usual by sharing their screen.

Microphones: Please mute your microphone on Zoom unless you have to say something. If you want to say something over Zoom, then please do not forget to unmute yourself before you do so.

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COVID-19 Travel Restrictions

It is each participant's own responsibility to check which travel restrictions apply to them. Please see the official travel guidelines provided by the German Federal Ministry of the Interior, Building and Community on their website. Please also check the website

<https://www.einreiseanmeldung.de/>

COVID-19 Health and Safety Measures

- Only those people who officially registered for the conference will be able to participate. Moreover, only those people will be allowed to participate in person whom the organizers have explicitly informed that they can do so. This applies to everyone, including students and employees of the university. They all have to be fully vaccinated.
- Please wear your name tag when you enter the lecture room to show that you are a registered participant.
- It is compulsory to wear a mask of grade FFP2 in all buildings on university grounds, but there is no obligation to wear a face mask in the outdoor areas. University employees may wear medical-grade face masks instead of FFP2-grade face masks. We ask everyone to bring their own face mask. If you have forgotten or lost yours, you can ask the organizers for one.
- We ask all participants to keep a minimum distance of 1.5m to each other at all times. Please use only the marked seats in the lecture room.
- Any participant who is feeling ill, has corona symptoms, or even has a positive corona test will not be able to participate.
- **We take violations against the above health and safety rules very seriously.**
- If you see someone who does not respect these rules, please ask them kindly to reconsider; if they do not change their behaviour, please report them immediately to the organizers, José Burgos, Walter Gubler or Klaus Künnemann.
- The above rules may be updated at any time in accordance with university and government regulations.

Abstracts

François Ballaÿ: Arithmetic Okounkov bodies and positivity in Arakelov geometry

The theory of Okounkov bodies, developed by Lazarsfeld and Mustaa and by Kaveh and Khovanskii, permits to use methods from convex analysis in the study of projective varieties by attaching convex bodies to line bundles. Boucksom and Chen introduced an analogous construction in the context of Arakelov geometry, and defined arithmetic Okounkov bodies attached to an adelic line bundle on a projective variety over a number field. In this talk, I will recall the definition of arithmetic Okounkov bodies and explain how they encode the positivity of an adelic line bundle. This result is an arithmetic counterpart to a theorem of Kronya and Lozovanu in algebraic geometry. It generalizes to arbitrary projective varieties criteria for the positivity of toric metrized line bundles on toric varieties established by Burgos Gil, Moriwaki, Philippon and Sombra.

Federico Bambozzi: The sheafiness problem for Banach rings

A major obstacle for an abstract theory of analytic spaces over any Banach ring (or affinoid in the sense of Huber) is that the canonical structure pre-sheaf is not a sheaf in general. I propose a solution to this problem using the methods of derived geometry obtaining a well-defined sheaf of dg-Banach algebras for all Banach rings on a modified version of the Huber's spectrum. Applications of this result will be presented as well as relations to a similar result obtained by Clausen and Scholze used condensed mathematics. This work has been done in collaboration with Kobi Kremnizer.

Alexander Borisov: Bi-Euclidean spaces and coherent sheaves on Arakelov curves

It is well-known that lattices in Euclidean spaces are arithmetic analogs of locally free sheaves over the compactified spectrum of the ring of integers. The main obstacle to generalizing this analogy to coherent sheaves is to understand what to do at infinity. We propose a natural, and essentially elementary, construction, that has the potential to greatly enhance Arakelov Geometry in several ways. The main object at infinity is, roughly speaking, a pair of positive quadratic functions on a real vector space, one greater than the other. The morphisms are linear maps that are non- expanding with respect to both functions, and our objects are formal quotients of two Euclidean spaces. The resulting category is a natural target for the direct image map from the category of Hermitian sheaves on an Arakelov variety. This is work in progress, joint with Jaiung Jun.

Ana Botero: Chern-Weil theory for positive singular metrics with an application to the line bundle of Siegel-Jacobi forms

Let X be a complex manifold and E a holomorphic vector bundle on X . The Chern classes $c_i(E)$ are topological invariants that measure, in some sense, how far E is from being trivial. Chern-Weil theory says that Chern classes can be represented by means of geometric data (connections and their curvatures). If h is a smooth hermitian metric on E then its Chern forms $c_i(E, h)$ represent the Chern classes $c_i(E)$ and it has been of great interest to extend Chern-Weil theoretical results to more singular metrics. However, it has been shown by Burgos-Kramer-Khn in an example, that in cases of high arithmetic interest, for automorphic line bundles together with its natural invariant metric, Chern- Weil theory does not hold. In this talk we propose to consider instead of line bundles so called b -line bundles, i.e. tower of line bundles over all proper modifications of X . We then show a Chern-Weil theoretical result and apply it to the line bundle of Siegel-Jacobi forms over the universal abelian variety.

Jan Bruinier: Arithmetic volumes of unitary Shimura varieties (joint work with Ben Howard)

Alexander Carney: Variation of canonical heights via adelic line bundles

In this talk I'll explain how adelic line bundles provide a geometric realization of canonical height functions, then use this to show that, given a family of abelian varieties over a curve, the canonical height of a section across each fiber produces a Weil height on the base. I'll then show how this provides partial results for more general families of dynamical systems, and discuss the possibilities and difficulties of this approach to higher dimensional families.

Huayi Chen: Arithmetic intersection theory over adelic curves

In this talk, I will present a joint work with Atsushi Moriwaki on the construction of arithmetic intersection number of adelic divisors on a projective scheme over an adelic curve. The arithmetic intersection number is realized as an integral of local intersection number of metrized divisors. The key point to show the integrability is an equality between the local intersection number and the logarithmic length of the resultant.

Dennis Eriksson: Genus one mirror symmetry and the arithmetic Riemann–Roch theorem

Mirror symmetry, in a crude formulation, is usually presented as a correspondence between the Gromov–Witten theory of a Calabi–Yau variety X , and some invariants extracted from the degeneration of Hodge structures of a mirror family of Calabi–Yau varieties. After the physicists Bershadsky–Cecotti–Ooguri–Vafa (henceforth BCOV), this is organised according to the genus of the curves in X we wish to enumerate, and gives rise to an infinite recurrence of differential equations. In this talk, I will present a rigorous mathematical formulation of the BCOV conjecture at genus one, in terms of a lifting of the Grothendieck–Riemann–Roch. I will explain a proof of the conjecture for Calabi–Yau hypersurfaces in projective space, based on the Riemann–Roch theorem in Arakelov geometry. Our results generalise from dimension 3 to arbitrary dimensions previous work of Fang–Lu–Yoshikawa. This is joint work with G. Freixas and C. Mourougane

Gerd Faltings: Estimates related to Arakelov

Arakelov theory is usually concerned with showing equalities, not inequalities. Here I give upper bounds for the Arakelov metric (and variants for different curvature conditions) on differentials of Riemann surfaces, in terms of embedded discs or of the hyperbolic metric. I also bound Weierstrass sections (whose zeroes are Weierstrass points) in the hyperbolic metric.

Gerard Freixas: Flat vector bundles and Arakelov geometry

In this talk I will discuss joint work with Dennis Eriksson and Richard Wentworth, on a formalism of direct images of characteristic classes of flat vector bundles, in the setting of families of compact Riemann surfaces. This is inspired by Deligne's functorial approach to Arakelov geometry, and relies on Chern-Simons classes as opposed to Bott-Chern classes, and ideas from non-abelian Hodge theory. The main application is the construction of a complex Chern-Simons line bundle on relative moduli spaces of representations of fundamental groups, which fulfils the expected axioms from TQFT. This object is key to proving that the Cappell-Miller torsion of flat vector bundles on Riemann surfaces satisfies a Riemann-Roch formula, analogous to the arithmetic Riemann-Roch theorem of Gillet-Soulé.

Ziyang Gao: A proof of the Uniform Mordell–Lang Conjecture

Let A be an abelian variety and let X be a subvariety, both defined over $\bar{\mathbb{Q}}$. For any finite rank subgroup Γ of $A(\bar{\mathbb{Q}})$, the famous Mordell–Lang Conjecture predicts that each component of $X \cap \Gamma$ is a coset of A . This conjecture is proved by Faltings and one also needs a result of Hindry to handle division points.

The Uniform Mordell–Lang Conjecture predicts that the number of irreducible components concerned above is bounded solely in terms of $\dim A$, $\deg X$ and the rank of Γ . The question was posed by Mazur and David–Philippon. Recently this conjecture is proved in a series of work (Dimitrov–Gao–Habegger, Kühne; Gao–Ge–Kühne). In this talk I will report this proof. I will focus on the case of rational points on curves and then explain how to generalize this method to the general case. This is a joint project with Vesselin Dimitrov, Philipp Habegger; Tangli Ge, Lars Kühne.

Ariyan Javanpeykar: Recent progress on conjectures of Campana, Lang and Vojta

I will survey recent progress on the various aspects of the conjectures of Campana, Lang, and Vojta, which relate the behaviour of entire curves on a variety to the density of integral/rational points.

Robin de Jong: Asymptotics of Arakelov invariants, and applications

We investigate the degenerate behavior, near the boundary, of certain canonical smooth hermitian metrics on certain canonical line bundles on the moduli spaces of complex curves and of complex principally polarized abelian varieties. These canonical metrics furnish the archimedean contributions to certain canonical heights in Arakelov geometry. As we will see, unfortunately our canonical smooth metrics do not in general extend continuously, and not even as a “good” metric, over the boundary of moduli space. Following work of Hain, we introduce the “height jump” as an obstruction to continuous extendability near a given point of the boundary. We state several new results on the height jump for each of the metrized line bundles of our interest. The proofs of these results rely on asymptotic results, in one-parameter degenerations, for several natural Arakelov invariants on moduli space. Interestingly, the leading terms in these asymptotics are controlled by natural tropical analogues of the invariants. This makes for an explicit combinatorial approach to the height jump. Based on joint work with Farbod Shokrieh.

K. Köhler: Asymptotics of equivariant torsion on homogeneous spaces

In recent years, increased research has been completed concerning the asymptotics of analytic torsion (forms) expanding the classical results by Bismut–Vasserot. We contribute explicit computations for any complex homogeneous space in the equivariant case, which has not yet been covered.

J. Kramer: A note on Kudla’s modularity conjecture

Roughly speaking, Kudla’s modularity conjecture states that generating series constructed by means of arithmetic special cycles on Shimura varieties associated to orthogonal groups give rise to Siegel modular forms. The proof of this conjecture can be reduced to the statement that the corresponding formal Fourier–Jacobi expansions, which satisfy a natural symmetry condition, originate indeed from Siegel modular forms. In the complex setting, a proof of this statement was given by J. Bruinier and M. Raum. In our talk we will revisit this problem using the arithmetic compactifications of the moduli space of principally polarized abelian varieties established by G. Faltings and C.-L. Chai.

U. Kühn: Multiple q-zeta values and Hilbert schemes of points on a surface

Li Yao: Categorification of Harder-Narasimhan Theory

The notion of Harder-Narasimhan filtration was firstly introduced by Harder and Narasimhan in the setting of vector bundles on a non-singular projective curve. Curiously, analogous constructions have been discovered in other branches of mathematics which motivate categorical constructions of Harder-Narasimhan filtration. In this talk, we will introduce a categorical construction of Harder-Narasimhan filtration via slope method which does not need a degree function. With a theorem of existence and uniqueness of Harder-Narasimhan filtration in our categorical setting, we give a categorical interpretation of Stuhler-Grayson filtration in the case of non-necessarily Hermitian normed lattice.

Yifeng Liu: A Gross-Zagier type formula for higher dimensional Shimura varieties

In this talk, we introduce the arithmetic inner product formula for unitary groups of even ranks, which relates central L-derivatives and Beilinson-Bloch heights of cycles over number fields. This can be regarded as a higher dimensional Gross-Zagier formula on unitary Shimura varieties. This is a joint work with Chao Li.

Xianon Ma: Quotient of Bergman kernels on punctured Riemann surfaces

We consider a punctured Riemann surface endowed with a Hermitian metric that equals the Poincaré metric near the punctures, and a holomorphic line bundle that polarizes the metric. We introduce a new method to compare the Bergman kernels of high tensor powers of the line bundle and of the Poincaré model near the singularity and show that their quotient tends to one uniformly on a neighborhood of the singularity up to arbitrary negative powers of the tensor power.

Esteban Gomezllata Marmolejo: A cube structure in Arakelov Theory

Given a flat projective morphism $f : X \rightarrow S$, of relative dimension n , and a set of $n + 2$ line bundles $L_1, \dots, L_{n+2}$, there is due to Ducrot a canonical trivialization of the line bundle $\bigotimes_{J \subset I} \det(R^\bullet f_* \bigotimes_{i \in J} L_i)^{\otimes (-1)^{|J|}}$, where J ranges over the subsets of $I = \{1, \dots, n + 2\}$, and $\det(R^\bullet f_*)$ is the determinant of cohomology defined by Grothendieck and Knudsen-Mumford. We then suppose that f is smooth on the generic fiber, that $f_{\mathbb{C}}$ is a smooth proper map of complex manifolds, we endow X with a Kähler metric, the line bundles with a hermitian metric, the determinants of cohomology with the Quillen metric, and conclude that in this situation the canonical trivialization is an isometry.

Jan Steffen Müller: p -adic Arakelov theory on abelian varieties and quadratic Chabauty

I will discuss a new construction of p -adic height functions on abelian varieties over number fields using Besser's p -adic Arakelov theory. In analogy with Zhang's construction of Néron-Tate heights via adelic metric, these heights are given in terms of canonical p -adic adelic metrics on line bundles. As an application, I will describe a new and simplified approach to the quadratic Chabauty method for the computation of rational points on certain curves. This is joint work in progress with Amnon Besser and Padmavathi Srinivasan.

Alberto Navarro: A proof of the Riemann-Roch theorem via Spanier-Whitehead duality

The usual proof of the Grothendieck-Riemann-Roch theorem relies on the decomposition of the morphism into a closed immersion followed by a projection of a projective space onto its base. In this talk we will show how to prove the Riemann-Roch theorem out of the theory

of duality in the motivic setting. More concretely, the proof relies on a lesser statement of the theory of duality—the so called Spanier-Whitehead duality—and it does not require to factor the morphism. The strategy of the proof holds as well in the topological setting for smooth manifolds, thus leading for the analogue statement between complex K-theory and rational cohomology. In addition, this proof applies to the motivic version of the arithmetic Riemann-Roch of Holmstrom and Scholbach.

Fabien Pazuki: Northcott property for special values of L-functions

Pick an integer n . Consider a natural family of objects, such that each object X in the family has an L-function $L(X, s)$. If we assume that the collection of special values $L(X, n)$ is bounded, does it imply that the family of objects is finite? We will first explain why we consider this question, in link with Kato’s heights of mixed motives, and give two recent results. This is joint work with Riccardo Pengo.

Gregory Pearlstein: Archimedean height pairings for higher cycles

By the work of Richard Hain, the archimedean height pairing on ordinary algebraic cycles can be interpreted as an invariant of an associated mixed Hodge structure. In this talk, we will present a similar construction for higher cycles in the Bloch complex. Families of higher cycles produce admissible variations of mixed Hodge structure. We will describe the asymptotic behavior of the height pairing in the case where the associated variation of mixed Hodge structure is Hodge-Tate. This is joint work with J. Burgos Gil and S. Goswami.

Riccardo Pengo: Mahler measure of successively exact polynomials

The relation between Mahler measures of polynomials and special values of L-functions has been widely explored since the seminal works of Boyd, Deninger and Rodriguez-Villegas in the late ’90s. Sometimes, as in the earliest examples computed by Smyth, these relations occur between Mahler measures of n -variable polynomials and special values associated to geometric objects of dimension strictly less than $n-1$. This phenomenon has found a first explanation in the notion of exactness, put forward by Maillot and Lalín. In this talk, based on joint work in progress with François Brunault, we will give an introduction to these questions, and explain how one can interpret them using new cohomological approaches, which provide a notion of successive exactness, predicted by Lalín, that explains the observed drops in the dimension of the geometric objects used to construct the L-functions whose special values should be related to the Mahler measure of the polynomial in question.

Emre Can Sertöz: Arithmetic self-intersections and canonical limit mixed Hodge structures

Since 1980’s, the height of certain mixed Hodge structures are known to be related to the Archimedean component of arithmetic intersections of disjoint cycles. These results do not apply to self-intersections because then the Archimedean component blows-up. We show that, if the limiting family is defined over an arithmetic ring in a strong sense, then the height of the associated limit mixed Hodge structure equals exactly the arithmetic self-intersection of a cycle related to the resolution of the singular fiber. This creates an opportunity to compute regulators in higher dimensions. This is ongoing joint work with Spencer Bloch and Robin de Jong.

Farbod Shokrieh: Jumps in the height of the Ceresa cycle

This talk is based on joint work with Robin de Jong. We give an explicit combinatorial formula for the “height jump” of the Ceresa cycle at a given stable curve in terms of the “slope” of the dual graph. We then characterize those stable curves for which the height jump vanishes. (This talk is closely related to Robin de Jong’s talk, but it will be self-contained.)

Martín Sombra: The mean height of a subvariety of a torus

Bernstein's theorem allows to predict the number of solutions of a typical system of Laurent polynomial equations in terms of combinatorial invariants. When the coefficients of the system are algebraic numbers, we can ask about the height of these solutions. In an on-going project with Roberto Gualdi, we approach this question using tools from Arakelov geometry of toric varieties.

Robert Wilms: A uniform Bogomolov type result for tautological cycles in Jacobians of curves in one-parameter families

For one-dimensional families of curves over number fields we show that the geometric points of Néron-Tate height bounded by a positive uniform constant on a tautological cycle Z in the Jacobian lie all on a divisor of Z , whose degree is also uniformly bounded. Using previous work by David-Philippon and also Pazuki, we will reduce this to a comparison of the adelic self-intersection of the relative dualizing sheaf and the Faltings height. This can be studied by local invariants at every place and we reduce it by a result by Cinkir to the study of asymptotics of invariants on Riemann surfaces. The general uniform Bogomolov conjecture has recently been obtained by Kühne and Gao-Ge- Kühne. Also our result is only a special case of it, we are using a completely different approach, which may be of independent interest. Moreover, our constants seem easier to make explicit, although our result is still inexplicit by a compactness argument.

Xinyi Yuan: Bigness of the admissible canonical bundle of a family of curves

In this talk, we introduce a generalization of Zhang's admissible canonical bundle from a single curve to a family of curves, based on the new theory of adelic line bundles of Yuan-Zhang. The key property of this admissible canonical bundle is its bigness. Then we use it to obtain a uniform Bogomolov conjecture extending the recent uniform Bogomolov conjecture of Dimitrov-Gao-Hebeger and Kühne.

Shou-Wu Zhang: Adelic line bundles over quasi-projective varieties

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