Quantum Fields and Cosmology: Local Thermal Equilibrium

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30 Sep 2010
Local Thermal Equilibrium on Cosmological Spacetimes

This talk reports on joint work with students of my group:

- Alexander Knospe
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Quantum Fields and Cosmology

General setting – QFT in curved spacetimes at two levels:

- (Local covariant) QFT on adynamical (kinematical) background:

  \[ M \rightarrow \phi_M \]

  assigns to each globally hyperbolic spacetime \( M \) a QFT on that spacetime, subject to condition of *local covariance*

  None of the \( M \) is influenced by the dynamics of \( \phi_M \) – “no backreaction”

- Semiclassical Einstein equations – dynamical backreaction of QFT on spacetime geometry:

  \[
  (\star) \quad G_{M\mu\nu}(x) = 8\pi \langle T_{\mu\nu}^{[\phi_M]}(x) \rangle_{\omega_M}
  \]

  For a given local covariant QFT, a solution to \((\star)\) is a spacetime \( M = (M, g_{\mu\nu}) \) and a state \( \langle . \rangle_{\omega_M} \) such that \((\star)\) is fulfilled.
Quantum Fields and Cosmology

General setting – \( \langle T_{\mu\nu}^{[\phi^M]}(x) \rangle_{\omega_M} \) (1)

We specialize in the following to a situation relevant for cosmology, with a simple QFT model (that we can handle):

- spacetime is flat FRW: \( M = \mathbb{R} \times \mathbb{R}^3 \),

\[
d s_g^2 = d t^2 - a(t)^2 (d x_1^2 + d x_2^2 + d x_3^2)
\]

- conformally coupled Klein-Gordon field:

\[
(\Box_g + \frac{1}{6} R + m^2) \phi_M = 0
\]

+ CCR quantization

- The expectation value \( \langle T_{\mu\nu}^{[\phi^M]}(x) \rangle_{\omega_M} \) is a renormalized quantity and therefore its value is not uniquely determined intrinsically within the theory.
Quantum Fields and Cosmology

General setting – \( \langle T^{\phi M}_{\mu \nu}(x) \rangle_{\omega_M} (2) \)

- For a large class of physically relevant states (Hadamard states), \( \langle T^{\phi M}_{\mu \nu}(x) \rangle_{\omega_M} \) can be defined by point-slitting regularization and Hadamard parametrix subtraction renormalization. Together with Wald’s axioms, this determines the renormalized expectation value of the energy-momentum tensor up to some state-independent metric-dependent terms:

\[
\langle T^{\phi M}_{\mu \nu}(x) \rangle_{\omega_M,[2]} = \langle T^{\phi M}_{\mu \nu}(x) \rangle_{\omega_M,[1]} + \tilde{C}_{[1][2]_{\mu \nu}}(x)
\]

- \( \tilde{C}_{[1][2]_{\mu \nu}} \) is a linear combination of 4 terms built out of \( g_{\mu \nu} \). A freedom of 4 constants remains.

- Note that the appearance of these terms can be interpreted as **dark energy** coming from renormalization ambiguity in definition of the renormalized energy-momentum tensor (lack of intrinsic concept of state of “zero energy”).
Quantum Fields and Cosmology

General setting – \( \langle T_{\mu
u}^{[\phi M]}(x) \rangle \omega_M \) (3)

- Dappiaggi, Fredenhagen and Pinamonti have shown (2008): One can choose the renormalization constants such that for \( m = 0 \) there is a solution to the semiclassical Friedmann equations with an asymptotic inflationary phase at large \( t \); remaining renormalization freedom determines expansion rate (“dark energy”)

- Pinamonti (2010) proves a local existence theorem for solutions of the semiclassical Friedmann equations for the \( m \neq 0 \) case.

- Dappiaggi, Hack and Pinamonti (2010) show that “dark energy effects” in observational data can be accounted for by the model.


- $\phi$ a quantum field on Minkowski-spacetime

- $\langle . \rangle_{\beta,e_0}$ thermal equilibrium state (KMS-state) w.r.t. a Lorentz frame

- Certain observables $s(x)$ composed out of $\phi$ are sensitive to thermal behaviour at spacetime point $x$, example:

  - $\phi =$ massless Klein-Gordon field, $s(x) =: \phi^2 : (x)$, then:

\[
\langle s(x) \rangle_{\beta,e_0} = \langle : \phi^2 : (x) \rangle_{\beta,e_0} = \frac{1}{12\beta^2} = \frac{k_B^2 T^2}{12}
\]
Candidates for $s(x)$ are balanced derivatives of Wick-squares:

$$\bar{\partial}_{\mu_1, \ldots, \mu_n} \cdot \phi^2 \cdot (x)$$

$$= \lim_{\zeta \to 0} \partial_{\zeta_{\mu_1}, \ldots, \zeta_{\mu_n}} \left( \phi(x + \zeta)\phi(x - \zeta) - \langle \phi(x + \zeta)\phi(x - \zeta) \rangle_{\text{vac}} \right)$$

For a linear quantum field $\phi$ (Klein-Gordon, Dirac, Maxwell,...), define:

$$S^{(n)}(x) = \text{space spanned by all balanced derivatives of Wick-square up to order } n,$$

$$S^{(\infty)} = \bigcup_{n \in \mathbb{N}} S^{(n)}(x)$$
Definition

Let $\phi$ be a (linear) quantum field in Minkowski spacetime. A state $\langle \cdot \rangle_\omega$ is an $[x, \beta, e_0]$ LTE state of degree $n$ if

$$\langle s(x) \rangle_\omega = \langle s(x) \rangle_{\beta, e_0} \quad \forall \ s(x) \in S^{(n)}(x)$$

* $G =$ spacetime region,
* $\beta : x \mapsto \beta(x)$,
* $e_0 : x \mapsto e_0(x)$,

a state $\langle \cdot \rangle_\omega$ is an $[G, \beta, e_0]$ LTE state of degree $n$ if

$$\langle s(x) \rangle_\omega = \langle s(x) \rangle_{\beta(x), e_0(x)} \quad \forall \ s(x) \in S^{(n)}(x), \ x \in G$$
Local Thermal Equilibrium in QFT (flat spacetime)

Remarks (apply for massless linear scalar field):

- $S^{(\infty)}(x)$ is dense in the space of all thermal observables at $x$
- LTE states of $\infty$ order with non-constant temperature can only exist on spacetime regions which are intersections of lightlike half-spaces

Example: The hot bang state

$$\langle \phi(x)\phi(y) \rangle_{hb} = \int \frac{dp}{(2\pi)^3} \frac{e^{-i(x-y)p}e(p_0)\delta(p^2)}{1-e^{-\gamma(x-y)p}} \beta(x) = [4\gamma^2x^2]^{1/2}$$
For a KMS-state $\langle . \rangle_{\beta,e_0}$, let

$$\varphi(\tau) = \varphi_x(\tau) = \langle \phi(x - \frac{1}{2} \tau e_0) \phi(x + \frac{1}{2} \tau e_0) \rangle_{\beta,e_0}$$

By the KMS-condition: There is a function

$$f = f_x : S_\beta = \{ \tau + iq : \tau \in \mathbb{R}, \ 0 < q < \beta \} \rightarrow \mathbb{C}$$

which is analytic on the open strip, defined and continuous on the closed strip except at the boundary points with $\tau = 0$, such that

$$\lim_{q \rightarrow 0} (\varphi(\tau) - f(\tau + iq)) = 0 \quad \text{and} \quad \lim_{q' \rightarrow \beta} (\varphi(-\tau) - f(\tau + iq')) = 0$$
Local Thermal Equilibrium in QFT (flat spacetime)

Alternative Characterization (N. Pinamonti, RV, in preparation)

Let $\langle . \rangle_\omega$ be a (sufficiently regular) state for the quantum field $\phi$, and let

$$\psi(\tau) = \psi_x(\tau) = \langle \phi(x - \frac{1}{2}\tau e_0) \phi(x + \frac{1}{2}\tau e_0) \rangle_\omega.$$  

$\langle . \rangle_\omega$ is an $[x, \beta, e_0]$ LTE state of degree $n$ iff there is a function

$$f = f_x : S_\beta = \{ \tau + iq : \tau \in \mathbb{R}, \ 0 < q < \beta \} \rightarrow \mathbb{C}$$

which is analytic on the open strip, defined and continuous on the closed strip except at the boundary points with $\tau = 0$, such that

$$\lim_{\tau \to 0} \partial^m_{\tau} \lim_{q \to 0} (\psi(\tau) - f(\tau + iq)) = 0 \text{ and }$$

$$\lim_{\tau \to 0} \partial^m_{\tau} \lim_{q' \to \beta} (\psi(-\tau) - f(\tau + iq')) = 0 \ (m \leq n)$$
Local Thermal Equilibrium on Curved Spacetime
Some Basic Considerations

Let \((M, g_{\mu\nu})\) be a curved Lorentzian spacetime, causally very regular, i.e. orientable, time-orientable, globally hyperbolic.

Let \(\phi\) be a (linear) quantum field on \((M, g_{\mu\nu})\) regular, e.g. compatible with local general covariance.

To generalize the concept of LTE states in this situation, observe:

- \((M, g_{\mu\nu})\) in general has no time symmetries, hence: there are no KMS states of \(\phi\) for RHS of LTE condition.
- What to choose for the spaces \(S^{(n)}(x)\)?
Some Basic Considerations

- $\phi$ be compatible with local general covariance implies:
  - there is a Minkowski space variant $\phi_0$ of $\phi$
- Balanced derivatives of the Wick-square $\delta_{\mu_1,\ldots,\mu_n} : \phi^2$ can also be defined for quantum fields on curved spacetime using point-splitting regularization and Hadamard parametrix subtraction renormalization

- **Attempt of LTE-definition:**
  Let $e$ be a future-pointing timelike vector at $x \in M$.
  A state $\langle . \rangle_\omega$ of $\phi$ is an $[x, \beta, e]$ LTE state of degree $n$ if:
  - there is an $[x_0, \beta, e_0]$ LTE state $\langle . \rangle_{\beta,e_0}$ of $\phi_0$ of degree $n$ such that
  $$\langle s(x) \rangle_\omega = \langle s_0(x_0) \rangle_{\beta,e_0}$$
  for $s(x) = \text{balanced derivatives of Wick-squares up to order } n$. 

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Note:

- $(x, e)$ and $(x_0, e_0)$ are identified by geodesic normal coordinates.
- The balanced derivatives $s(x)$ and $s_0(x)$ actually carry tensor indices; identification of tensors again by geodesic normal coordinates.
- Point-slitting regularization and Hadamard parametrix subtraction renormalization provides a local general covariant definition of $\phi^2 : (x)$ and of balanced derivatives $\delta_{\mu_1,\ldots,\mu_n} : \phi^2 : (x)$. 
Local Thermal Equilibrium on Curved Spacetime

Some Basic Considerations

‘But’:
The definition of $\phi^2 : (x)$ and $\bar{\partial}_{\mu_1,\ldots,\mu_n} : \phi^2 : (x)$ is not unique: There are renormalization ambiguities

$$(\bar{\partial}_{\mu_1,\ldots,\mu_n} : \phi^2 : (x))[2] = (\bar{\partial}_{\mu_1,\ldots,\mu_n} : \phi^2 : (x))[1] + \xi^{(n)}_{[1][2]_{\mu_1,\ldots,\mu_n}(x)}$$

A second ‘But’:
The process of point-splitting renormalization and Hadamard parametrix subtraction renormalization can lead to anomalies, i.e. in

$$\bar{\partial}_{\mu_1,\ldots,\mu_n} : \phi^2 : (x)$$

and quantities derived thereof (derivatives, traces) there may appear curvature-dependent (and state-independent) terms which vanish in the case of flat spacetime (and hence, for $\bar{\partial}_{\mu_1,\ldots,\mu_n} : \phi_0^2 : (x)$ and related derived quantities).
Consequently:

Must try to use the renormalization ambiguity in order to cancel the anomaly-creating terms at least in such a way as to set up an LTE condition which is formally consistent

Formally Consistent LTE-State Definition:

Given a choice of $\bar{\phi}_{\mu_1,\ldots,\mu_m} : \phi^2 : , \ m \leq n$, $\langle . \rangle_\omega$ is $[x, \beta, e]$ LTE of order $n$ if

$$\langle \bar{\phi}_{\mu_1,\ldots,\mu_m} : \phi^2 : (x) \rangle_\omega + \xi^{(m)}_{\mu_1,\ldots,\mu_m}(x) = \langle \bar{\phi}_{\mu_1,\ldots,\mu_m} : \phi_0^2 : (x_0) \rangle_{\beta,e_0}$$

for $m \leq n$ with suitable metric-built tensors $\xi^{(m)}_{\mu_1,\ldots,\mu_m}$ such that the condition is formally consistent (e.g. for $m = 2$: if RHS is trace-free, then this holds also for LHS)
Local Thermal Equilibrium on Curved Spacetime

FAQs

- gravity / curved spacetime QFT
- Hadamard parametrix renormalization mutually consistent?
- LTE condition

Three levels to look at this question:

(I) Can a formally consistent LTE condition be set up upon suitable choice of $\xi^{(m)}_{\mu_1,\ldots,\mu_m}$?

(II) Do LTE states exist; on which spacetimes/spacetime regions?

(III) Are there solutions to the semiclassical Einstein equations,

$$G_{\mu\nu}(x) = 8\pi \langle T^{[\phi]}_{\mu\nu}(x) \rangle_{\beta(x),\epsilon(x)}$$

with LTE states of a quantum field on the right hand side?
Local Thermal Equilibrium on Curved Spacetime

Results:

- A formally consistent LTE condition can be given for:
  - linear scalar field up to order $n \leq 2$
  - linear Dirac field up to order $n \leq 1$
  - free Maxwell field up to order $n = 0$ in the field strength on generic curved spacetimes.

- In general, it is not expected that states fulfilling the exact LTE condition exist – in the above cases – on open regions in arbitrary curved spacetimes.

  It is likely that there exist states fulfilling the LTE condition approximately.

  In what follows, we present some partial results on this issue.
Local Thermal Equilibrium on Curved Spacetime

On Existence of LTE states in FRW spacetimes

Model:

- Spatially flat FRW spacetime with arbitrary $C^\infty$ scale-factor $a(t)$,
  metric: $ds^2 = dt^2 - a(t)^2 dx^2$
- Quantized neutral linear Klein-Gordon field with mass $m$ and curvature coupling $\xi$,

$$(\Box + \xi R + m^2)\phi = 0$$

Theorem (Jan Schlemmer)

For $t_*$ given, there is some (sufficiently small) $\beta_*$ such that there exist Hadamard states $\langle . \rangle_{[t_*;\beta]}$ which fulfill the $[x, \beta, \partial_t]$ LTE condition up to order 2,

for all $x = (t_*, x), \beta \leq \beta_*$. 

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This means: There are Hadamard states fulfilling the LTE condition up to second order for high enough temperatures at all spacetime points of constant time $t = t_\ast$.

The following numerical example for $a(t) \sim e^{H_0 t}$ shows that an LTE state at some $t = t_\ast$ will not maintain the exact LTE condition for all other times:

\[
\theta = \langle : \phi^2 : \rangle[t_\ast, \beta]
\]

\[
\varepsilon_{\mu\nu} = -\frac{1}{4} \langle \tilde{\partial}_{\mu\nu} : \phi^2 : \rangle[t_\ast, \beta]
\]
Local Thermal Equilibrium on Curved Spacetime

On Existence of LTE states in FRW spacetimes: Scattering Situation

FRW-metric in conformally flat coordinates:

$$ds^2 = \Omega(t)^2(dt^2 - dx^2), \quad \Omega^2(t) = \lambda + \frac{\Delta \lambda}{2}(1 + \tanh(\varrho t))$$

Conformally coupled massive linear scalar field:

$$(\Box + \frac{1}{6}R + m)\phi = 0$$

Parameters: $m, \lambda, \Delta \lambda, \varrho$

$$A + B = \lambda + \Delta \lambda$$

$$A - B = \lambda$$
This spacetime has asymptotically flat regions for $t \to \pm \infty$; $\phi_{\text{in}}$ and $\phi_{\text{out}}$ are the asymptotic flat space “limit QFTs” of $\phi$:

$$
\begin{align*}
\phi_{\text{in}} \quad &\quad \leftrightarrow \quad \phi \quad \longrightarrow \quad \phi_{\text{out}} \\
\quad t \to -\infty \quad &\quad \quad \quad \quad t \to \infty
\end{align*}
$$

The asymptotic fields are related by a Bogoliubov transformation,

$$
\phi_{\text{out}} = Bog(\phi_{\text{in}})
$$

which can be exactly calculated in this case.

Take a state $\langle . \rangle_{\beta_{\text{in}}}$ which is a KMS state at inverse temperature $\beta_{\text{in}}$ on the $\phi_{\text{in}}$ fields.

Is this state an LTE-state on the $\phi_{\text{out}}$ fields?
To check on the LTE-property of $\langle . \rangle_{\beta_{\text{in}}}$, define

- $\vartheta(\lambda, \Delta \lambda, m, \varrho, \beta_{\text{in}}) = \langle : \phi_{\text{out}}^2 : \rangle_{\beta_{\text{in}}}$
- $\varepsilon_{\mu \nu}(\lambda, \Delta \lambda, m, \varrho, \beta_{\text{in}}) = -\frac{1}{4} \langle \delta_{\mu \nu} : \phi_{\text{out}}^2 : \rangle_{\beta_{\text{in}}}$

If there are KMS states $\langle . \rangle_{\beta_{\vartheta}}$ and $\langle . \rangle_{\beta_{\varepsilon_{\mu \nu}}}$ for $\phi_{\text{out}}$ so that

- $\vartheta(\lambda, \Delta \lambda, m, \varrho, \beta_{\text{in}}) = \langle : \phi_{\text{out}}^2 : \rangle_{\beta_{\vartheta}}$
- $\varepsilon_{\mu \nu}(\lambda, \Delta \lambda, m, \varrho, \beta_{\text{in}}) = -\frac{1}{4} \langle \delta_{\mu \nu} : \phi_{\text{out}}^2 : \rangle_{\beta_{\varepsilon_{\mu \nu}}}$

then $\langle . \rangle_{\beta_{\text{in}}}$ is LTE on $\phi_{\text{out}}$ iff $\beta_{\vartheta} = \beta_{\varepsilon_{\mu \nu}}$

Relations $\#$ and $\#\#$ are inverted numerically to obtain $\beta_{\vartheta}, \beta_{\varepsilon_{\mu \nu}}$ as functions of $\lambda, \Delta \lambda, m, \varrho, \beta_{\text{in}}$
Typical behaviour of $\beta_s(\beta_{\text{in}})$ for small $\varrho$:
a line with slope 1 going to a constant
Local Thermal Equilibrium on Curved Spacetime
On Existence of LTE states in FRW spacetimes: Scattering Situation
Numerical Results by Falk Lindner

Left: The function $\beta_\vartheta$ over $\beta_\text{in}$: there are turning points $\beta_c$.
Right: The value of $\beta_c$ over the mass parameter $m$
Local Thermal Equilibrium on Curved Spacetime
On Existence of LTE states in FRW spacetimes: Scattering Situation
Numerical Results by Falk Lindner

Left: The function $\beta_\vartheta$ over the mass $m$: $\beta_\vartheta \rightarrow \beta_{in}$ for $m \rightarrow 0$, is consistent with analytic calculation.

Right: same result for $\varrho = 1$. 

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On Existence of LTE states in FRW spacetimes: Scattering Situation
Numerical Results by Falk Lindner

Bad behavior of 2-point functions $\vartheta$: If $\beta_{\text{in}}$ gets big enough, the positivity of $\vartheta$ cannot be granted, independent of the choice of the other parameters.
Local Thermal Equilibrium on Curved Spacetime

On Existence of LTE states in FRW spacetimes: Scattering Situation

Numerical Results by Falk Lindner

Left: Deviations in a graph of small $\varrho$ with turning point.

Right: Deviations in the oscillating case for larger $\varrho$.

$\hat{\beta} = 1/3(\beta_\varphi + \beta_{\varepsilon_{00}} + \beta_{\varepsilon_{xx}})$ is a reference function.
The semiclassical Einstein equation is

\[ G_{\mu\nu}(x) = 8\pi \left\langle T^{[\phi]}_{\mu\nu}(x) \right\rangle_{\omega} \quad (x \in M) \]

- \( G_{\mu\nu} \) = Einstein tensor of a spacetime \((M, g_{\mu\nu})\)
- \( T^{[\phi]}_{\mu\nu} \) = Stress-energy-tensor of a quantum field \(\phi\) on \((M, g_{\mu\nu})\)
- \( \left\langle . \right\rangle_{\omega} \) = a state of the quantum field (suitably regular)

Our programme:

Assume that semiclassical Einstein equation has solutions of a special form,

then analyze the consequences
We assume that there are solutions of the semiclassical Einstein equations of the following form:

- \((M, g_{\mu\nu})\) is a FRW cosmological spacetime
- \(\phi\) is a linear quantum field
- \(T_{\mu\nu}^{[\phi]}\) is obtained from \(\phi\) by point-splitting regularization and Hadamard parametrix subtraction renormalization, plus corrections to reach consistency with Wald’s axioms
- \(\langle . \rangle_\omega\) is a homogeneous, isotropic LTE state
Wald’s axioms require that $T^{[\phi]}_{\mu\nu}$ is local covariant, divergence free, contains no derivatives of $g_{\mu\nu}$ higher than order 2.

As with the Wick-square and its balanced derivatives, the $T^{[\phi]}_{\mu\nu}$ obtained by point-splitting regularization and Hadamard parametrix subtraction renormalization is not unique, but subject to renormalization ambiguities and anomalies.

$$T^{[\phi]}_{\mu\nu}(x) = T^{(SHP)}_{\mu\nu}(x) - g_{\mu\nu} Q(x) + C_{\mu\nu}(x)$$

$$\left( T^{[\phi]}_{\mu\nu}(x) \right)_{[2]} = \left( T^{[\phi]}_{\mu\nu}(x) \right)_{[1]} + \tilde{C}_{[1][2]}_{\mu\nu}(x)$$
Local Thermal Equilibrium and Gravity

LTE and Semiclassical Einstein Equation

**LTE-state stress-energy-tensor**

(i) build $\langle T_{\mu\nu}^{(SHP)} \rangle_{\beta,e}$ from linear combinations of $\langle \partial_{\ldots} : \phi^2 : \rangle_{\beta,e}$, their derivatives and curvature terms:

$$\langle T_{\mu\nu}^{(SHP)} \rangle_{\beta,e} = F(\langle \partial_{\ldots} : \phi^2 : \rangle_{\beta,e}, \nabla, g_{\mu\nu})$$

Note:

$$\langle T_{\mu\nu}^{(SHP,\phi)}(x) \rangle_{\beta,e} = \langle T_{\mu\nu}^{(SHP,\phi_0)}(x) \rangle_{\beta,e_0} + A_{\mu\nu}[\beta,e,g_{\mu\nu}](x)$$

(ii) correct by a divergence-compensating term:

$$\langle T_{\mu\nu}^{(SHP,cor)}(x) \rangle_{\beta,e} = \langle T_{\mu\nu}^{(SHP)}(x) \rangle_{\beta,e} - g_{\mu\nu} Q(x)$$

(iii) choose — if possible — $C_{\mu\nu}$ such that the resulting $\langle T_{\mu\nu}^{[\phi]} \rangle_{\beta,e}$ fulfills Wald’s axioms — in particular, such that it contains no higher $\nabla$ orders than 2 [Dappiaggi, Fredenhagen, Pinamonti]:

$$\langle T_{\mu\nu}^{[\phi]}(x) \rangle_{\beta,e} = \langle T_{\mu\nu}^{(SHP,cor)}(x) \rangle_{\beta,e} + C_{\mu\nu}^{(min)}(x)$$
Put all this in place – with

- $\phi$ = massless Dirac field on spatially flat FRW spacetime,
  
  \[ ds^2 = dt^2 - a^2(t)dx^2 \]

- by assumption:

\[
G_{\mu\nu}(t, x) = 8\pi \left\langle T^{[\phi]}_{\mu\nu}(t, x) \right\rangle_{\beta(t), \partial_t}
\]

$C_{\mu\nu}^{(min)}$ is fixed by requirement that Wald's axioms be fulfilled

The trace $\left\langle T^{[\phi]}_{\mu\mu} \right\rangle_{\beta(t), \partial_t}$ is state-independent and non-zero (trace-anomaly)
Local Thermal Equilibrium and Gravity

LTE and Semiclassical Einstein Equation

Results by Alexander Knospe

\( a(t) \) is determined by \( \nabla^\mu \left\langle T^{[\phi]}_{\mu\nu} \right\rangle \beta(t), \partial_t = 0 \) (state-independent) together with an initial condition at \( t = t_* \).

Same analysis as in [Dappiaggi, Fredenhagen, Pinamonti PRD77 (2008)] yields differential eqn for \( H(t) = \dot{a}(t)/a(t) \):

\[
\dot{H}(t) = \frac{-H^4(t) + 2A_0^2H^2(t)}{H^2(t) - A_0^2}, \quad A_0 = \text{universal constant}
\]
Local Thermal Equilibrium and Gravity
LTE and Semiclassical Einstein Equation
Results by Alexander Knospe

Left: $t(H)$, $t$ as function of $H$ for solution of deqn for $H(t)$ (solid line) (dashed line: FRW solution for classical radiation)

Right: $\rho = \langle \phi^2 \rangle_{\beta, \partial_t}$ as function of $H$ (solid line). For large $H$, it deviates from the solution for classical radiation, indicating a much higher increase in temperature for small $t$ than predicted for classical radiation.
Conclusion

- The LTE condition in its present form is likely too idealized as a characterization for local thermal equilibrium for QFT on curved spacetime.
- There seems to be a class of states which comes very close to fulfilling the LTE condition — need a characterization of an approximate LTE condition.
- Analyzing LTE states in semiclassical gravity indicates deviations from classical treatment — in combination with implementing Wald’s axioms, it gives new types of solutions to the semiclassical Friedmann equations which promise to give new insights into inflation and dark energy.