New paths in the PDEs of fluid mechanics

Peyresq, August 31th, September 4th, 2026

Danica Basarić. Global weak solutions to a compressible Navier-Stokes/Cahn-Hilliard system with singular entropy of mixing. In this talk, we are going to study the Navier-Stokes/Cahn-Hilliard system modelling the evolution of a compressible binary mixture of viscous fluids undergoing phase separation. With respect to previous works, where only regular potentials with polynomial growth were considered, we include in our analysis the physically relevant Flory-Huggins (logarithmic) entropy. We will establish the existence of global-in-time weak solutions in three-dimensional bounded domains for arbitrarily large initial data. The heart of the proof relies on the derivation of new estimates for the chemical potential and the Flory-Huggins entropy arising from a density-dependent Cahn-Hilliard equation. In addition, we prove that the phase variable, which represents the difference of the mass concentrations, takes value within the physical interval $(-1, 1)$ almost everywhere on the set where the density is positive. The talk is based on a joint work with Andrea Giorgini from the Department of Mathematics of Politecnico di Milano.

Edoardo Bocchi. Sharp asymptotics of the 3D Stokes problem near solid contacts. An important issue in fluid-solid interactions is the possible collision of multiple solids or of a solid with the boundary of the fluid domain, which requires studying the behavior of the fluid in the narrow gap between the solid surfaces, whether rigid or elastic. In this context, it is usual to consider the stationary Stokes problem, supplemented with boundary conditions encoding the motion of the surfaces. I will present a variational method that allows to compute and rigorously justify the asymptotic expansion of the 3D Stokes solution when the solid surfaces approach isolated contacts. This talk is based on an ongoing collaboration with D. Bonheure and M. Hillairet.

Felix Brandt. On a fully averaged poroelastic plate model and its interaction with a fluid. We study a fluid-structure interaction problem between an incompressible viscous fluid and a thin poroelastic structure modeled by a new fully averaged Kirchhoff-type poroelastic plate equation. The fluid is governed by the time-dependent Stokes equations, while the structure is described by a codimension-one system for the transverse displacement and the averaged pore pressure. The coupling is imposed through kinematic and dynamic interface conditions enforcing continuity of normal velocities, a Beavers-Joseph-Saffman-type slip condition for tangential velocities, and balance of forces at the fluid-structure interface.

The main focus of the talk is the analysis of this coupled problem. We briefly discuss the derivation and structure of the fully averaged plate model, emphasizing its natural coupling with incompressible viscous fluids, and then prove the existence of weak solutions by energy methods. For a regularized version of the system, we establish global-in-time existence and uniqueness of strong solutions. The key step is to formulate the problem as an abstract Cauchy problem governed by an operator matrix with a non-diagonal domain and to prove sectoriality of the associated spatial operator, yielding maximal L^p regularity.

We conclude with a brief discussion of numerical simulations and comparisons with the full Biot-Stokes system in the thin-structure regime. This talk is based on joint work with S. Čanić, A. Scharf, and J. Tambača.

Marco Bravin. On the collective dynamics of a cloud of small particles in viscous incompressible fluid. In this talk I will present some resent results on the dynamics of a cloud of small particles in viscous incompressible fluid modeled by the incompressible Navier-Stokes equation. In this setting, we study the effects that infinitely many rigid bodies have on the fluid and the limiting dynamics of the rigid bodies. We show that the limiting behavior depends on the mass of rigid bodies. To show these results, we introduce the concept of cluster test functions, which is a new tool to avoid assuming minimal distance conditions in these types of works.

This is joint work with Feireisl, Roy, Zarnescu.

Didier Bresch. Homogenization and compressible flows. In this talk, I will review various homogenization results for compressible viscous flows. I will present some examples related to multiphase models recently derived using mathematical methods related to young measures.

Frédéric Charve. 3D-2D asymptotics for the rotating MHD. In this joint work with Van-Sang Ngo, we consider the 3D-rotating magnetohydrodynamic (MHD) system. We begin this talk by providing a few examples of penalized geophysical models similar to the incompressible Navier- Stokes system, and which converge (when the small penalization parameter goes to zero) towards a limit system that can be easily seen to be incomplete. Reaching a more complete limit requires adding to some classical initial data a non-conventional component linked to the special structure of the limit system. Then we study (for both weak and strong solutions) the asymptotics when the Rossby number goes to zero (i.e. for strong rotation) of the 3D-rotating (MHD) system when the initial velocity and magnetic field both feature some 2D-part (i.e. depending only on the horizontal space variables). We show this limit is the 2D-MHD system with three components supplemented with an additional 3D magnetic field transported by the 2D limit velocity.

Dimitri Cobb. Critical Solutions for a Euler-Chemotaxis System. The purpose of this talk is to study solutions of an Euler-Chemotaxis system. This is a system of PDEs which describe the evolution of active particles (e.g. micro-swimmers) suspended in an ideal fluid. The motion of the particles, in addition to fluid advection, is a response to the presence of a chemical stimulus, for instance a nutrient dissolved in the fluid. This problem is related to the well-known family of Keller-Segel equations, although it bears major differences which will be crucial for our study.

After a brief description of the model and the related literature, we will discuss the scaling properties of the system, which are key to understanding whether global estimates may be found or not. In particular, we will see that the subcritical scaling of the fluid/micro-swimmer coupling will allow us to repeatedly upgrade 'basic' low-regularity estimates to higher order bound.

We will then discuss the functional setting required for establishing existence of global solutions. It turns out that closing L^2 estimates leads to the Besov space $\dot{B}^0_{\infty,2}$ as the most natural choice of regularity for the initial nutrient concentration. If time allows, we will also explain how real interpolation plays a key role for deriving Chemin-Lerner bounds for the solutions.

Timothee Crin-Barat. he compressible Euler system with damping: strong relaxation limit in L^p spaces. In this talk, I will present recent results on the global well-posedness of the compressible Euler system with damping and on its relaxation toward the porous medium equation in L^p-based critical spaces, for $2 \leq p < \infty$. Particular emphasis will be placed on the links between this singular relaxation problem, hypocoercive phenomena and hyperbolic approximations of diffusive systems. We will see that the spectral structure of the linearized dynamics reveals hidden dissipative mechanisms governing both the large-time behavior of solutions and their convergence to the diffusive regime. I will then discuss nonlinear stability and convergence results obtained through a refined Littlewood–Paley frequency analysis.

Thomas Eiter. On existence and selection of energy-variational solutions for evolution equations For many evolution equations, general global-in-time existence results are not available in a setting of weak solutions, which motivates to introduce further generalized solution concepts. In this talk, we study the concept of energy-variational solutions. It relaxes the weak formulation by an error term and combines it with the energy-dissipation inequality in such a way that the formulation is stable under convergence in natural topologies. For a wide class of evolution equations, we construct energy-variational solutions by a minimizing-movement scheme and study their properties. Moreover, we discuss admissible selection criteria in this framework. We exemplify the results by applications to fluid equations in low-regularity settings, like the Euler and Euler-Korteweg equations.

Francesco Fanelli. On the incompressible limit for a class of two-fluid models. This talk is concerned with the asymptotic behaviour of solutions to a certain class of two-fluid models in the regime of small compressibility of the fluid.

After reviewing classical results on the incompressible limit of weakly compressible flows, we propose a new method to perform the incompressible limit in the context of two-fluid systems with one velocity, based on the penalisation of only one of the two pressure gradients appearing in the momentum equation. The result crucially exploits an underlying transport structure, well-known in the context of two-fluid models, and a hidden compactness property (which seems not to have been pointed out before) for a new scalar quantity related to the two density functions.

The talk is based on a joint work with Y.-S. Kwon (Dong-A University) and A. Wróblewska-Kamińska (Polish Academy of Sciences).

Théo Fradin. Shallow-water limit for oceanic flows with small density variations. The goal of this talk is to present a well-posedness result on the incompress- ible Euler equations, in the presence of a free surface and density variations. Such a setting is inspired by oceanic flows near the coast, for which the main quantity of interest is the free surface. The main difficulty is to provide the dependency of the result with respect to the aspect ratio, which is often small in practice. This allows us to justify the non-linear shallow-water equations, as a reduced model for the evolution of the free surface, in the case of small density variations. If time allows, we will see how to use the structure of the non-linear shallow-water equations to improve the well-posedness result on the free-surface Euler equations, in some regimes.

David Gérard-Varet. Fluid-bubbles interaction. This talk will be about the mathematical analysis of reduced models for the evolution of N bubbles of gas in a liquid. We will explain how to derive such models through a least action principle, and will give elements about their well-posedness in both cases: the case of an inviscid curl-free flow and the case of a viscous liquid. The talk is based on joint works with Cosmin Burtea.

Jiao He. The vanishing limit of a rigid body in 3D viscous incompressible fluid. We consider the evolution of a small rigid body in an incompressible viscous fluid filling the whole space $\mathbb{R}^3$. When the small rigid body shrinks to a point in the sense that its density is constant, we prove that the solution of the fluid-rigid body system converges to a solution of the Navier–Stokes equations in the full space. Based on some $L^p - L^q$ estimates of the fluid–structure semigroup and a fixed point argument, we obtain a uniform estimate of velocity of the rigid body. This allows us to construct admissible test functions which plays a key role in the procedure of passing to the limit. This is a joint work with Pei Su.

Pascal Hadré. Homogenization of the Stokes equations with critically scaled connected holes In this talk, we derive macroscopic equations for a fluid inside of a domain perforated by critically scaled connected holes. That is, we remove from a domain $\Omega \subset \mathbb{R}^3$ cylinders of length $L \gg 1$ and width $a_\epsilon \ll \epsilon$, that are oriented in each coordinate direction and placed on periodic grids of size 2ϵ. We show that for the critical scaling $(2\epsilon)^{-2}|\log(a_\epsilon)|^{-1} \to C_0 \in \mathbb{R}_{>0}$, the solution u_ϵ to the Stokes equations in this perforated domain Ω_ϵ converges to the solution u of the Stokes equations with an additional drag term Mu, where $M \in \mathbb{R}^{3\times 3}$ is a positive isotropic matrix. To this end, we will first introduce the problem and the notations to formulate the theorem. We then illustrate the strategy of the proof and present the main steps containing the construction of appropriate cut-off functions and proving the existence of an appropriately bounded Bogovskii operator for the domain Ω_ϵ. Moreover, we explicitly determine the macroscopic drag matrix M.

Marco Inversi. Dissipation regularity and intermittency for incompressible Euler In the setting of the incompressible Euler equations, we derive fine properties of the Duchon—Robert distribution, which is known to describe the (possible) failure of the local energy balance. In particular, we focus on the relation between lower dimensional dissipation and intermittency. More precisely, for solutions with spatial Besov regularity, it turns out that the Duchon—Robert distribution has improved regularity in a negative Besov space and, in the case it is a Radon measure, it is absolutely continuous with respect to a reference Hausdorff measure. This imposes quantitative constraints on the dimension of the, possibly fractal, dissipative set and the admissible structure functions exponents, relating to the phenomenon of "intermittency" in turbulence.

Christophe Lacave. Leapfrogging of Vortex Rings. In this talk, I will discuss recent progress on the dynamics of concentrated vortex rings. I will begin with the vortex filament conjecture for the three-dimensional Euler equations, which predicts that vorticity concentrated around a filament remains concentrated and evolves according to the binormal curvature flow. Although this problem remains open in full generality, several positive results have been obtained in specific geometric settings, including the 2D Euler equations, the axisymmetric 3D Euler equations without swirl, the lake equations, and the helical 3D Euler equations without swirl. I will highlight the connections between the corresponding Biot–Savart kernels and the classical 2D Euler kernel, and explain the main ingredients used to prove the persistence of concentration around a single filament. In the second part of the talk, I will present a recent result showing that, in an appropriate asymptotic regime, two concentrated coaxial vortex rings separated by a small distance undergo the classical leapfrogging motion. The proof relies on a new approach to establishing the persistence of mass concentration, combined with a refined iterative scheme and sharp estimates controlling the growth of the vorticity support. This is joint work with Martin Donati, Lars Eric Hientzsch, and Evelyne Miot.

David Lannes. On F. John's model for floating objects. We consider the linearization around the rest state of the $2D$ irrotational free surface Euler equations in the presence of a partially immersed object. Several difficulties arise to prove the well posedness of this model: - Functional framework. One needs to caracterize the space of traces of functions in the Beppo-Levi $\dot{H}^1(\Omega)$ (naturally associated with fluids of finite kinetic energy) on the part of the boundary that corresponds to the free surface, and which is therefore non connex. We show that the good framework is furnished by the screened Sobolev spaces introduced by Leoni and Tice and exhibit nonlocal terms relating the traces on the different connected components. - Realization of homogeneous spaces. Since we work with semi-normed spaces, we are led to work in quotient spaces in order to be in a Banach framework. A good realization of these quotient spaces is needed, that is, a good choice of a representative in the equivalence class. This leads us to a new interpretation of the Bernoulli constant. - Elliptic problems in corner domains. The velocity potential satisfies a Laplace equation in the fluid domain. This fluid domain contains corners corresponding to the contact points between the object and the free surface. These singularities limit the regularity of the solution. - Dynamics of the object. We show several interesting phenomena, such as the different nature of vertical and horizontal displacements of the object, or stability conditions for the equilibrium state. This is based on joint works with M. Ming and M. Paulsen.

Amina Mecherbet. Homogenization of the Steady Navier–Stokes Equations with Prescribed Flux or Pressure Drop in a Perforated Pipe. Homogenization of the (Navier) Stokes equations in perforated domains has seen significant development over the past few decades. The characterization of the effectif model depending on the particles configuration and their rigid motion is well knwon understood in the case of bounded domain with homogeneous Dirichlet boundary condition on the container.

In this talk I will present a recent result regarding such a derivation in the case of a flow in a perforated bounded pipe. The non standard boundary type conditions that we consider are one of the following : - Prescribed net flux at the inlet or outlet of the pipe - Prescribed pressure drop of the flow In this framework, a first result has been proved by G. Sperone in the subcritical regime i.e. when the particles are too small and their effect is negligible asymptotically. We extend this result to the critical case resulting in the derivation of a Brinkman force. This is a joint work with Richard R. Höfer and G. Sperone.

David T. Meyer. Vortex-sheet desingularization for three-dimensional ideal fluids. We prove a desingularization theorem for analytic vortex sheets of the 3D incompressible Euler equations. Starting from an analytic solution of the corresponding Birkhoff-Rott system, we construct, for every sufficiently small thickness parameter $\varepsilon > 0$, an exact Euler vorticity supported on a tubular neighborhood of width $O(\varepsilon)$ around the sheet, and defined on a time interval that does not shrink to 0 as $\varepsilon \to 0$. We show that these vorticities converge, in the sense of distributions, to the prescribed vortex sheet. In particular, we conclude that analytic 3D vortex sheet motions arise as limits of exact Euler flows with lifespan bounded from below independently of ε. The proof hinges on the study of vorticities defined in terms of a time-dependent foliation by almost parallel surfaces and of divergence-free vector fields tangent to these surfaces.

Elena Salguero. Stable and unstable behavior of two-phase flow under gravity. We study the influence of gravity on density-dependent incompressible fluids in a regime where inertial forces are neglected. In particular, we consider piecewise constant densities, leading to the formation of a sharp interface and reducing the evolution to the dynamics of this interface. Depending on the regime and the initial configuration, a rich variety of phenomena arises, including both stable and unstable behaviors. This talk is based on a series of joint works with F. Gancedo, R. Granero, Z. Hu, and Y. Yao.

Richard Schubert. Mean-field limits and propagation of chaos in systems with non-attractive interaction: implications for transport-Stokes. We will discuss mean-field limits in the context of suspensions, where particles interact via a mediating fluid. The governing macroscopic model, the transport-Stokes equation, has so far been derived under far-from-random assumptions on the distribution of the particles. In this talk I present progress in this direction, in particular a general mean-field result for binary systems, that allows propagation of chaos for non-attractive kernels just below the singularity of the Stokes kernel while maintaining control of individual particle trajectories. The result can be used to derive the transport-Stokes equation under significantly relaxed, close to random conditions on the distribution of the particles. This is based on joint work with Richard Höfer, Regensburg and Manuel Heger, Bonn.

Sebastian Schwarzacher. Time periodic solutions for partially dissipative systems. Time-periodic weak solutions for a coupled hyperbolic-parabolic system are discussed. A linear heat and wave equation are considered on two respective spatial domains that share a common interface. The parabolic-hyperbolic system is only partially damped, leading to an indeterminate case for existing theory. We construct periodic solutions by obtaining novel a priori estimates for the coupled system, reconstructing the total energy via the interface, provided that the wave domain satisfies suitable geometric constraints. These conditions are reminiscent of classical boundary control conditions for wave stabilizability. We note a "loss" of regularity between the forcing and solution which is greater than that associated with the heat-wave Cauchy problem and that depends on the geometric constraints. Further spectral properties for an explicit wave domain that violates the geometric constraints are discussed, which allow to construct examples with an arbitrary regularity loss. The lecture is based on a joint work with S. Mosny, B. Muha and J. Webster.

Christian Seis. Steady motions of air bubbles and bubble rings. Air bubbles and air bubble rings in water can be modeled by the capillary two-phase Euler equations and are characterized by the presence of a vortex sheet on their surface. In this talk, I will present several families of steady solutions describing the rectilinear motion of thin bubble rings and near-spherical bubbles, as well as the spiraling motion of near-spherical bubbles.

Stefan Skondrić. The density patch problem for the inhomogeneous, incompressible $2D$ Navier-Stokes equations. We are interested in the density patch problem for the inhomogeneous, incompressible Navier-Stokes equations in $\mathbb{R}^2$ at a critical level of regularity. More precisely, we assume that the initial density of the fluid is the indicator function of a bounded Lipschitz domain and that the initial velocity u_0 lies in the Besov space $\dot{B}^0_{2,1}(\mathbb{R}^2)$. We first introduce a suitable class of solutions and give an overview of the most important a priori estimates. This allows us to prove the global existence and uniqueness of solutions in the critical regularity framework above and to conclude that the Lipschitz regularity of the patch is preserved over time. Compared to previous works related to the density patch problem, the main novelty is an L^1-global-in-time Lipschitz estimate on the velocity field. This enables us to fully describe the long-time behavior of the patch, which is the rigid motion of an emerging Lipschitz domain.

This is based on joint work with Alessandro Violini.

David Wiedemann. Evolving Microstructures in Fluid Flow and Transport Many processes in porous media involve an evolution of the pore geometry caused, for example, by dissolution or precipitation. At the same time, these geometric changes can strongly affect fluid flow and transport at the macroscopic scale.

In this talk, we first present a general framework for the homogenisation of problems involving evolving microstructures – the two-scale transformation method. We then apply this method to several PDEs posed on evolving microscopic domains and investigate how the evolution of the microstructure affects the resulting macroscopic models.

For Stokes flow through an evolving porous medium, we derive a Darcy-type law as the effective model. It features a time- and space-dependent permeability as well as a new source term for the pressure arising from changes in the local effective porosity.

We further address the homogenisation of a free-boundary problem in which the geometry evolves through reactions coupled to an advection–reaction–diffusion problem and Stokes flow. The resulting limit is a strongly coupled micro–macro model in which fluid flow, concentration, and pore evolution mutually influence one another.

Finally, we consider microstructures that pulsate rapidly in time. Such oscillations can generate effective macroscopic advection, even when the underlying microscopic model contains only diffusion. This demonstrates how temporal changes in the microscopic geometry can produce qualitatively new effects in macroscopic fluid and transport equations.