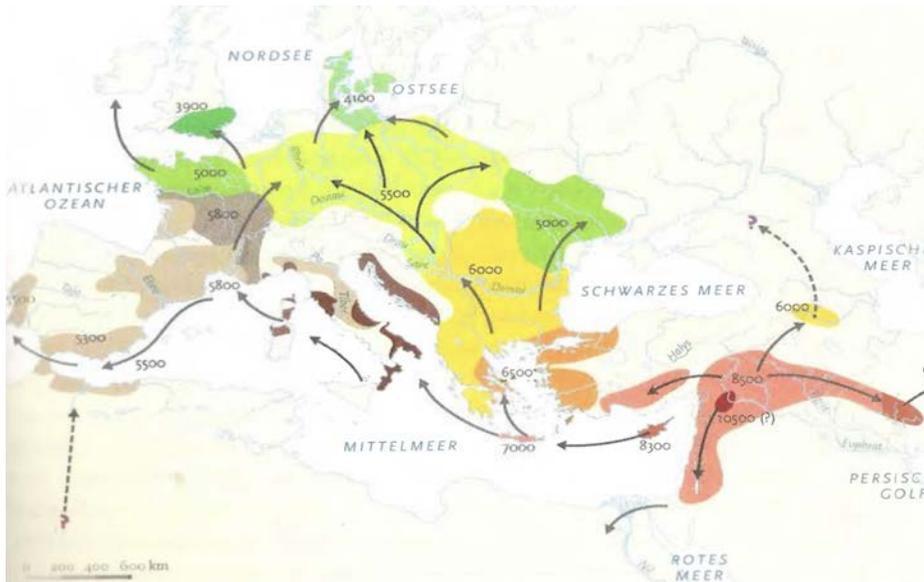


Regional Economics

Lecture Notes (Jan. 28th, 2026) ¹



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¹These are the notes for „Regional Economics“. Please only use them yourself; do not share them outside this class. Title picture: Frankfurt's skyline.

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6 Urban Form

Be. 64: Battista Alberti

In 15th century Italy, Battista Alberti asserted the dual role of a house. A house's four walls, Alberti explained, not only define the owner's private space but also contribute to defining the streets and squares of the city's public space. A similar point could also be made for a house's windows. A window allows a neighboring yard to send daylight into the house. This is Le Corbusier's (1929) interest when calling for open space to provide *light and air*. But a window also allows one to look out into the street. This is Jacobs' (1961) focus when calling for *eyes on the street*. If it is to be complete, the economic analysis of urban daylight should account for both of daylight's aspects, i.e. daylighting of private space and visibility in public space. Double entry bookkeeping is not specific to the analysis of daylight. It is a "micro-foundation" (Schelling (1978)). This chapter's analysis follows Dascher/Haupt (2026).

Be. 65: Streets I

Brighter housing certainly benefits the household during the time that is spent at home (Fleming et al. (2018)). At the same time, all this extra daylight also means that houses need to retreat from each other. Then fewer houses align with the street. And if streets fail to have "eyes" watching over them (Jacobs (1961)), they become less safe. The motorized commuter, the adult commuter, or the male commuter may not notice. But pedestrians, children, or women surely will (Yavuz/Welch (2010)), as will their parents or partners. Less safety may actually mean more crime. But less safety is no less relevant when it is merely imagined. And whether real or only feared, less safety also registers in a drop of all activities that are complementary to commuting.

Be. 66: Streets II

There are numerous such activities. Households may work less just to avoid commuting. They may forego career or educational choices, only to be able to reduce commuting. Borker (2024) finds that women restrict themselves to a second-best option in education if that is safer to reach than their first-best choice. Children may be taken to school by car, when really they would prefer to walk or bike to school (McDonald (2007)). Workers accept lower wages – and



(a) rue de Mazagran

(b) Paris Periphery

Figure 6.1: Urban Paradigms

Notes: Houses in Rue de Mazagran in Fig. (6) tend to be dark and occluded. Yet there are many eyes on that street, and the street feels safe at any time of day. Houses in the Paris high-rises in Fig. (6) are bright. Yet these high-rises are not aligned with any one street in particular.

hence forego more productive jobs – in exchange for living in a more compact city (Harari (2020)), possibly signaling a preference for safer streets. Households may generally walk less to avoid street level hazards. On-street retailers, cafés, restaurants, dry-cleaners, newsagents, opticians etc. find fewer commuters to sell to. Once these businesses have closed down, commuting safety deteriorates further.

Be. 67: Paris

To illustrate this paper’s premise (of our cities turning their houses away from the street), we offer an example from Paris, France. (Infinitely many other pictures could be shown, each with its own backdrop and hence the potential to distract from the paper’s theme, which is why we restrict ourselves to one example only). So consider the typical tenement in Paris’ *rue de Mazagran* (Fig. (6)). None of its apartments appear to be daylit well. But houses do attune, and provide eyes, to the street. Contrast this with more recent houses in the suburbs of Paris (Fig. (6)). Those houses fail to align with any street. They overlook green open space instead. Apartments in these houses are bright. Yet while they have “eyes”, those just as certainly are no “eyes on the street”.

Be. 68: Zoning

Proponents of de-zoning (e.g. Fischel (2005), Glaeser (2009), Gallagher et al. (2021)) argue for its potential land price decreasing effects. Proponents of land

assembly (Brooks/Lutz (2016)), Nakashima/Teshima (2021)) argue for its density increasing, and hence supply increasing, effects. Our model is consistent with neither proposition. In our model, land not built on is purposely not built on, to daylight neighboring lots. Building up on that land, as the de-zoning literature suggests, may increase density; but as long as it fails to make commutes safer, too, that comes at the expense of the aggregate value of the block. A daylighting perspective helps us understand both the welfare loss from, and the fierce opposition against, de-zoning. A similar point applies to land assembly, which amalgamates lot developers into larger developers. In our context (of given building height), block developers reduce, rather than increase, density.

6.1 City Spatial Structure

Be. 69: Spatial Structure

The economy's land is a subset of the two-dimensional plane. It is large enough to accommodate a monocentric city and its large hinterland. Each tenant-household has two members, the homemaker and the commuter. Utility in hinterland is given. Perfect interregional mobility ties utility in the city to that in the hinterland. The city's center is point O in the plane. It accommodates all jobs, shops, and schools. This framework corresponds to the *open city* (Brueckner (1987)). This is the framework we build on. We extend the open city to allow for the two "dual" fundamental expressions of daylight: daylighting and visibility. Daylighting plays out within the urban block. Visibility is negotiated within the urban street. We need to make room for blocks and streets.

Be. 70: Daylit Open City

Daylighting allows the household to see. Daylighting enhances the quality of the home in many ways; imagine cooking, cleaning, reading in the dark, or under artificial light only. Daylighting also often comes along with a view, with more privacy or intimacy, or even with electricity production. Visibility allows the household to be seen. Visibility strengthens the safety of the commuter's path, deterring potential hazards along the way or encouraging participation in the labor market. We extend the open city by adding daylight specific language, such as windows, daylightings, occlusions, frontage, opposition, field-of-vision, safety, etc. At the end, a *daylit open city* emerges.

Be. 71: Cadastre

Land is subdivided into columns of unit width and rows of unit height. Fig. (6.1) shows the resulting cadastre of land parcels. Roads are rows reserved for traffic, and spaced n rows apart. Streets are columns reserved for traffic, and spaced

n columns apart. We assume $n \geq 5$ and n odd. Indices $s = \pm 0, \pm 1, \pm 2, \dots$ and $r = \pm 0, \pm 1, \pm 2, \dots$ index streets and roads, respectively (again Fig. (6.1)). Streets and roads ± 0 bisect at the city center (recognizable as the black square in Fig. (6.1)). They are the city's central traffic arteries. We collapse streets ± 0 into a single street named 0, and roads ± 0 into a single road named 0. When we speak of streets only, we mean streets and roads. While our exposition often assumes $s, r \geq 0$, this is no restriction we impose on the full model. Streets and roads (the "traffic grid") are owned by the city. Grid use is non-rivalrous.

Be. 72: Streets and Roads

A parcel sr that interlaces street s with road r we call junction sr . E.g., junction 21 is where street 2 intersects with road 1 (Fig. (6.1)). Two neighboring junctions bookend the street or road section between them. Sections are the city's public space. (Households' interest in public space will be whether it is safe to travel through it.) Junctions and sections are vertices and edges, respectively, of an (undirected) traffic network. A commuting path P in that network is an alternating sequence of junctions and sections, starting at junction sr and ending at center O (or vice versa), such that no junction or section is visited more than once. A shortest path between junction sr and O winds along precisely $s+r$ sections. So its length is $s+r$. This is junction sr 's (Manhattan) distance to O,

$$\delta(sr) = |s| + |r| \quad (\text{distance to center O}). \quad (6.1)$$

Travel cost is t per distance unit. Commuting costs along the shortest path are $t\delta$.

Be. 73: Rings and Contours

Ring δ collects all junctions with given distance δ to center O. From a distance, a ring looks "diamond-shaped". *Contour* δ collects all sections linking some junction in ring $\delta - 1$ to another in ring δ . A contour is "staircase-shaped". Fig. (6.1) illustrates ring 0 (coincident with junction O, black), ring 1 (four junctions, dark grey) and ring 2 (eight junctions, light grey). It also illustrates the twelve sections that belong to contour 2 (in grey). Each of them links one junction from ring 1 to some neighboring junction in ring 2. We also see an example of a path from junction 11 to O (the dashed line).

Be. 74: Blocks

Street and road sections envelope connected square subsets of land, called *blocks*. Each square block has four *fronts*. A block is privately owned, by absentee landlord-developers. Blocks within city limits are the city's private space.

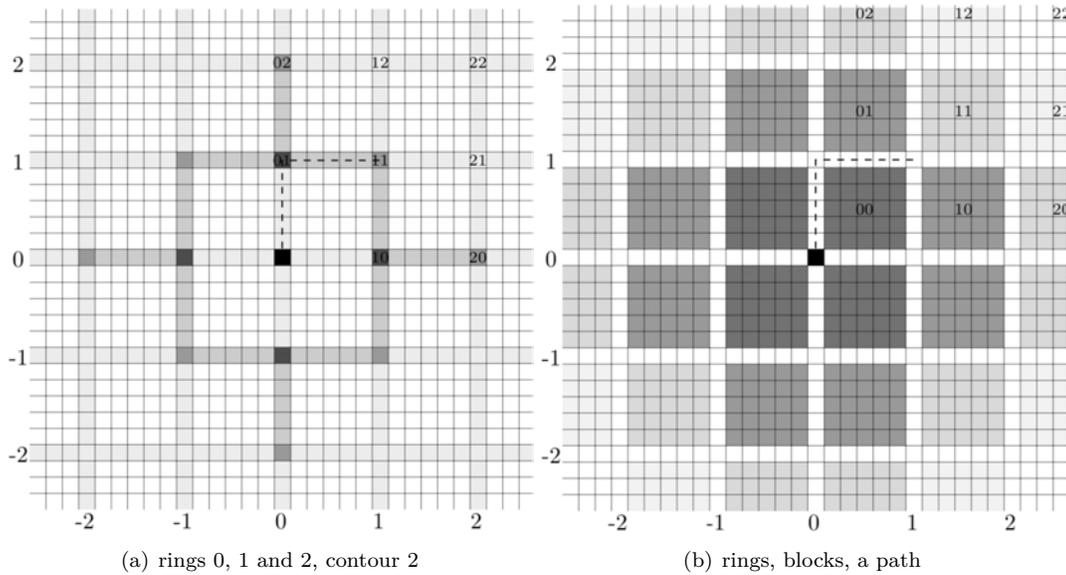


Figure 6.2: *Public vs. Private Space*

Notes: (i) Fig. (6.1) shows the cadastre and the traffic grid. (ii) Streets run North-South, roads East-West. Streets and roads are numbered $\pm 0, \pm 1, \pm 2, \dots$, where streets and roads ± 0 bisect at the city center (black square). (iii) Junctions are labeled by the street and road that intersect in them. (iv) Fig. (6.1) illustrates ring 0 (city center junction O, in black), ring 1 (junctions in dark grey) and ring 2 (junctions in light grey) as well as contour 2 (sections that link a junction from ring 1 to another in ring 2, in light grey). (v) Fig. (6.1) shows the blocks whose residents depart from ring 0 (dark grey), from ring 1 (dark grey), and from ring 2 (light grey). (vi) The dashed line in Fig. (6.1) is a shortest path for a resident in ring 2, starting at junction 10, traversing one section in contour 2 and another in contour 1.

(Households' interest in private space will be the degree to which it is daylit.) We cannot yet say which path a block's tenant takes to center O. But whatever that path, we assume that a block's tenants, and only they, may travel within their block, at no cost. On their commute to the city center, a block's tenants never see (in fact, never need to see) any of their block's fronts. We assume that a block's tenants start their commute to O at *that* of the four junctions bordering on the block that is *nearest* to O. It is after that junction that we label the block. With that convention, a ring becomes a set of equidistant blocks.

Be. 75: Blocks and Sections

Fig. (6.1) and (6.1) jointly illustrate this labeling convention, showing junctions 10, 20, 01 etc. and blocks 10, 20, 01 etc. The rectangular grid and the Manhattan metric impose their own specific structure on blocks, rings, contours, and on the relationships between them. This is due to blocks actually claiming city space now, rather than just collapsing into points. For one, a block in ring δ contributes its fronts to sections in contours $\delta + 1$ and $\delta + 2$, and to those

only. More specifically, it contributes no front to contour δ . Instead, a section in contour δ is escorted either by blocks from the preceding ring $\delta - 1$ only, or by blocks from the two preceding rings $\delta - 1$ and $\delta - 2$, and by those only. It is never escorted by any block from ring δ .

6.2 Daylit Windows

Be. 76: Blocks, houses, yards, . . . windows

Because streets are spaced n units apart, each block is n units wide and n units long. A block is a rectangular $n \times n$ array of n^2 equal-sized square parcels (e.g. Fig. (6.5) when n is 5). (We can easily generalize our analysis of the block. For example, the block may also be rectangular. Then its dimensions are $m \times n$, with m not necessarily equal to n .) Its parcels we call its *lots*. Let us fix some block sr , drop its indices for now, and assign its “Northwest” corner the role of the origin of another, i.e. block, coordinate system. The block may be represented by an $n \times n$ matrix, with the entry in lot ij equal to 1 if lot ij has a *house*, and 0 if it is a *yard*. So a lot is either of two *types*. Each lot has 4 faces. If it is built up faces become *walls*. Embedded into each wall is a *window*.

Be. 77: Configuration, design, block

We stack all n rows of our block matrix into one long $n^2 \times 1$ vector and call it \mathbf{x} . Thus

$$\mathbf{x}' = (x_{11}, \dots, x_{nn}) \quad (\text{block}). \quad (6.2)$$

We call \mathbf{x} a *configuration* if it is an equilibrium, and a *design* if it is someone’s plan. We also call \mathbf{x} a *block* when no distinction is needed. Here is an example of a configuration for the 2×2 -block:

$$\mathbf{x}' = (0, 1, 1, 0) \quad (6.3)$$

Fig. (6.3) illustrates. The first lot (in the top left corner) is not built up, i.e. is a yard. The second lot (top right corner) is a house. The third lot (bottom left corner of the block) has no yard; while the last lot (the bottom right corner lot) is a yard again. We will repeatedly use this example to illustrate this section’s concepts. Two trivial blocks are the zero block (ZO), which has no houses, and the full block (FU), which has only houses. Full blocks do occur. Cerdà’s Eixample in Barcelona or Haussmannian blocks in Paris (e.g. Fig. (6)) come to mind. Zero blocks are zoned as parks, or are outside the city. If ι denotes an $n^2 \times 1$ -vector of 1’s, then: the number of lots is $\iota' \iota$ or n^2 , houses N are $\iota' \mathbf{x}$,

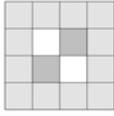


Figure 6.3: Configuration $\mathbf{x}' = (0, 1, 1, 0)$ on the 2×2 -block

the number of windows is $4\iota'\mathbf{x}$, and that of yards is $\iota'(\iota - \mathbf{x})$. Fig. (6.5) further down gives 16 examples of configurations on the 5×5 block.

Be. 78: Adjacency matrix

A symmetric $n^2 \times n^2$ adjacency matrix \mathbf{D} records each lot's neighbors. Its entry in row ij and column kl is 1 if lots ij and kl are adjacent, and 0 otherwise, where $i, j, k, l = 1, \dots, n$. denoted by ε . The technology embodied in \mathbf{D} skips over the intricate issues of azimuth angle, cloud cover, orientation, choice of hemisphere, etc. Instead, it expresses a lot's daylightings as the sum of its two, three or four neighboring lots' deflections. Daylightings equal any one of the five numbers in $\{0, 1, 2, 3, 4\}$. Daylight is never received from further afar than its four neighbors, and it is never received from above. Houses are single floor. Inasmuch these "daylight axioms" are reasonable, they are so for uniformly higher houses. Scaling building height up to 5 stories seems defensible.

Be. 79: Adjacency matrix for the 2×2 block

Consider a 2×2 block. Its adjacency matrix is

$$\mathbf{D} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}. \quad (6.4)$$

The adjacency matrix here is a 4×4 matrix! It has four rows and four columns. After all, there are four lots on the 2×2 block. Each of them can be neighbored by any of the four (?) others. Yes, in principle it could be neighbored by itself. Actually, we allow for that possibility, then deny it. In the adjacency matrix, the main diagonal has 0 entries only.

Be. 80: Daylightings

Let us multiply the adjacency matrix \mathbf{D} by a configuration \mathbf{x} . This gives

$$\mathbf{D}\mathbf{x} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 2 \end{pmatrix} \quad (6.5)$$

Attribute	Notation
How many lots?	ι'
How many houses?	$\iota' \mathbf{x}$
How many yards?	$\iota'(\iota - \mathbf{x})$
How many occlusions?	$\mathbf{A} \mathbf{x}$
How many interior daylight windows?	$\mathbf{A}(\iota - \mathbf{x})$
How many exterior daylight windows?	\mathbf{f}
How many (interior or exterior) daylight windows?	$\mathbf{A}(\iota - \mathbf{x}) + \mathbf{f}$
How many daylight windows altogether?	$\mathbf{x}' \mathbf{A}(\iota - \mathbf{x}) + \mathbf{x}' \mathbf{f}$
How many windows?	$4 \iota' \mathbf{x}$
How many windows are occluded?	$\mathbf{x}' \mathbf{A} \mathbf{x}$
Is the block separable?	Yes/No
If it is separable, how so?	ϕ
Is the block multiply separable?	$\phi_1, \phi_2, \dots?$

Table 6.1: *Characteristica of the Urban Block*

Look at the 1's in the configuration. Wherever there is a 1, there is a house, throwing shadows on its (up to 4) neighbors. We call \mathbf{x} the *occluder*. When we multiply the adjacency matrix with that occluder, then we in fact compute the *occlusions* on the block. In our example we see that lots (1, 1) and (2, 2) are occluded twice, lots (1, 2) and (2, 1) not at all. Of course, the same thing you see when consulting your earlier sketch of the configuration.

Be. 81: Interior daylight windows

There are two types of daylight windows: windows daylight from inside the block (interior daylight windows) and windows daylight from outside the block, the street (exterior daylight windows). Since \mathbf{x} is the occluder, then $\iota - \mathbf{x}$ is the *daylighter*. After all, ι , pronounced „jota“, simply is a list of $mn \times 1$ ones,

$$\iota' = (1, 1, \dots, 1), \quad (\text{vector of ones}) \quad (6.6)$$

Now let us multiply the adjacency matrix with the daylighter. Exploiting eq. (6.7), then in the continuation of our example,

$$\mathbf{D}(\iota - \mathbf{x}) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \\ 0 \end{pmatrix}. \quad (6.7)$$

This, too, is what you see in your sketch of the block. Lots (1, 2) and (2, 1) are daylight twice from within the block.

Be. 82: Exterior daylit windows

To assess the windows daylit by the street, we introduce vector \mathbf{f} . In our example in Fig. (6.3),

$$\mathbf{f}' = (2, 2, 2, 2), \quad (\text{frontage vector}). \quad (6.8)$$

This vector gives the number of times a lot is daylit by the street. Since in a small block, all lots are corner lots, this is very simple. Then \mathbf{f} is just a list of 2's. In larger blocks, the streetfront or frontage vector will also have 1's. A 1 simply is a streetfront lots that is no corner lot. If we multiply \mathbf{f}' with \mathbf{x} , we get $\mathbf{x}'\mathbf{f}$, the total of exterior daylit windows. Summing interior daylit windows and exterior daylit windows gives

$$D(\iota - \mathbf{x}) + \mathbf{f} \quad (\text{total daylit windows}). \quad (6.9)$$

Here we have a list of total daylit windows for each lot.

Be. 83: (Total) Daylit Windows

Vector (or list) $\mathbf{A}(\iota - \mathbf{x}) + \mathbf{f}$ lists daylightings for every lot, whether house or yard. This is important to see: A yard can be daylit, too, if it is surrounded by neighboring yards. Daylit yards are not very interesting. One can sit there in the summer, which is nice. But it is more important to put daylight into the house (where the household, typically, spends more time). We need to focus on daylit houses, i.e. on daylit windows. How can we extract, and even then sum over, daylit windows? This is done by pre-multiplying the previous list with \mathbf{x} , as in

$$\Lambda = \mathbf{x}'\mathbf{A}(\iota - \mathbf{x}) + \mathbf{x}'\mathbf{f}. \quad (\text{total daylit windows}) \quad (6.10)$$

This gives total daylit windows, now denoted Λ . This is an important attribute of the block. Blocks often differ in their number. From the perspective of block supply, a block with more daylit windows should receive a higher price. Its houses have, on average, greater quality. Let us compute this figure for our example in Fig. (6.3). Clearly, from inspecting the figure, Λ equals 8. But we can also confirm this using our formulas. There are $\mathbf{x}'\mathbf{A}(\iota - \mathbf{x})$ interior daylit windows:

$$\mathbf{x}'\mathbf{A}(\iota - \mathbf{x}) = (0, 1, 1, 0) \cdot \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = 4, \quad (6.11)$$

and there are $\mathbf{x}'\mathbf{f}$ or 4 exterior daylit windows. So Λ really is 8.

Be. 84: Occlusions on the Block

We recall that it is \mathbf{Ax} the occluder vector. Pre-multiplying occlusions \mathbf{Ax} gives occlusions of houses, i.e. gives occluded windows.

$$\mathbf{x}'\mathbf{Ax} \quad (\text{Zahl verschatteter Fenster im Block}). \quad (6.12)$$

Let us revisit our example in Fig. (6.3). There we find that $\mathbf{x}'\mathbf{Ax}$ just equals 0. After all,

$$\mathbf{x}'\mathbf{Ax} = (0, 1, 1, 0) \cdot \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = 0. \quad (6.13)$$

This is obvious from consulting Fig. (6.3). No house neighbors on another. This makes this particular configuration “bright”. We might characterize it as a *chess block*, even if the 2×2 block clearly has no chess board dimensions.

Be. 85: Total Daylit Windows

We finally add another, different way of counting daylit windows. This alternative departs from the simple fact that windows can only be either daylit or occluded; there is nothing else. So let us first take all windows, $4l'\mathbf{x}$, then subtract all occluded windows, $\mathbf{x}'\mathbf{Ax}$. This gives total daylit windows, too:

$$\Lambda = 4l'\mathbf{x} - \mathbf{x}'\mathbf{Ax} \quad (\text{daylit windows}) \quad (6.14)$$

Again let us try this out for the example in Fig. (6.3). It is $\Lambda = 8 - 0 = 8$, and that is actually the same result as in Bem. (83). Eqs. (6.10) and (6.14) are two alternative ways of computing total daylit windows – for any block and configuration.

6.3 Lot Developers (Nash-Equilibrium)

Be. 86: Small Developer Profit

Let there now be many small real estate developers. (We are in a market economy with many suppliers.) It is \mathbf{Ax} the list of occlusions. If we are interested in the occlusions of lot ij , we pre-multiply that list simply by \mathbf{e}_{ij} , which is a list of zeros except at position ij , where there is a 1. This product is $\mathbf{e}'_{ij}\mathbf{A}$. It is an important concept that we will use many times. It looks difficult, but it is



Figure 6.4: The “Bauhaus” (Walter Gropius) in Dessau emphasizes daylight

simple. It is a number. It is the number of times lot ij is occluded (by neighboring houses). Clearly that number can only be an integer from set $\{0, 1, 2, 3, 4\}$. From this it follows that that lot ij is daylit

$$4 - \mathbf{e}'_{ij} \mathbf{A} \mathbf{x} \quad (\text{Belichtung der Parzelle } ij). \quad (6.15)$$

times. Now we account for the value of a daylit window. This value we denote by v . And we account for the location of the house. That location carries a premium of p . Then lot developer profit becomes

$$\pi_{ij} = (4 - \mathbf{e}'_{ij} \mathbf{A} \mathbf{x})v + p, \quad (\text{lot developer's profit}) \quad (6.16)$$

if the developer builds a house, and 0 if not.

Be. 87: Best Responses

Consider the developer of lot 21. If π_{21} positive, the developer will build up (so

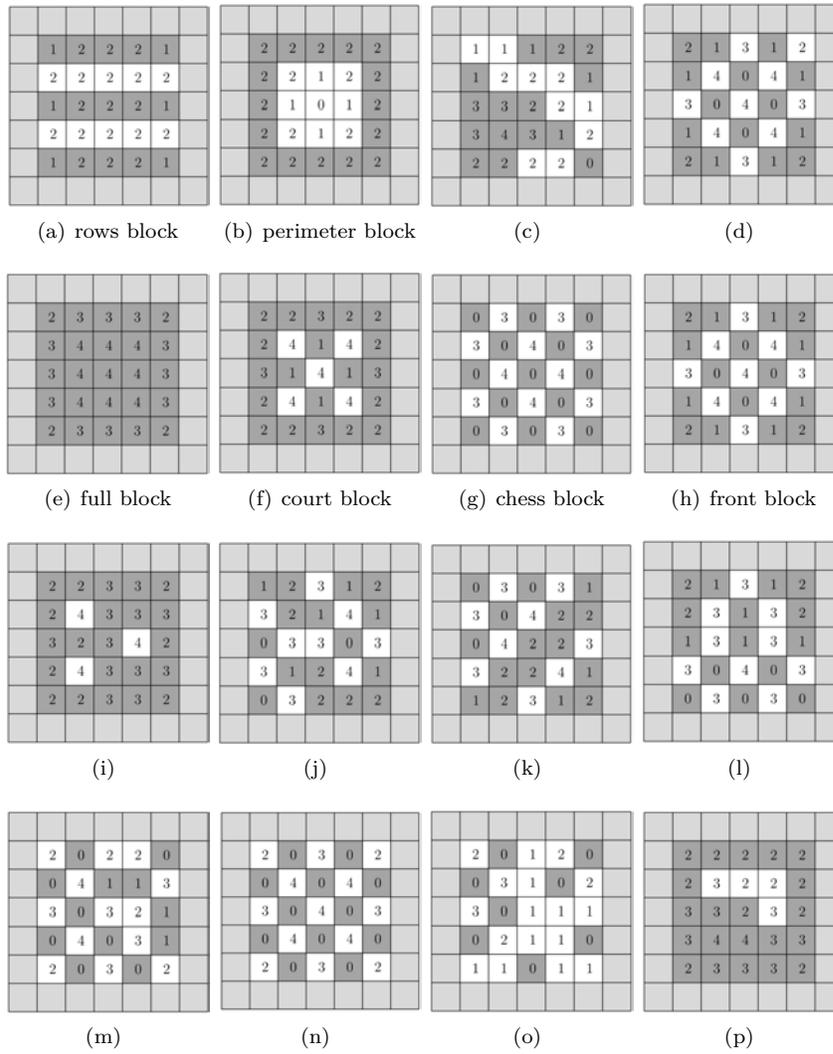


Figure 6.5: Various configurations on the 5×5 block

that $x_{21} = 1$); else he will not build up (so that then $x_{21} = 0$). to summarize

$$p + 4v - e'_{21} \mathbf{A} \mathbf{x} v \geq 0 \quad \Leftrightarrow \quad x_{21} = 1 \quad (6.17)$$

$$p + 4v - e'_{21} \mathbf{A} \mathbf{x} v < 0 \quad \Leftrightarrow \quad x_{21} = 0. \quad (6.18)$$

Now let us focus on an important point here. The decision of the developer, as just described, depends on the decisions of all the other developers! Why? Because the others' decisions (on whether they build up or not) is contained in configuration \mathbf{x} . So here we have a description of developer behavior as a function of all other developers' behaviors. We have a *best reponse* in a context of strategic interaction, i.e. in the context of a non-cooperative game.

Be. 88: (Nash-) Equilibrium

Not just this one developer thinks the way we just set out. Every developer does. And all developers decide on their construction decisions simultaneously. So actually we have

$$p + 4v - e'_{ij} \mathbf{A} \mathbf{x} v \geq 0 \quad \Leftrightarrow \quad x_{ij} = 1 \quad \text{for all } i, j \quad (6.19)$$

$$p + 4v - e'_{ij} \mathbf{A} \mathbf{x} v < 0 \quad \Leftrightarrow \quad x_{ij} = 0 \quad \text{for all } i, j. \quad (6.20)$$

A difficult question (for which we have a simple solution in a moment) is whether a configuration \mathbf{x} exists that solves *all* these conditions? If such a configuration exists, it is the “intersection” of all the best response functions. And we know that such an intersection is called Nash-Equilibrium. In Nash-Equilibrium, each player chooses the best strategy for himself – given all other players' strategy choices.

Be. 89: Maximum tolerable Occlusions for the Lot Developer

Let us rewrite equilibrium conditions a little. Let us write

$$\phi^{**} \geq e'_{ij} \mathbf{A} \mathbf{x} \quad \Leftrightarrow \quad x_{ij} = 1 \quad \text{for all } i, j \quad (6.21)$$

$$\phi^{**} < e'_{ij} \mathbf{A} \mathbf{x} \quad \Leftrightarrow \quad x_{ij} = 0 \quad \text{for all } i, j. \quad (6.22)$$

On the l.h.s. of the inequalities we find the occlusion of lot ij . On the r.h.s. $4 + p/v$ is always the same number (itself dependent on daylight valuation v and location rent p). This number is an occlusions *threshold*. If houses are to be built, then occlusions must be at most this number. At the same time, if no house is to be built, occlusions must exceed that number. We call this threshold ϕ^{**} ,

$$\phi^{**} = 4 + \frac{p}{v} \quad (\text{maximum tolerable occlusions}) \quad (6.23)$$

it is ϕ^{**} the occlusion that a house tolerates maximum, if it is to be built.

Be. 90: Block Separability

In Nash-Equilibrium, houses are occluded by at most ϕ^{**} , while yards are occluded by strictly more than ϕ^{**} . Hence *all* houses are always less occluded than *all* yards. Put yet more differently, the largest occlusion among houses must be strictly smaller than the largest occlusion among yards. This is a necessary condition for a configuration to satisfy Nash-Equilibrium. Moreover, it is a simple criterion to test for. You scan all houses and look for the largest occlusion among them. Then you scan all yards and look for the smallest occlusion among them. And when you are finished doing this, the first number must be strictly smaller than the second. (The two numbers are not allowed to be equal.) We now say that a configuration or a block is *separable* if the two sets of numbers can be separated in the way we just set out.

6.4 Large Developer (Maximum Profit)

Be. 91: Jungle vs Garden

Having a large developer on the block is like a gardener. She tends to the entire garden (block), and makes decisions on which plants are allowed to grow by receiving daylight, and which are not. Note the analogy of a daylit house and a daylit plant here. Plants need daylight, too! Small developers are like a jungle. Each plant decides on its own whether and how much it wants to grow, depending on which room surrounding plants allow for. The jungle is characterized by intense competition for daylight. This is true for the real jungle; it is also true for those many lot developers competing on the block. – We should not be surprised that the best design with having a large gardener is more structured than the equilibrium configuration with having many small developers. At the same time, even the jungle has structure: note the separability of Nash-equilibrium we uncovered above (Be. (??)).

Be. 92: Profit Maximization

In the previous section, each lot was developed individually, by mn different small landlord-developers. Now we consult the polar case of a single landlord-developer who owns all mn lots. That developer we call *block developer*. We will later see that the large landlord-developer should choose his design \mathbf{x} such that

$$\phi^{**}/2 \geq \mathbf{e}'_{ij} \mathbf{A} \mathbf{x} \quad \Leftrightarrow \quad x_{ij} = 1 \quad \text{for all } i, j \quad (6.24)$$

$$\phi^{**}/2 < \mathbf{e}'_{ij} \mathbf{A} \mathbf{x} \quad \Leftrightarrow \quad x_{ij} = 0 \quad \text{for all } i, j. \quad (6.25)$$

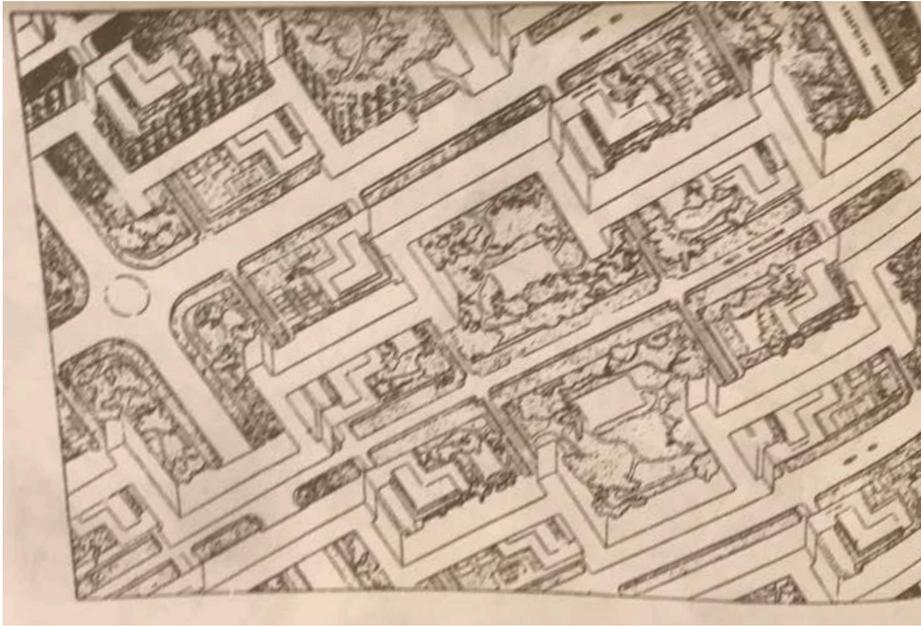


Figure 6.6: Le Corbusier's designs emphasize daylight, too. (Le Corbusier (1929))

conditions are satisfied. So please note that these conditions are identical to those characterizing Nash-equilibrium. The only difference is marked in red, i.e. division by 2. We discussed in class why the large developer divides by 2. Rather than observing maximum tolerable occlusions of ϕ^{**} , the block developer observes maximum tolerable occlusions of *only half as much*. The block developer is more hesitant to build up. Why? Because she knows that by building up she occludes $2e'_{ij} \mathbf{A} \mathbf{x}$ of daylight windows *that all belong to her, irrespective to which house they are assigned*, rather than just one (i.e. as with the lot developer who ignores the effect on the neighboring houses).

Be. 93: Counting Rules: Faces and Windows

An intermediate step towards maximizing block developer profit, Π , is to identify maximum daylight windows, Λ . And yet an intermediate step towards maximizing daylight windows is to maximize interior daylight windows

$$\max_{\mathbf{x}} \Lambda_i = \mathbf{x}' \mathbf{A} (\iota - \mathbf{x}) \quad (6.26)$$

We can show that if you want to maximize daylight windows, Λ , you need to have maximized interior daylight windows, Λ_i . Why? Each lot has four faces. Yet only

2, 3 or 4 of those faces face the interior of the block. The remainder faces the street. The number of faces a lot offers to neighboring lots of the block equals

$$d_{ij} = 4 - f_{ij} \quad (\text{degree}), \quad (6.27)$$

and it is also called lot ij 's *degree*. Here are two simple but fundamental accounting rules on counting faces (degrees) and houses:

$$\begin{aligned} \text{house interior faces} + \text{yard interior faces} &= \text{interior faces (fixed)} \\ \text{interior windows (house interior faces)} + \text{exterior windows} &= 4l'x \text{ (fixed)} \end{aligned}$$

The first of these rules applies to any block. The total number of interior faces (degrees) is given, and it decomposes into interior faces (degrees) of houses and interior faces (degrees) of yards. The second of these rules applies only to the special (but still interesting) case of when the number of houses is given. The total number of windows simply is the sum of windows opening up towards the block interior and of windows looking into the street.

Be. 94: Fundamental Identity

We now combine the two previous identities or counting rules. This gives

$$\text{exterior windows} = \underbrace{(4l'x - \text{interior faces})}_{\text{fixed}} + \text{yard interior faces} \quad (6.28)$$

This is a fundamental identity that holds for *any* block as long as the number of houses is held fixed. It places an aggregate constraint on how yard interior faces and exterior windows must move together. When yard interior faces increase, exterior windows must increase. When yard interior faces decrease, exterior windows must decrease. This is relevant for daylight because (i) yard interior faces are a precondition for any interior daylit window (it needs a yard interior face opposite) and (ii) exterior windows are exterior daylit windows always.

Be. 95: Max. Daylit Windows \Rightarrow Max. Interior Daylit Windows

Suppose interior daylit windows Λ_i are not maximum. Then clearly it is possible to have even more interior daylit windows. The extra interior daylit window also means that there is an extra yard degree. By identity (6.28), the extra yard degree automatically adds an extra exterior daylit window. Thus now we have an extra interior window and an extra exterior window. So we have more total daylit windows. *Wait, we now have an alternative block that has more daylit windows?!* But then surely it is not true that x already maximizes Λ . This shows something very important: If we are looking for a block x that maximizes daylit windows, we should look no further than to a block that maximizes interior



Figure 6.7: Chess block in Dortmund: Contemporary Real Estate (2025)

daylit windows. Now, blocks that achieve the latter are simple to identify. Moreover, they are *few*. And so now we have almost solved the problem of finding maximum daylit windows.

Be. 96: Adjacencies

Each interior daylit window is a combination of both: a yard interior face and an adjacent house interior face (window). But the number of adjacencies is limited. There are only ε adjacencies to the block, and it is straightforward to show that

$$\varepsilon = m(n - 1) + n(m - 1) \quad (\text{number of adjacencies}) \quad (6.29)$$

So there can be no more adjacencies than those, and so

$$\Lambda_i(\mathbf{x}) \leq \varepsilon \quad (6.30)$$

Now let us consider one specific block, the *chess block* (shown in Fig. (1)). The chess block actually turns every adjacency between two neighboring lots into a daylit window. So for the chess block, now denoted \mathbf{c} , interior daylit windows are maximum.

$$\Lambda_i(\mathbf{c}) = \varepsilon \quad (6.31)$$

The chess block maximizes interior daylit windows, as required in Be. (95). Is

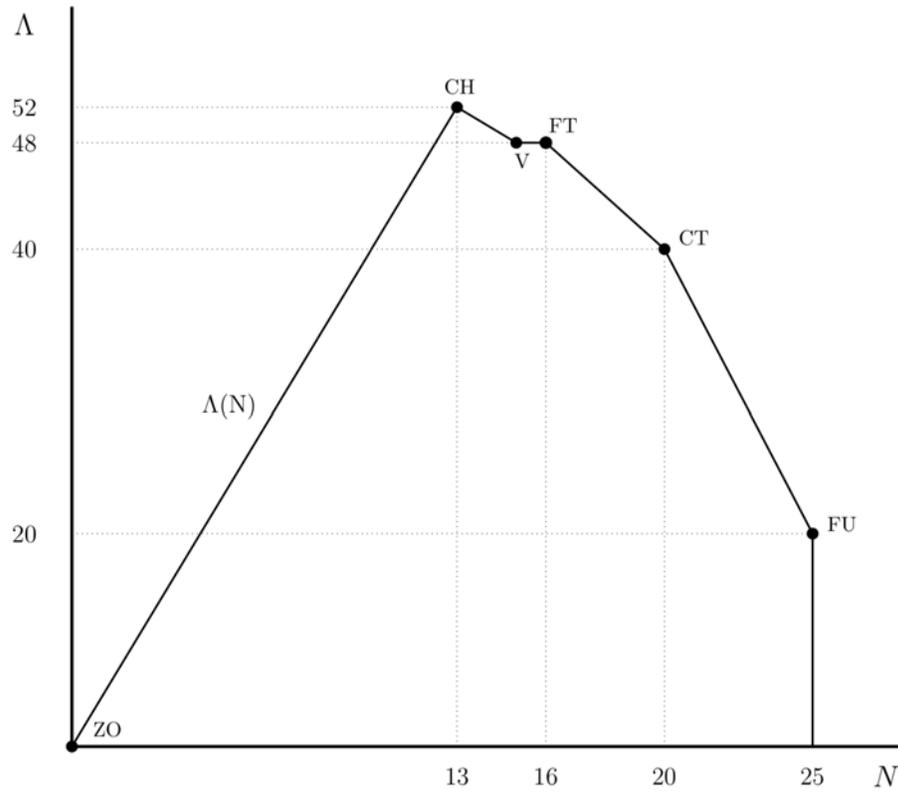


Figure 6.8: The 5×5 -block's daylight frontier $\Lambda(N)$

it the only one that does so? No. There is one other block that also maximizes interior daylight windows. This is the complement of the chess block. That block is shown in Fig. (6.3). That block simply reverses the roles of houses and yards.

Be. 97: Maximum Daylit Windows

Let us summarize: We have two blocks that maximize interior daylight windows (Be. (96)). And we know that a daylight windows maximizing block needs to maximize interior daylight windows (Be. (95)). So either the chess block or its complement are daylight windows maximizers. Of those two, we simply pick the block that has more daylight windows, and that is the chess block. (It has one house, and four daylight windows, more.) Thus,

$$\Lambda(c) \geq \Lambda(x) \text{ for all } x \neq c. \quad (6.32)$$

This is a fundamental result. The chess block maximizes daylight windows on the

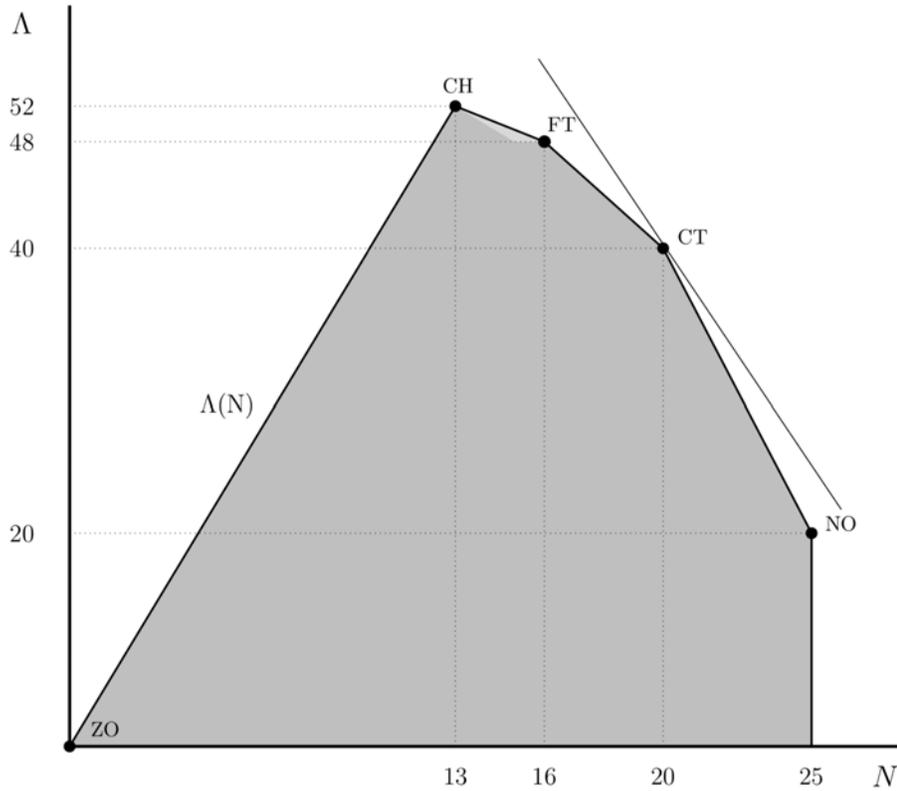


Figure 6.9: Daylight frontier, profit contours, and optimal block design

block. There is no design or configuration that yields more daylight. This may help its popularity in modern real estate.

Be. 98: Daylight Frontier

When we successively add houses to, or subtract houses from, we will no longer be able to attain maximum daylight windows. But we still want to attain maximum daylight windows for that larger or smaller number of houses. As we graph the maximum number of daylight windows against the number of houses, we obtain the *daylight frontier* (Fig. (6.8)). The term “frontier” indicates that we achieve the maximum possible, i.e. the “frontier”, of daylight at given number of houses N . Formally,

$$\Lambda(N) = \max_{\mathbf{x}} \Lambda(\mathbf{x}) \quad \text{s.t.} \quad \iota' \mathbf{x} = N \quad (6.33)$$

The graph in Fig. (6.8) traces the frontier based on a simple principle. As we

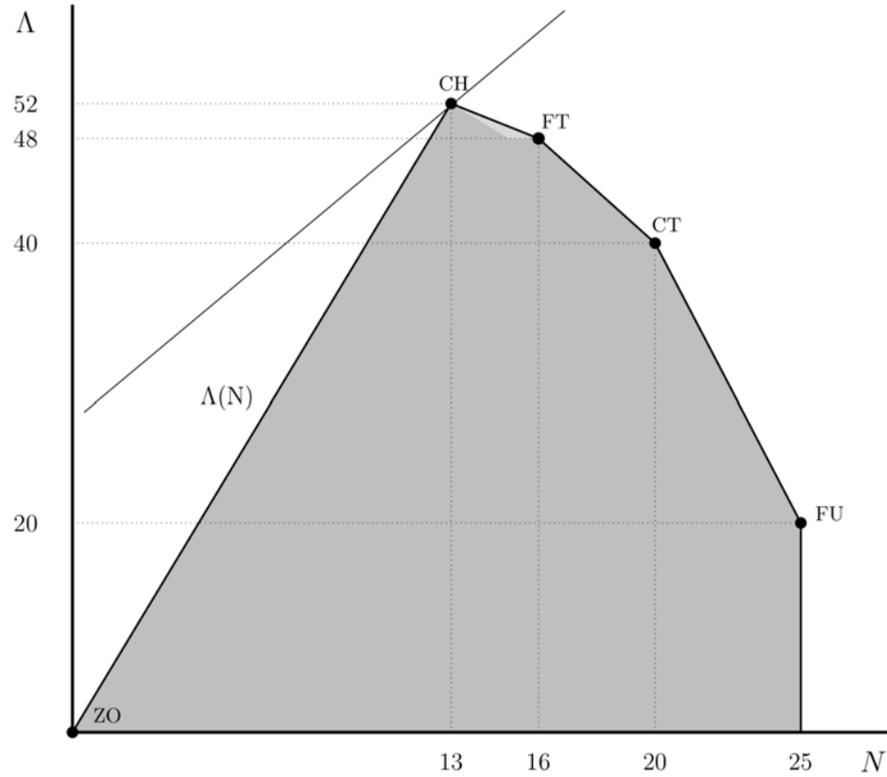


Figure 6.10: Daylight frontier, profit contours, and optimal block design

increase houses by 1, we always try to add as many daylit windows as possible; while we decrease houses by 1, we always try to avoid reducing daylit windows as much as possible. This (essentially) yields the graph of the frontier. The top of the frontier is the chess block, CH in short. There are a number of vertices. These are ZO, which stands for the zero block (with no houses at all), and FU, which stands for the full block (with only houses on the block and no yards). Further, there is the vertex for the court block CT and the vertex of the front block FT. The blocks that underly those vertices are shown in Figs. (1) and (1), respectively.

Be. 99: Profit Maximization

Block developer profit Π is a linear objective,

$$\Pi = Np + \Lambda v \quad (\text{block profit}) \quad (6.34)$$

So contour sets (N, Λ) -space are straight lines, too. Thus no point below straight line CH-V can maximize profit. Nothing is lost if we “convexify” the menu by adding the area of the lightly shaded triangle to it. In between CH and FT, the frontier thus “concavified” slope becomes $-4/3$. To find a point on the frontier that yields maximum profit, the block developer solves

$$\max_{N, \Lambda} Np + \Lambda v \quad \text{s.t.} \quad \Lambda = \Lambda(N) \quad (\text{maximum profit}) \quad (6.35)$$

In (N, Λ) -space the contours of profit, $Np + \Lambda v$, have slope $-p/v$. Fig. (6.9) adds an isoprofit contour (also drawn continuously for convenience) with slope between -4 and -2 and passing through vertex CT. That contour selects vertex CT, and hence the court block design. Alternatively Fig. (6.10) illustrates a profit contour that represents negative real rent. That contour selects vertex CH, and so here it is the chess block that maximizes profit. Etc. More generally, as real rent p/v falls, the block developer successively *pivots* across optimal designs FU, CT, FT, CH and, ultimately, ZO. Pivoting creates sudden, large swings in design in the vicinity of the four rental thresholds.

6.5 Block vs. Lot Developers

Be. 100: Urban form

Now we can analyze urban form for both lot and block developers. First lot developers: As we move further out from the city center, dark rent p falls. Hence real dark rent p/v falls, too. But then occlusions threshold ϕ^{**} falls, and houses need to be brighter and brighter. A pattern of the type shown in Fig. (6.11) emerges. The same in fact now is turn with block developers. As we move further out from the city center, again p/v falls. Isoprofit lines (which are negative in the beginning) ultimately turn positive, and block developers pivot away from dark designs (FU and CT) to ever brighter designs (such as FT and even CH). Again houses become brighter and brighter. So with both lot and block developers there is a “brightness gradient”. Only, the city gets brighter *fast* with block developers. Consider the case where p is zero, for instance. Block developers will want to build the very bright chess block CH, while lot developers still produce the full block FU.

Be. 101: Consolidation: Economies of scope

No greater profit is made than with the block developer. In short,

$$\Pi(\mathbf{x}) \geq \sum_{ij} \pi(\mathbf{x}) \quad \text{for all } \mathbf{x} \quad (6.36)$$

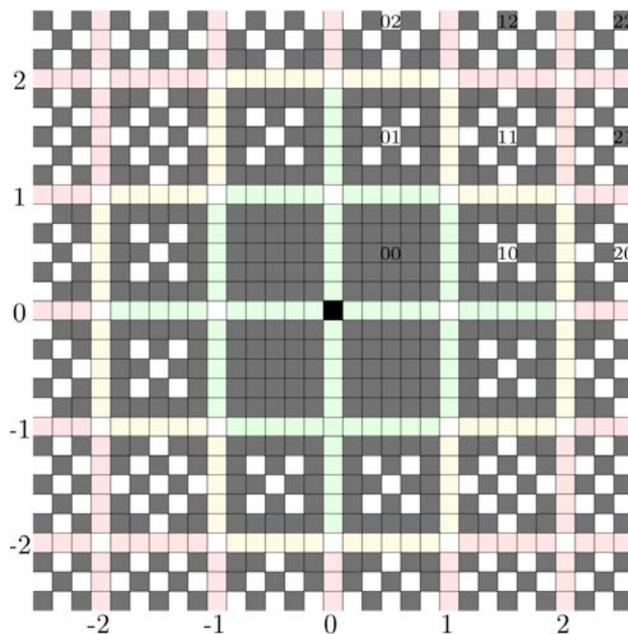


Figure 6.11: Developer city (ignore the heatmap)

This follows from the definition of maximization: “The maximum of a sum is never worse than the sum of all the individual maxima”. This suggests that lot developers even have an *incentive* to amalgamate, to merge into one single big block developer. They jointly receive a greater profit. Suitably dividing up that pie always yields as much as when going it alone. Let \mathbf{s} denote the list of payments paid out to lot developers when consolidating into a block developer,

$$\mathbf{s} = (s_{11}, \dots, s_{mn}) \quad \text{with} \quad \iota' \mathbf{s} = \Pi(\mathbf{c}) \quad (6.37)$$

Then because of inequality (6.36), we have

$$\begin{aligned} 0 &\leq \Pi(\mathbf{c}) - \sum_{ij} \pi_{ij} \\ &= \iota' \mathbf{s} - \iota' \boldsymbol{\pi} = (s_{11} - \pi_{11}) + \dots + (s_{mn} - \pi_{mn}) \end{aligned} \quad (6.38)$$

So it is possible to offer each lot developer a positive amount $s_{ij} - \pi_{ij}$ above and beyond of what he makes when developing his lot on his own, rather than developing it jointly. This is an example of *economics of scope* in real estate development.

Be. 102: Regional Economics and Economics of Federalism

Much of what we discussed here can be reinterpreted from a regional economics perspective. Lots in an urban block are regions in a country. Negative occlusion spillovers (shadows) are environmental inter-regional externalities (such as polluting rivers upstream to the detriment of the downstream region, building high smokestacks to make the wind carry pollution elsewhere, constructing nuclear power plants on a state border, lowering social welfare payments so that potential recipients migrate into neighboring regions, prohibiting abortion access with similar effect, etc.). Positive daylighting spillovers (daylightings) become positive inter-regional externalities (raising welfare payments, thereby attracting neighboring states' poor, replacing nuclear power plants by green energy, building a university with access to neighboring states' students, etc.) Moreover, the incentives are similar. Lot developers have an incentive to ignore the full societal cost of building up; they ignore the daylight windows lost on neighboring lots and thus overbuild. State policy makers have an incentive to ignore the full societal cost of the negative externalities they produce, and hence overproduce them. Where amalgamation of lot developers into a single block developer raises profits, centralizing certain decisions at the central, federal level (taking them away from the state) level is welfare improving.

Exercises

6.1 (**Daylit Windows**) The following Fig. (6.5) shows 16 5×5 blocks altogether. In each block, houses are given in black, yards in grey.

- (a) Identify the blocks for which you see a pattern. Group them into the group "patterned blocks".
- (b) Group the rest into the group "non-patterned blocks".
- (c) **For every block in either group, now sketch a 3×5 (!) block.** This is straightforward for the group of patterned blocks; for the other group try something similar. **Then assign the number of occlusions (overshadowed) windows to each lot in your block.**
- (d) Discuss, jointly, properties of those blocks. (This is not just a leisurely exercise. It is important to have some preference ranking oneself before you come into class on Tuesday.) Here are some examples for potential criteria:
 - would you like to live there yourself?
 - will it be cheap to live there?
 - is the block sustainable?
 - would you later want to send your children out to play?
 - could this be a profit maximizing block for a developer?

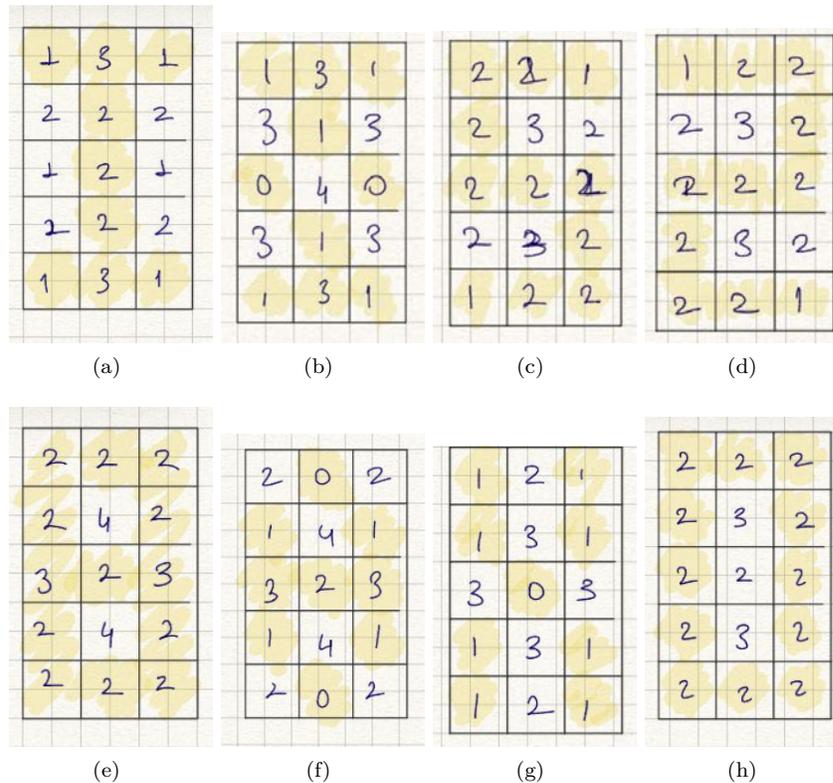


Figure 6.12: Various configurations on the 5×3 block

- have you seen something like this being built, anywhere?
 - what is your favored block?
 - how many blocks are there?
 - ...
- (e) (Please each of you sketch two or three of those blocks on the blackboard in the Uebung on Monday, January 12th.)
- (f) Do not forget to take a picture of every block drawn once you're done (so it can be projected onto the screen on Tuesday.)
- (g) Explain the relationship to real estate development.

6.2 (Visual Block Analysis I) We want to get to know a few important blocks. Consider the rows configuration, a perimeter block configuration, and the chess block configuration for the 5×5 block from Fig. (6.5).

- (a) Write down the configuration vector \mathbf{x} .

- (b) Compute the total numbers of houses $l'\mathbf{x}$, windows $4l'\mathbf{x}$, occluded windows $\mathbf{x}'\mathbf{A}\mathbf{x}$, and daylit windows Λ .
- (c) Check all three configurations for whether they are separable (i.e. could be a Nash-equilibrium). If so, what is the number it is separable by?
- (d) If separable, for which prices p and v could the block be a Nash-Equilibrium?
- (e) Rank all blocks by criterion Λ . Which block wins?

6.3 (**Visual Block Analysis II**) We want to get to know a few important blocks. First write down the adjacency matrix for the 5×3 -block.

- (a) What is the adjacency matrix \mathbf{D} for the 5×3 -block?
- (b) Draw the three blocks mentioned in the previous question (chess block, rows block, perimeter block). (You might have already done this in last week's tutorial.)
- (c) Give the configuration vector \mathbf{x} .
- (d) Compute the total numbers of houses $l'\mathbf{x}$, windows $4l'\mathbf{x}$, occluded windows $\mathbf{x}'\mathbf{A}\mathbf{x}$, and daylit windows Λ .
- (e) Check all three configurations for whether they are separable (i.e. could be a Nash-equilibrium). If so, what is the number it is separable by?
- (f) Analyze all other blocks in Fig. (6.12).
- (g) Rank all blocks by criterion Λ . Which block wins? Do you see a pattern?

6.4 (**Separability for the Block Developer**) We saw that Nash-equilibrium blocks are separable. Briefly show block developer-built blocks are separable, too.

6.5 (**Regional Economics**) If the block's lots are regions, and the block is a country, explain the two different institutional setups:

- (a) many lot developers
- (b) a single block developer

